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KEY TO

ALGEBRA MADE EASY, VOL. I.

(MATRICULATION ALGEBRA.)

BY

KALI PADA BASU, M.A.,

Late Professor of Mathematics, Dacca College.
Fellow and Examiner, Calcutta University

AND

Author of "Matriculation Geometry" "Intermediate Algebra", "Intermediate Solid Geometry"
&c. &c.

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KEY

TO

ALGEBRA MADE EASY, VOL. I.

Exercise 1

1 1 maund and 25 seers = 65 seers Hence it contains the unit of weight (1 seer) 65 times, and the required measure is **65**

2 65 seers contains the unit of weight (5 seers) 13 times, and the required measure is **13**

3 300 miles contains the unit of distance 25 times, the required unit is $\frac{300 \text{ miles}}{25}$, or **12 miles**

4 300 miles contains the unit of distance 40 times, the required unit is $\frac{300 \text{ miles}}{40}$, or **$7\frac{1}{2}$ miles**

5 Rs 400 contains the unit of money 16 times, the unit is $\frac{400 \text{ Rupees}}{16}$, or Rs 25 Rs 225 contains the unit of money (Rs 25) 9 times, and the required measure is **9**

6 8 ft 4 in = 100 inches, and as it contains the unit of length 25 times, the unit is $\frac{100 \text{ inches}}{25}$, or 4 inches 4 ft being equal *to 48 inches, it contains the unit of length (4 in) 12 times*, the required measure is **12**

7 2 hrs 15 mts, *i e* 135 mts, contains the unit of time 3 times, the required unit of time is $\frac{135 \text{ mts}}{3}$, or **45 mts**

8 The required time contains the unit of time (15 seconds) 60 times, it is (15 seconds) × 60, or **15 minutes**

9 $2\frac{1}{7}$ cwt $= \frac{15}{7}$ cwt $= (\frac{15}{7} \times 4 \times 28)$ lbs $= 240$ lbs, the number of times that $7\frac{1}{2}$ pounds is contained in 240 lbs is $\frac{240}{7\frac{1}{2}}$, or **32**

10 (*a*) 9 sq in $= \frac{9}{144}$ sq ft $= \frac{1}{16}$ sq ft, and $\frac{1}{16}$ sq ft $=$ (4 sq ft) $\times \frac{1}{64}$, ∴ the required number is $\frac{1}{64}$

(*b*) 9 sq yds $=$ 81 sq ft, and 81 sq ft $=$ (4 sq ft) $\times 20\frac{1}{4}$, ∴ the required number is $20\frac{1}{4}$

11 125 sq ft is $8\frac{1}{3}$ times the unit of area, ∴ the unit of area is (125 sq ft) $\div 8\frac{1}{3}$, or 15 sq ft. Hence 3 times the unit of area $=$ **45 sq ft**

12 The unit of money $=$ (£3 7*s* 6*d*) $\div 9 =$ **7s 6d**

13 The required measure $=$ (£7 13*s* 4*d*) $\div$ (7*s* 8*d*) $=$ (1840*d*) $\div$ (92*d*) $=$ **20**

14 The required measure $=$ (Rs 25 10*a* 3*p*) $\div$ (Rs 2 13*a* 7*p*) $=$ (4923*p*) $\div$ (547*p*) $=$ **9**

15 The required measure $=$ (16 mds $12\frac{3}{4}$ seers) $\div$ (23 seers 5 chattacks) $=$ (2611×16 ch) $\div$ (4×373 ch) $=$ **28**

16 The old unit $=$ (Rs 20 10*a*) $\div 5\frac{1}{2} =$ (Rs 41 4*a*) $\div 11 =$ Rs 3 12*a* and the new unit $=$ (Rs 3 12*a*) $\times 3 =$ Rs 11 4*a*. Hence the required measure $=$ (Rs 45) $\div$ (Rs 11 4*a*) $=$ **4**

17 The old unit $=$ (19 cwt 2 qr) $\div 273 =$ (78 qrs) $\div 273 = \frac{2}{7}$ qr, and the new unit $=$ ($\frac{2}{7}$ qr) $\times \frac{1}{16} = \frac{1}{56}$ qr. Hence the required measure $=$ (1 ton) $\div$ ($\frac{1}{56}$ qr) $=$ (80 qrs) $\div$ ($\frac{1}{56}$ qr) $=$ **4480**

18 The old unit $=$ (39 yds 2 ft) $\div 84 =$ (119 ft) $\div 84 = \frac{17}{12}$ ft, and the new unit $=$ ($\frac{17}{12}$ ft) $\times \frac{3}{17} = \frac{1}{4}$ ft. Hence the required number $=$ (75 yds) $\div \frac{1}{4}$ ft $=$ (225 ft) $\div$ ($\frac{1}{4}$ ft) $=$ **900**

19 The old unit $=$ (26 days 10 hrs 26 mts) $\div 120 =$ (634 hrs 26 mts) $\div 120 =$ 5 hrs 17 mts 13 secs, and the new unit $=$ (5 hrs 17 mts 13 secs) $-$ (47 mts 13 secs) $=$ 4 hrs 30 mts. Hence the required number $=$ (366 days) $\div$ ($4\frac{1}{2}$ hrs) $=$ (366×24 hrs) $\div$ ($4\frac{1}{2}$ hrs) $=$ ($366 \times 24 \times 2$) $\div 9 = 122 \times 8 \times 2 =$ **1952**

20 The new unit $=$ (5 hrs 17 mts 13 secs) $+$ (6 hrs 54 mts 47 secs) $=$ 12 hrs 12 mts $= 12\frac{1}{5}$ hrs. Hence the required number $=$ (366×24 hrs) $\div$ ($12\frac{1}{5}$ hrs) $=$ ($366 \times 24 \times 5$) $\div 61 = 6 \times 24 \times 5 =$ **720**

Exercise 2

1 $a + b \times c = 2 + 4 \times 8 = 2 + 32 = \mathbf{34}$

2 $c - a \times b = 8 - 2 \times 4 = 8 - 8 = \mathbf{0}$

3 $c \div b \times a = 8 \div 4 \times 2 = 2 \times 2 = \mathbf{4}$

4 $c \div ba = 8 \div (4 \times 2) = 8 \div 8 = \mathbf{1}$

5 $c-3\times a=8-3\times 2=8\times\frac{1}{3}\times 2=\frac{16}{3}=5\frac{1}{3}$

6 $c-3a=8-(3\times 2)=8\times\frac{1}{3\times 2}=\frac{4}{3}=1\frac{1}{3}$

7 $c-b-a=8-4-2=8-2=6$

8 $a+c-b=2+8-4=2+2=4$

9 $3c-4b+2a=(3\times 8)-(4\times 4)+(2\times 2)=24-16+4=12$

10 $c-b-a+c-b=8-4-2+8-4=8-2+2=8$

11 $c-b-2\times a=8-4-2\times 2=8\times\frac{1}{4}\times\frac{1}{2}\times 2=2$

12 $c-b-2a=8-4-(2\times 2)=8\times\frac{1}{4}\times\frac{1}{2\times 2}=\frac{1}{2}$

13 $5c-2b=(5\times 8)-(2\times 4)=5\times 8\times\frac{1}{2\times 4}=5$

14 $5\times c-2\times b=5\times 8-2\times 4=5\times 8\times\frac{1}{2}\times 4=80$

15 $4ab-c-4\times a+b-2a=(4\times 2\times 4)-8-4\times 2+4-(2\times 2)$
$=32-8\times\frac{1}{4}\times 2+4\times\frac{1}{2\times 2}=32-4+1=29$

16 $80-b\times ca+80-bc\times a=80-4\times(8\times 2)+80-(4\times 8)\times 2$
$=80\times\frac{1}{4}\times 8\times 2+80\times\frac{1}{4\times 8}\times 2=320+5=325$

17 $64-c\times b\times a-64-cba=64-8\times 4\times 2-64-(8\times 4\times 2)$
$=64\times\frac{1}{8}\times 4\times 2-64\times\frac{1}{8\times 4\times 2}=64-1=63$

18 $3bc-16a+5c-16\times a-c-2b\times b-a\times 4$
$=(3\times 4\times 8)-(16\times 2)+(5\times 8)-16\times 2-8-(2\times 4)\times 4-2\times 4$
$=3\times 4\times 8\times\frac{1}{16\times 2}+5\times 8\times\frac{1}{16}\times 2-8\times\frac{1}{2\times 4}\times 4\times\frac{1}{2}\times 4$
$=3+5-8=8-8=0$

19 $10c-ab\times 2+32b-2ac+15c-3b\times c-5=(10\times 8)-(2\times 4)\times 2+(32\times 4)-(2\times 2\times 8)+(15\times 8)-(3\times 4)\times 8-5=10\times 8\times\frac{1}{2\times 4}\times 2+32\times 4\times\frac{1}{2\times 2\times 8}+15\times 8\times\frac{1}{3\times 4}\times 8\times\frac{1}{5}=20+4+16=40$

20 $48c-b-a\times 6-4b-3c-2b-4\times 3-a\times 8+6a-c-2\times b-3\times 5=(48\times 8)-4-2\times 6-(4\times 4)-(3\times 8)-(2\times 4)-4\times 3-2\times 8+(6\times 2)-8-2\times 4-3\times 5=48\times 8\times\frac{1}{4}\times\frac{1}{2}\times 6\times\frac{1}{4\times 4}-3\times 8\times\frac{1}{2\times 4}\times\frac{1}{4}\times 3\times\frac{1}{2}\times 8+6\times 2\times\frac{1}{8}\times\frac{1}{2}\times 4\times\frac{1}{3}\times 5=18-9+5=14$

21 $8m-3p-mn+q\times 3r+5s-2\times p=(8\times 2)-(3\times 4)-(2\times 3)+0+(5\times 10)-2\times 4=16-12\times\frac{1}{6}+50\times\frac{1}{2}\times 4=16-2+100=114$

22 $s\times 6-5m\times 8p-16n=10\times 6-(5\times 2)\times(8\times 4)-(16\times 3)=10\times 6\times\frac{1}{5\times 2}\times 8\times 4\times\frac{1}{16\times 3}=4$

23 $24-3p\times 4s-5r\times 14m=24-(3\times 4)\times(4\times 10)-(5\times 7)\times(14\times 2)=24\times\frac{1}{3\times 4}\times 4\times 10\times\frac{1}{5\times 7}\times 14\times 2=64$

24 $mnr+5qs-3s-m-5n+4r-3p\times6m=2\times3\times7+0-(3\times10)-2-(5\times3)+(4\times7)-(3\times4)\times(6\times2)=42-3\times10\times\frac{1}{2}\times\frac{1}{5\times3}+4\times7\times\frac{1}{3\times4}\times6\times2=42-1+28=$ **69**

25 $3\times r-5\times s-7\times p-8rs-m-3\times n-7p+5m-2r\times7=3\times7-5\times10-7\times4-(8\times7\times10)-2-3\times3-(7\times4)+5\times2-(2\times7)\times7=3\times7\times\frac{1}{5}\times10\times\frac{1}{7}\times4-8\times7\times10\times\frac{1}{2}\times\frac{1}{3}\times3\times\frac{1}{7\times4}+5\times2\times\frac{1}{2\times7}\times7=24-10+5=$ **19**

Exercise 3

1 $3a^3=3\times2^3=3\times8=$ **24**

2 $7x^2-y=(7\times8^2)-12=7\times8\times8\times\frac{1}{12}=\frac{112}{3}=$ **37⅓**

3 $2a^7-7m^2=2\times2^7-7\times6^2=2\times128-7\times36=256-252=$ **4**

4 $8b^3c-ab^2x=8\times3^3\times4-2\times3^2\times8=8\times27\times4-2\times9\times8$
$=864-144=$ **720**

5 $5c^5-3x^3=(5\times4^5)-(3\times8^3)=5\times4\times4\times4\times4\times4-(3\times8\times8\times8)$
$=5\times4\times4\times4\times4\times4\times\frac{1}{3\times8\times8\times8}=\frac{10}{3}=$ **3⅓**

6 $7a^3b^2y-m^4n=(7\times2^3\times3^2\times12)-(6^4\times7)$
$=7\times2\times2\times2\times3\times3\times12\times\frac{1}{6\times6\times6\times6\times7}=$ **⅔**

7 $x^a-c^b=8^2-4^3=8\times8\times\frac{1}{4\times4\times4}=$ **1**

8 $9c^4x^3y^2-8a^8b^2m^2-by^2-a=(9\times4^4\times8^3\times12^2)-(8\times2^8\times3^2\times6^2)-(6\times12^2)-2=9\times4\times4\times4\times4\times8\times8\times8\times12\times12\times\frac{1}{8\times2\times2\times2\times2\times2\times2\times2\times2\times3\times3\times6\times6}-6\times12\times12\times\frac{1}{2}$
$=256-216=$ **40**

9 $2a^xy-x^2y^a=(2\times2^8\times12)-(8^2\times12^2)=2\times2\times2\times2\times2\times2\times2\times2\times2\times12\times\frac{1}{8\times8\times12\times12}=$ **⅔**

10 $3c^ma^y-x^nb^3=(3\times4^6\times2^{12})-(8^7\times3^3)=3\times4\times4\times4\times4\times4\times4\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$
$\times\frac{1}{8\times8\times8\times8\times8\times8\times8\times3\times3\times3}=$ **$\frac{8}{9}$**

11 The given expression

$=40+66\times8\times8-21\times8+8\times8\times8\times8\times8\times8-65\times8\times8\times8\times8$

$=40+4224-168+262144-266240=4096+262144-266240$

$=266240-266240=0$

12 The given expression

$=8\times(7\ 5)^4+6\times(7\ 5)^3+11\times(7\ 5)^2+13\times(7\ 5)+29$

$=8\times\ 7\ 5\times\ 7\ 5\times\ 7\ 5\times\ 7\ 5+6\times\ 7\ 5\times\ 7\ 5\times\ 7\ 5$

$+11\times\ 7\ 5\times\ 7\ 5+13\times\ 7\ 5+29=6\times\ 7\ 5\times\ 7\ 5\times\ 7\ 5$

$+6\times\ 7\ 5\times\ 7\ 5\times\ 7\ 5+8\ 25\times\ 7\ 5+9\ 75+29$

$=4\ 5\times\ 56\ 25+4\ 5\times\ 56\ 25+6\ 1875+38\ 75$

$=2\ 53125+2\ 53125+44\ 9375=50$

13 $m^2=\frac{16}{49}$, $m^3=\frac{64}{343}$, $m^4=\frac{256}{2401}$, $m^5=\frac{1024}{16807}$

Hence $35m^6-4m^2+7m+15m^3-34m^4-3$

$=\frac{5\times1024}{2401}-\frac{4\times16}{49}+4+\frac{15\times64}{343}-\frac{31\times256}{2401}-3$

$=\frac{5\times1024-4\times16\times49+15\times64\times7-34\times256}{2401}+4-3$

$=\frac{5120-3136+6720-8704}{2401}+1=0+1=1$

14 The given expression

$=25\times(2\ 6)^8-27+20\times2\ 6+78\times(2\ 6)^6-199\times(2\ 6)^6$

$=25\times2088\ 27064576-27+52+78\times118\ 81376$

$-199\times308\ 915776$

$=52206\ 766144-27+52+9267\ 47328-61474\ 239424$

$=52231\ 766144+9267\ 47328-61474\ 239424$

$=61499\ 239424-61474\ 239424=25$

15 The given expression

$=50\times(3\ 4)^7-51\times(3\ 4)^4+35\times(3\ 4)-563\times(3\ 4)^6-19$

$=50\times5252\ 3350144-51\times135\ 6336$

$+35\times3\ 4-563\times454\ 35424-19$

$=262616\ 75072-6815\ 3136+119-255801\ 43712-19$

$=255801\ 43712+119-255801\ 43712-19=100$

16 The given expression

$$=64\times(\tfrac{11}{8})^{10}-55\times(\tfrac{11}{8})^{4}+32\times(\tfrac{11}{8})^{6}-121\times(\tfrac{11}{8})^{8}$$
$$+64\times(\tfrac{11}{8})^{2}-4\times(\tfrac{11}{8})^{5}+79$$
$$=\frac{11^{10}}{8\times8\times8\times8\times8\times8\times8\times8}-\frac{55\times(11)^4}{8\times8\times8\times8}+\frac{(11)^6}{8\times8\times8\times8}$$
$$-\frac{121\times(11)^8}{8\times8\times8\times8\times8\times8\times8\times8}+(11)^2-\frac{(11)^5}{2\times8\times8\times8\times8}+79$$
$$=\frac{11\times11\times11\times11\times11\times11\times11\times11\times11\times11}{8\times8\times8\times8\times8\times8\times8\times8}-\frac{55\times14641}{8\times8\times8\times8}$$
$$+\frac{1771561}{2\times8\times8\times8\times8}-\frac{11\times11\times11\times11\times11\times11\times11\times11\times11\times11}{8\times8\times8\times8\times8\times8\times8\times8}$$
$$+121-\frac{161051}{2\times8\times8\times8\times8}+79$$
$$=200-\frac{805255}{8\times8\times8\times8}+\frac{1771561-161051}{2\times8\times8\times8\times8}$$
$$=200-\frac{805255}{8\times8\times8\times8}+\frac{805255}{8\times8\times8\times8}=200$$

17 $a^3+b^3+c^3-3abc=(24)^3+(27)^3+(29)^3-3\cdot24\cdot27\cdot29$
$$=13824+19683+24389-56376$$
$$=57896-56376=1520$$

18 The given expression

$$=(3\cdot625)^3+(4\cdot625)^3+(5\cdot625)^3-3\times3\cdot625\times4\cdot625\times5\cdot625$$
$$=47\cdot634765625+98\cdot931640625+177\cdot978515625-$$
$$282\cdot919921875=324\cdot544921875-282\cdot919921875$$
$$=41\cdot625$$

19 The given expression

$$=\left(\frac{311}{7}\right)^3+\left(\frac{360}{7}\right)^3+\left(\frac{409}{7}\right)^3-3\cdot\frac{311}{7}\cdot\frac{360}{7}\cdot\frac{409}{7}$$
$$=\frac{30080231}{343}+\frac{46656000}{343}+\frac{68417929}{343}-\frac{137374920}{343}$$
$$=\frac{145154160-137374920}{343}=\frac{7779240}{343}=22680$$

20 The given expression

$$=(1659)^3+(1667)^3+(1674)^3-3\cdot1659\cdot1667\cdot1674$$
$$=4566034179+4632407963+4691010024-13888607166$$
$$=13889452166-13888607166=845000$$

Exercise 4.

1 $\sqrt[3]{bdp} = \sqrt[3]{2 \cdot 4 \cdot 8} = \sqrt[3]{64} = 4$

2 $\sqrt[5]{pd} = \sqrt[5]{8 \cdot 4} = \sqrt[5]{32} = 2$

3 $\sqrt[3]{p}\,n = \sqrt[3]{8} \times 3 = 2 \times 3 = 6$

4 $6\sqrt[6]{(cn^4)} = 6\sqrt[6]{9 \cdot 3^4} = 6\sqrt[6]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$
$= 6 \times 3 = 18$

5 $4\sqrt[4]{4d} = 4\sqrt[4]{4 \cdot 4} = 4\sqrt[4]{16} = 4 \times 2 = 8$

6 $4\sqrt[4]{4^4} = 4 \times 4 = 16$

7 $2\sqrt{4p^2} = 2 \times 2p = 2 \times 2 \cdot 8 = 32$

8 $2\sqrt{4p^4} = 2 \times 2p^2 = 2 \cdot 2 \cdot 8^2 = 4 \times 64 = 256$

9 $a + b\sqrt{c} = 5 + 2\sqrt{9} = 5 + 2 \cdot 3 = 5 + 6 = 11$

10 $\overline{a+b}\sqrt{c} = \overline{5+2}\sqrt{9} = 7 \times 3 = 21$

11 $3\sqrt{c} + p = 3\sqrt{1} + 8 = 3 \cdot 1 + 8 = 3 + 8 = 11$

12 $3\sqrt{c+p} = 3\sqrt{1+8} = 3\sqrt{9} = 3 \cdot 3 = 9$

13 $\sqrt[3]{3(c+p)} = \sqrt[3]{3(1+8)} = \sqrt[3]{3 \cdot 9} = 3$

14 $3\sqrt[3]{8}\,(n+3p) = 3 \cdot 2(3+24) = 6 \cdot 27 = 162$

15 $3\sqrt[3]{8(n+3p)} = 3\sqrt[3]{8(3+24)} = 3\sqrt[3]{8 \cdot 27} = 3(2 \cdot 3) = 18$

16 $f\sqrt{m} + c = 0 + 9 = 9$

17 $f\sqrt{m+c} = 0 \times \sqrt{7+9} = 0$

18 $3c - (2d - b) = 27 - (8 - 2) = 27 - 6 = 21$

19 $3c - 2(d - b) = 27 - 2(4 - 2) = 27 - 4 = 23$

20 $3(c - 2d) - b = 3(9 - 8) - 2 = 3 - 2 = 1$

21 $(3c - 2)d - b = (27 - 2)4 - 2 = 25 \cdot 4 - 2 = 100 - 2 = 98$

22 $(3c - 2)\,\overline{d-b} = (27 - 2)\,\overline{4-2} = 25 \cdot 2 = 50$

23 $3\{c - (2d - b)\} = 3\{9 - (8 - 2)\} = 3 \cdot 3 = 9$

24 $3(c - 2)(d - b) = 3(9 - 2)(4 - 2) = 3 \cdot 7 \cdot 2 = 42$

25 $7p - (n^2 - b^2) = 7 \cdot 8 - (3^2 - 2^2) = 56 - (9 - 4) = 56 - 5 = 51$

26 $(7p - n)^2 - b^2 = (56 - 3)^2 - 2^2 = (53)^2 - 4$
$= 2809 - 4 = 2805$

27 $7p - (n^2 - b)^2 = 56 - (9 - 2)^2 = 56 - 49 = 7$

28 $7(p - n)^2 - b^2 = 7(8 - 3)^2 - 2^2 = 7 \cdot 25 - 4 = 175 - 4 = 171$

29 $\{7p - (n^2 - b)\}^2 = \{56 - (9 - 2)\}^2 = (56 - 7)^2 = (49)^2 = 2401$

30 $\sqrt[3]{\overline{p+3c+4d}}\,(c+n)^3 = \sqrt[3]{\overline{8+3+16}}\,(1+3)^3$
$= \sqrt[3]{27}\,(4)^3 = 3\ 64 = 192$

31 $\sqrt[3]{p}+3c+4d\,(c+n)^3 = \sqrt[3]{8}+3+16(1+3)^3 = 2+3+16\ 64$
$= 5+1024 = 1029$

32 $\sqrt[3]{p}+\overline{3c+4d}\,(c+n)^3 = \sqrt[3]{8}+(3+16)(1+3)^3 = 2+19\ 64$
$= 2+1216 = 1218.$

33 $\sqrt[3]{p}+(3c+4d)c+n^3 = \sqrt[3]{8}+(3+16)+3^3 = 2+19+27 = 48$

34 $\sqrt[3]{p}+3\{(c+4)dc+n^3\}$
$= \sqrt[3]{8}+3\{(1+4)4+3^3\} = 2+3(5\ 4+27)$
$= 2+3(20+27) = 2+3\ 47 = 2+141 = 143$

35 $a(x+y)^2(a-\overline{c-z})^3 = 6(2+3)^2(6-\overline{5-4})^3$
$= 6\ 5^2\ 5^3 = 6\ 25\ 125 = 150\ 125 = 18750.$

36 $4\{n-a(d-\overline{a+p})\}\sim 4\{\overline{n-a}(d-a)+p\}$
$= 4\{9-6(8-\overline{6+1})\}\sim 4\{\overline{9-6}(8-6)+1\}$
$= 4\{9-6(8-7)\}\sim 4\{3\ 2+1\}$
$= 4(9-6)\sim 4(6+1) = 12\sim 28 = 16$

37 $5\{c+\overline{x^2+y}(n-\overline{d-z})\}\sim 5\{(\overline{c+x^2}+y)\overline{n-d}-z\}$
$= 5\{5+\overline{4+3}(9-\overline{8-4})\}\sim 5\{(\overline{5+4}+3)\overline{9-8}-4\}$
$= 5(5+7(9-4)\}\sim 5\{(9+3)\ 1-4\}$
$= 5(5+7\ 5)\sim 5(12\ 1-4)$
$= 5(5+35)\sim 5(12-4)$
$= 5\ 40\sim 40 = 200-40 = 160$

38 $[x+y^2\{ap-z(c-\overline{a-x})\}]\sim[\overline{x+y}^2\{(ap-z)c-a\}-x]$
$= [2+3^2\{6-4(5-\overline{6-2})\}]\sim[\overline{2+3}^2\{(6-4)5-6\}-2]$
$= [2+9\{6-4(5-4)\}]\sim[5^2\{2\ 5-6\}-2]$
$= [2+9\{6-4\}]\sim[25\{10-6\}-2]$
$= [2+9\ 2]\sim[25\ 4-2]$
$= [2+18]\sim[100-2]$
$= 20\sim 98 = 78$

Exercise 5

1 A's loss = + 50 or £100

2 Loss of £1 is evidently represented by = 1, *gain* of £1 must be represented by − 1, and a gain of £70 by − 70

3 Income of £1 is represented by $+\frac{1}{4}$, *a debt* of £1 must be represented by $-\frac{1}{4}$, and a debt of £100 by $-\frac{1}{4}\times 100$ or − 25

4 A debt of £1 is represented by $+\frac{1}{8}$ an income of £1 must be represented by $-\frac{1}{8}$, and an income of £800 by $-\frac{1}{8}\times 800$ or − 100

5 A distance of 1 mile to the north is represented by $+\frac{1}{5}$ a distance of 1 mile to the South must be represented by $-\frac{1}{5}$, and a distance of 150 miles to the South by $-\frac{1}{5}\times 150$ or − 30

6 The rise (8 in) on the 1st day $=\frac{8 \text{ in}}{2 \text{ in}}=4$

The fall (6 in) on the 2nd day $=\frac{6 \text{ in}}{2 \text{ in}}=3$;

the rise on the 2nd day must be represented by − 3

The rise (10 in) on the 3rd day $=\frac{10 \text{ in}}{2 \text{ in}}=5$

7 The gain in the 1st year $=\frac{\text{Rs } 30}{\text{Rs } 2}=15$

The loss in the 2nd year $=\frac{\text{Rs } 20}{\text{Rs } 2}=10$, the gain in the 2nd year must be represented by − 10

The loss in the 3rd year $=\frac{\text{Rs } 40}{\text{Rs } 2}=20$, the gain in the 3rd year must be represented by − 20

The gain in the 4th year $=\frac{\text{Rs } 60}{\text{Rs } 2}=30$

8 The gain in the 1st year = 15 ,
the loss in the 1st year = − 15
The loss in the 2nd year = 10,
and the loss in the 3rd year = 20

The gain in the 4th year = 30 ,
the loss in the 4th year = − 30

Exercise 6

1 $(-3)-(-7)+(-12)=-(3+7+12)=-22$

2 $a-(-b)+(-5c)=(-5)-(-3)+\{-5(2)\}$
$=-(5+3+10)=-18$

3 $-5-a-b=(-5)+(-6)-(-20)=-(5+6+20)$
$=-31$, and $(-10)-(-31)=-(10+31)=-41$

4 $-a-(b-c)=-5-\{(-8)+(-6)\}=-(5+8-6)=-19$

5 $(-a^4b^2)-(-b^4c^2)-\{-(a^2-b^2)\}$
$=(-256\cdot4)+(-16\cdot9)+\{-(16-4)\}$
$=(-1024)+(-144)+(-12)$
$=-(1024-144-12)=-1180$

6 $(-3a^2b^2)-d+e-(-20c^2)+(d+e)$
$=(-3\cdot1\cdot8)-(-4)-(-5)+(-20\cdot9)+\{(-4)+(-5)\}$
$=(-24)+(-4)-(-5)-(-180)-(-9)$
$=-(24+4+5-180-9)=-222$

7 $\{-a^4(b-c)\}-\{-b^4(c-a)\}-\{-c^4(b-a)\}$
$=\{-16(5-4)\}-\{-625(4-2)\}+\{-256(5-2)\}$
$=(-16)-(-1250)+(-768)$
$=-(16-1250-768)=-2034.$

8 $\{-(x^2-y^2)\}-\{-(x^3-y^3)\}-\{-(x^4-y^4)\}$
$=\{-(25-9)\}-\{-(125-27)\}+\{-(625-81)\}$
$=(-16)-(-98)+(-544)=-(16+98-544)=-658$

9 $\{-z^3(y^2-z^2)\}-\{-y^3(z^2-x^2)\}+\{-z^3(y^2-x^2)\}$
$=\{-27(36-25)\}+\{-216(25-9)\}-\{-125(36-9)\}$
$=\{-27\cdot11\}-\{-216\cdot16\}-\{-125\cdot27\}$
$=(-297)-(-3456)+(-3375)$
$=-(297-3456-3375)$
$=-7128$

10 $a^4-b^4-c^4=60^4-4^4-2^4$
$=60\times60\times60\times60\times\frac{1}{4\times4\times4\times4}-16$
$=50625-16=50609$;
$a^4-(b^4-c^4)=60^4-(256-16)$
$=60\times60\times60\times60\times\frac{1}{240}=54000$

$$a^4 - b^4 \times c^4 = 12960000 - 256 \times 16$$
$$= 12960000 - 4096 = 12955904\ ,$$
and $(a^4 - b^4) \times c^4 = (12960000 - 256) \times 16$
$$= 12959744 \times 16 = 207355904$$

Hence the given expression

$= (-50609) + (-54000) + (-12955904) + (-207355904)$

$= -(50609 + 54000 + 12955904 + 207355904)$

$= -220416417$

Exercise 7

1 $7 + (-4) = 7 - 4 = 3$

2 $8 + (-13) = 8 - 13 = -5$

3 $b - c + d = 13 - 25 + 8 = -12 + 8 = -4$

4 $3a + (-5b) + c + d + 4e$

$\quad = 6 - 15 + 4 - 7 + 12 = -9 - 3 + 12 = -12 + 12 = 0$

5 $3m - 5n + 6q + r = 12 - 30 + 12 - 8 = -18 + 4 = -14$

6 $2x^2y - 3y^3x - 5x^2y^2 + x^3y^4$

$= 2\;2^2\;2 - 3\;2^3\;2 - 5\;2^2\;2^2 + 2^3\;2^4$

$= 2\;4\;2 - 3\;8\;2 - 5\;4\;4 + 8\;16$

$= 16 - 48 - 80 + 128 = -32 + 48 = 16$

7 $a^2c = 4\;3 = 12\ ,\ bd^2 = 5\;36 = 180\ ,$

$cb^2 = 3\;25 = 75\ ,$ and $a^2d^2 = 4\;36 = 144$

Hence the given expression

$= (-12) + (180) + (-75) + (-144)$

$= -12 + 180 - 75 - 144 = 168 - 219 = -51$

8 The given expression

$= 3^3 + 3\;9\;5 + 3\;3\;25 - 5^3$

$= 27 - 135 + 225 - 125 = -108 + 100 = -8$

9 The given expression

$= 16 - 4\;8\;4 + 6\;4\;16 - 4\;2\;64 + 256$

$= 16 - 128 + 384 - 512 + 256$

$= -112 - 128 + 256 = -240 + 256 = 16$

10 The given expression

$=5^5-5\cdot625\cdot7+10\cdot125\cdot49-10\cdot25\cdot343+5\cdot5\cdot2401-7^5$

$=3125-21875+61250-85750+60025-16807$

$=-18750-24500+43218=-43250+43218=\mathbf{-32}$

Exercise 8

1 $a+b+c-2a-3b+4c$

$=(a-2a)+(b-3b)+(4c-c)=\underline{-a-2b+3c}$

2 $5a^2-7b^2+8c^2+5b^2-7c^2-6a^2$

$=(5a^2-6a^2)+(5b^2-7b^2)+(8c^2-7c^2)=\underline{-a^2-2b^2+c^2}$

3 $8a-ab-7a+5c-3a+5ab$

$=(8a-3a-7a)+(5ab-ab)+5c=\underline{-2a+4ab+5c}$

4 $5mnp^2-6abr-7c^2-8mnp^2+9c^2+4abr-5c^2$

$=(5mnp^2-8mnp^2)+(4abr-6abr)+(9c^2-5c^2-7c^2)$

$=\underline{-3mnp^2-2abr-3c^2}$

5 $-7a^3b-5b^2c^2+10a^3b-3b^2c^2+3df-a^3b-b^2c^2-5df$

$=(-7a^3b+10a^3b-a^3b)-(5b^2c^2+3b^2c^2+b^2c^2)+(3df-5df)$

$=\underline{2a^3b-9b^2c^2-2df}$

6 $8x^4y-5xyz-17x^4y+20x^2y^2-2xyz-35x^2y^2$

$+3x^4y-4xyz+5x^2y^2=(8x^4y-17x^4y+3x^4y)$

$-(5xyz+2xyz+4xyz)+(20x^2y^2-35x^2y^2+5x^2y^2)$

$=\underline{-6x^4y-11xyz-10x^2y^2}$

7 $-13a^2bc+15ab^2c-27abc^2-5a^2bc+13abc^2-23abc^2$

$+7a^2bc+6abc^2+19ab^2c$

$=(-13a^2bc-5a^2bc+7a^2bc)+(15ab^2c-23ab^2c+19ab^2c)$

$+(13abc^2-27abc^2+6abc^2)$

$=\underline{-11a^2bc+11ab^2c-8abc^2}$

8 $20x^3mn-23m^3nx+14n^3xm-37x^3mn-47n^3xm$

$+54m^3nx-8x^3mn+13n^3xm$

$-15m^3nx+20n^3xm=(20x^3mn-37x^3mn-8x^3mn)$

$+(54m^3nx-23m^3nx-15m^3nx)$

$+(14n^3xm-47n^3xm+13n^3xm+20n^3xm)$

$=\underline{-25x^3mn+16m^3nx}$

9 The required sum $=(-2+9+20)+(25-24-42)$
$=-2+9+20+25-24-42=(9+20+25)-(2+24+42)$
$=54-68=-14$

10 The required sum $=(15-14)+(-70-96)+(15+6-90)$
$=15-14-70-96+15+6-90$
$=(15+15+6)-(14+70+96+90)$
$=36-270=-234$

11 The required sum
$=(3\ 4)+(-5\ 9+7\ 16)+(-2\ 6+5\ 10-12)+(10\ 5-7\ 9)$
$=12-45+112-12+50-12+50-63=(12+112+50+50)$
$-(45+12+12+63)=224-132=92$

12 The required sum
$=(-4+9-16)+(-5-30+42)+(24-45-30+60)$
$=-4+9-16-5-30+42+24-45-30+60$
$=(9+42+24+60)-(4+16+5+30+45+30)$
$=135-130=5$

13 The required sum
$=(-6+72)+(120-60+7)+(-16-25+90)$
$+(48-35-42+24)=-6+72+120-60+7-16-25$
$+90+48-35-42+24=(72+120+7+90+48+24)$
$-(6+60+16+25+35+42)=361-184=177$

14 The required sum
$=(24-120)+(490-72-180)+(-80+243-810)$
$+(80-2800+72-1500)=24-120+490-72-180$
$-80+243-810+80-2800+72-1500$
$=(24+490+243+80+72)-(120+72+180+80$
$+810+2800+1500)=909-5562=-4653$

15 The required sum
$=(3\ 81\ 0-5\ 81\ 6-4\ 100\ 8)+(-13\ 4\ 10+4\ 64\ 5-7\ 25\ 4)+$
$(-5\ 144\ 4+8\ 7\ 100+9\ 125)+(5\ 64\ 9-7\ 100\ 12-4\ 216\ 10$
$+8\ 4\ 5\ 9)=(2430-2430-3200)+(-520+1280-700)$
$+(-2880+5600+1125)+(2880-8400-8640+1440)$
$=-3200+60+3845+(-12720)=(3845+60)-$
$(12720+3260)=3905-15920=-12015$

Exercise 9

1
$$\begin{array}{r} a-2b+5c \\ -7a+3b-8c \\ \hline -6a+b-3c \end{array}$$

2
$$\begin{array}{r} -3x+5y-9z \\ +5x-3y+7z \\ -2y+z \\ \hline 2x \qquad -z \end{array}$$

3
$$\begin{array}{r} x^3+3x^2-5x+4 \\ 2x^3-6x^2+7x-8 \\ -x^3+7x^2-2x+9 \\ +5x^2 \qquad +2 \\ \hline 2x^3+9x^2 \qquad +7 \end{array}$$

4
$$\begin{array}{r} 3a-2b+7c-8d \\ -5a \qquad +2c+6d \\ +3b-10c+d \\ a-4b+c \qquad \\ +5b \qquad -7d \\ \hline -a+2b \qquad -8d \end{array}$$

5
$$\begin{array}{r} x^2+2xy+3y^2-x+y+2 \\ -5x^2 \qquad +y^2+2x \qquad -5 \\ -3xy-7y^2 \qquad +3y+1 \\ 6x^2+xy \qquad -x-4y+2 \\ \hline 2x^2 \qquad -3y^2 \qquad\qquad \end{array}$$

6
$$\begin{array}{r} 2x^2-5xy+y^2 \qquad\qquad \\ -7x^2 \qquad +4y^2-5x+2y \\ 3xy-6y^2 \qquad +y-5 \\ 3x-4y+3 \\ \hline -5x^2-2xy-y^2-2x-y-2 \end{array}$$

7
$$\begin{array}{r} a^2b+abc-b^2c^2 \\ 5a^2b-3abc-12b^2c^2 \\ -4a^2b+2abc+8b^2c^2 \\ 2a^2b \qquad +5b^2c^2 \\ \hline 4a^2b \qquad\qquad \end{array}$$

8
$$\begin{array}{r} m^3n^2-3mnp+2m^2n^3+6m^2n^2 \\ 5m^3n^2+7mnp-m^2n^3-10m^2n^2 \\ -5mnp+3m^2n^3+2m^2n^2 \\ -7m^3n^2 \qquad -4m^2n^3+m^2n^2 \\ \hline -m^3n^2-mnp \qquad -m^2n^2 \end{array}$$

9
$$\begin{array}{r} 12a^3b^2x-29b^3x^2a+37x^3a^2b+45a^2b^2x^2 \\ -18a^3b^2x+25b^3x^2a-5x^3a^2b-16a^2b^2x^2 \\ 20a^3b^2x-28b^3x^2a-23x^3a^2b+32a^2b^2x^2 \\ -14a^3b^2x+32b^3x^2a-9x^3a^2b-60a^2b^2x^2 \\ \hline a^2b^2x^2 \end{array}$$

10
$$\begin{array}{r} -15a^4b^4c^4+7c^4a^3b^5-24b^4c^3a^5+27a^4b^3c^5 \\ 23a^4b^4c^4+19c^4a^3b^5-8b^4c^3a^5-15a^4b^3c^5 \\ 11a^4b^4c^4-16c^4a^3b^5+29b^4c^3a^5-9a^4b^3c^5 \\ -18a^4b^4c^4-10c^4a^3b^5+3b^4c^3a^5-3a^4b^3c^5 \\ \hline a^4b^4c^4 \qquad\qquad\qquad\qquad\qquad\qquad \end{array}$$

11
$$\begin{array}{r}
25a^3b^3 - 8b^3c^3 - 23c^3a^3 + 19a^2b^2c^2 \\
-19a^3b^3 - 12b^3c^3 + 16c^3a^3 - 14a^2b^2c^2 \\
13a^3b^3 - 20b^3c^3 + 17c^3a^3 + 27a^2b^2c^2 \\
-21a^3b^3 + 29b^3c^3 - 13c^3a^3 - 6a^2b^2c^2 \\
3a^3b^3 + 10b^3c^3 + 4c^3a^3 - 27a^2b^2c^2 \\
\hline
a^3b^3 - b^3c^3 + c^3a^3 - a^2b^2c^2
\end{array}$$

12
$$\begin{array}{r}
5a^3 - 18b^3 - 53c^3 - 25abc \\
-37a^3 + 29b^3 + 38c^3 - 7abc \\
43a^3 + 11b^3 - 17c^3 + 26abc \\
4a^3 + 13b^3 + 21c^3 - 18abc \\
-14a^3 - 34b^3 + 12c^3 + 21abc \\
\hline
a^3 + b^3 + c^3 - 3abc
\end{array}$$

13
$$\begin{array}{r}
3x^3 + 5y^5 - 20a^2 + 49b^2 \\
-23x^3 - y^5 + 17a^2 - 27b^2 \\
+20x^3 - 4y^5 - 3a^2 + 3b^2 \\
+ 7a^2 - 23b^2 \\
\hline
a^2 + 2b^2
\end{array}$$

Hence the required value
$=25+128=153$

14
$$\begin{array}{r}
10a^2 - 26x^3y^4 - 30x^3b^5 - 17a^6y^7 \\
-304a^2 + 35x^3y^4 - 28x^3b^5 + 16a^6y^7 \\
- 9x^3y^4 - 7x^3b^5 - 8a^6y^7 \\
+289a^2 + 5x^3b^5 - 25a^6y^7 \\
\hline
-5a^2
\end{array}$$

Hence the required value $=-125$

15
$$\begin{array}{r}
2a^2 - 7b^2 + 9x^2 - 13y^2 + 15ab - 21xy \\
-6a^2 + 8b^2 - 20x^2 + 5y^2 - 8ab + 17xy \\
+5a^2 - 2b^2 + 13x^2 - 10y^2 - 20ab - 16xy \\
- a^2 + 3b^2 - 2x^2 + 18y^2 + 13ab + 23xy \\
\hline
2b^2 + 3xy
\end{array}$$

Hence the required value $=32+168=200$

16
$$\begin{array}{r}
29abx - 39bxy + 49xya - 59yab \\
-19abx + 29bxy - 39xya + 49yab \\
2abx + 6bxy - 12xya + 24yab \\
-13abx + 4bxy + 3xya - 14yab \\
\hline
- abx + xya
\end{array}$$

Hence the required value $=-160+280+120$

17

$$\begin{array}{rrrr}
18a^2b^2 & -43b^2x^2 & +62x^2y^2 & -23abxy \\
-25a^2b^2 & +28b^2x^2 & -42x^2y^2 & +39abxy \\
37a^2b^2 & +19b^2x^2 & +35x^2y^2 & -25abxy \\
-29a^2b^2 & -4b^2x^2 & -55x^2y^2 & +9abxy \\ \hline
a^2b^2 & & &
\end{array}$$

Hence the required value $=25\times16=400$

18

$$\begin{array}{rrrr}
46a^4 & +38b^4 & -87abx^2 & -105y^4 \\
-56a^4 & -58b^4 & +47abx^2 & +85y^4 \\
+23a^4 & +75b^4 & +63abx^2 & +57y^4 \\
-39a^4 & -33b^4 & -27abx^2 & +8y^4 \\
+26a^4 & -22b^4 & +5abx^2 & -45y^4 \\ \hline
 & & abx^2 &
\end{array}$$

Hence the required value $=4\times5\times8\times8=1280$

19

$$\begin{array}{rrrrr}
35xy^4 & +207ab^4 & -98bx^4 & -62ya^4 & -83abx^2y \\
-65xy^4 & -87ab^4 & +68bx^4 & +102ya^4 & +53abx^2y \\
+53xy^4 & -75ab^4 & +43bx^4 & -25ya^4 & +26abx^2y \\
-29xy^4 & +45ab^4 & +26bx^4 & +28ya^4 & -65abx^2y \\
+6xy^4 & -89ab^4 & -39bx^4 & -43ya^4 & +69abx^2y \\ \hline
 & ab^4 & & &
\end{array}$$

Hence the required value $=5\times4\times4\times4\times4=1280$

20

$$\begin{array}{rrrrr}
57a^4bx & +25b^4xy & -143x^4ya & +37y^4ab & -253a^2b^2x^2 \\
-63a^4bx & -85b^4xy & +63x^4ya & -92y^4ab & +73a^2b^2x^2 \\
+96a^4bx & +132b^4xy & +36x^4ya & +35y^4ab & +82a^2b^2x^2 \\
-78a^4bx & -52b^4xy & -17x^4ya & +27y^4ab & -50a^2b^2x^2 \\
-12a^4bx & -20b^4xy & +61x^4ya & -7y^4ab & +148a^2b^2x^2 \\ \hline
\end{array}$$

Hence the required value $=0$

Exercise 10

1 $-3+(-5)-(-6)+(-8)=-3-5+6-8$

$$=-(3+5+8)+6=-10$$

2 $3-(-5)+(-6)-(-8)=3+5-6+8=(3+5+8)-6=10$

3 $-6-(-8)-\{-(-5)\}-3=-6+8-(5)-3=-6+8-5-3$

$$=-(6+5+3)+8=-6$$

4 $-6-\{-(-8)\}+(-5)-3=-6-(8)-5-3=-6-8-5-3$
$=-(6+8+5+3)=-22$

5 $-(-3)+(-5)-\{-(-6)\}-(-8)=3-5-(6)+8=3-5-6+8$
$=(3+8)-(5+6)=11-11=0$

6 $50-(-47)-\{-(-154)\}+(-234)=50+47-(154)-234$
$=50+47-154-234=(50+47)-(154+234)$
$=97-388=-291$

7 $-\{-(-47)\}+(-234)-(-50)-(-154)$
$=-(47)-234+50+154$
$=-47-234+50+154$
$=-(47+234)+(50+154)$
$=-281+204=-77$

8 $-\{-(-154)\}+(-47)-(-234)-(-50)$
$=-154-47+234+50=-(154+47)+(234+50)$
$=-201+284=83$

9 $-\{-(-234)\}-(-47)-(-154)-(-50)$
$=-234+47+154+50=-234+(47+154+50)$
$=-234+251=17$

10 $-(-50)-(-234)-\{-(-154)\}-(-47)=50+234-154+47$
$=(50+234+47)-154=331-154=177$

Exercise 11

1
$$\begin{array}{r} (2a+3b-c) \\ +(-a+b-c) \\ \hline a+4b-2c \end{array}$$

2
$$\begin{array}{r} (-a-2b+8c) \\ +(-2a+5b-4c) \\ \hline -3a+3b+4c \end{array}$$

3
$$\begin{array}{r} (2x+3y-4z) \\ +(x-y+z) \\ \hline 3x+2y-3z \end{array}$$

4
$$\begin{array}{r} (-5m^2-mn+4) \\ +(3m^2-2mn-5) \\ \hline -2m^2-3mn-1 \end{array}$$

5
$$\begin{array}{r} (3x^2-y^2+2z^2) \\ +(-x^2+2y^2-3z^2) \\ \hline 2x^2+y^2-z^2 \end{array}$$

6
$$\begin{array}{r} (2ax+xy-6y^2) \\ +(-3ax+4xy-5y^2) \\ \hline -ax+5xy-11y^2 \end{array}$$

7
$$\begin{array}{r} (a^2-5ab-8b^2) \\ +(3a^2-2ab+7b^2) \\ \hline 4a^2-7ab-b^2 \end{array}$$

8
$$\begin{array}{r} (5bc-c^2+2xy) \\ +(2bc-6c^2+8xy) \\ \hline 7bc-7c^2+10xy \end{array}$$

9
$$\begin{array}{r} (x^3-3x^2+6x+7) \\ +(-2x^3+4x^2-7x-5) \\ \hline -x^3+x^2-x+2 \end{array}$$

10
$$\begin{array}{r} (2x^4-2x^3y-3x^2y^2+4xy^3-y^4) \\ +(-x^4-2x^3y+3x^2y^2-6xy^3+y^4) \\ \hline x^4-4x^3y\qquad\qquad-2xy^3 \end{array}$$

11
$$\begin{array}{r} (2m^4-13m^2n+15mn^2-37n^4) \\ +(-3m^4+7m^2n-8mn^2+13n^4) \\ \hline -m^4-6m^2n+7mn^2-24n^4 \end{array}$$

12
$$\begin{array}{r} (5p^4-12p^3q+7p^2q^2-23pq^3+3q^4) \\ +(-8p^4+7p^3q-10p^2q^2+13pq^3-5q^4) \\ \hline -3p^4-5p^3q-3p^2q^2-10pq^3-2q^4 \end{array}$$

13
$$\begin{array}{r} (3x^5-5x^4y+2x^3y^2-7x^2y^3+6y^5) \\ +(7x^5-6x^4y+8x^3y^2+13x^2y^3-9y^5) \\ \hline 10x^5-11x^4y+10x^3y^2+6x^2y^3-3y^5 \end{array}$$

14
$$\begin{array}{r} (5m^3nx-17n^3xm+26x^3mn-13m^2n^2x-19n^2x^2m) \\ +(-3m^3nx+10n^3xm-14x^3mn+20m^2n^2x+27n^2x^2m) \\ \hline 2m^3nx-7n^3xm+12x^3mn+7m^2n^2x+8n^2x^2m \end{array}$$

15
$$\begin{array}{r} (48x^6-31x^5y-7x^4y^2-39x^3y^3-41x^2y^4+65xy^5-53y^6) \\ +(-37x^6+28x^5y-43x^4y^2+54x^3y^3+67x^2y^4-84xy^5+93y^6) \\ \hline 11x^6-3x^5y-50x^4y^2+15x^3y^3+26x^2y^4-19xy^5+40y^6 \end{array}$$

Exercise 12

1
$$\begin{array}{r} (2a+3b-5c) \\ +(-3a+4b-5c) \\ \hline -a+7b-10c \end{array}$$

2
$$\begin{array}{r} (7m-2n+5r) \\ +(3m-5n+7r) \\ \hline 10m-7n+12r \end{array}$$

3
$$\begin{array}{r} (2x^2-3xy+4y^2) \\ +(x^2-xy+y^2) \\ \hline 3x^2-4xy+5y^2 \end{array}$$

4
$$\begin{array}{r} (2x^3+3x^2-3x+5) \\ (-x^3+2x^2-5x-6) \\ \hline x^3+5x^2-8x-1. \end{array}$$

5
$$\begin{array}{r} (2a^2-3ab+3bc) \\ +(-a^2+5ab-7bc-2b^2) \\ \hline a^2+2ab-4bc-2b^2 \end{array}$$

6
$$\begin{array}{r} (3x^3-2x^2y+8xy^2-5y^3) \\ +(5x^3-6x^2y+4xy^2-7y^3) \\ \hline 8x^3-8x^2y+12xy^2-12y^3 \end{array}$$

7
$$\begin{array}{r} (4-6x+7x^2-9x^3) \\ +(2-3x+5x^2-7x^3) \\ \hline 6-9x+12x^2-16x^3 \end{array}$$

8
$$\begin{array}{r} (3x^4-2x^3-8x^2+7) \\ +(-x^4+7x^2-9) \\ \hline 2x^4-2x^3-x^2-2 \end{array}$$

9
$$\begin{array}{r} (x^5-6x^4-4x^2+5) \\ +(5x^5-2x^4+x^3-6x^2-9x-8) \\ \hline 6x^5-8x^4+x^3-10x^2-9x-3 \end{array}$$

10
$$\begin{array}{r} (x^3-4x^2y-8xy^2-5ab) \\ +(-2x^3+3x^2y-7xy^2+3ab+8y^3-a^3) \\ \hline -x^3-x^2y-15xy^2-2ab+8y^3-a^3 \end{array}$$

11
$$\begin{array}{r} (12x^4y-5x^3y^2+8xyz^2-9x^2yz) \\ +(2x^4y-2x^3y^2-3xyz^2+7x^2yz) \\ \hline 14x^4y-7x^3y^2+5xyz^2-2x^2yz \end{array}$$

12
$$\begin{array}{r} (2a^2-3b^2+5c^2-6ab+7bc-8ac) \\ +(-5a^2+4b^2+3c^2+7ab-2bc-ac) \\ \hline -3a^2+b^2+8c^2+ab+5bc-9ac \end{array}$$

13
$$\begin{array}{r} (2x^2-3xy+5y^2-6x-8y+9) \\ +(3x^2-2xy-4y^2+5x+7y-12) \\ \hline 5x^2-5xy+y^2-x-y-3 \end{array}$$

14
$$\begin{array}{r} (a^3-3a^2b+5ab^2-9b^3+3a^2-2ab+b^2) \\ +(-2a^3+2a^2b+7ab^2+7b^3+5a^2-5ab-4b^2) \\ \hline -a^3-a^2b+12ab^2-2b^3+8a^2-7ab-3b^2 \end{array}$$

15
$$\begin{array}{r} (3ax^4-5a^2x^3+6yzbc^2-7y^2zbc+8yz^2bc) \\ +(2ax^4-3a^2x^3+2yzbc^2+9y^2zbc-4yz^2bc) \\ \hline 5ax^4-8a^2x^3+8yzbc^2+2y^2zbc+4yz^2bc \end{array}$$

16
$$\begin{array}{r} (25-16x^3y^5z-17xy^3z^5+21x^3z^5y-6x^2y^2z^2+8xyz^4) \\ +(-27+15x^3y^5z+19xy^3z^5-19x^3z^5y+12x^2y^2z^2-11xyz^4) \\ \hline -2-x^3y^5z+2xy^3z^5+2x^3z^5y+6x^2y^2z^2-3xyz^4 \end{array}$$

17
$$\begin{array}{r} (29x^4y^3z^2-37x^3y^4z^2+54x^2y^3z^4-45x^3y^2z^4-67x^4y^2z^3+89x^2y^4z^3) \\ +(-25x^4y^3z^2-43x^3y^4z^2-26x^2y^3z^4+23x^3y^2z^4-35x^4y^2z^3+66x^2y^4z^3 \\ \hline 4x^4y^3z^2-80x^3y^4z^2+28x^2y^3z^4-22x^3y^2z^4-102x^4y^2z^3+155x^2y^4z^3 \end{array}$$

18
$$\begin{array}{l}(41x^3y^4z^5-87x^3y^5z^4-28x^4y^5z^3+63x^4y^3z^5-55x^5y^3z^4+37x^5y^4z^3)\\ +(-53x^3y^4z^5-13x^3y^5z^4+86x^4y^5z^3+29x^4y^3z^5+94x^5y^3z^4-75x^5y^4z^3),\\ \hline -12x^3y^4z^5-100x^3y^5z^4+58x^4y^5z^3+92x^4y^3z^5+39x^5y^3z^4-38x^5y^4z^3\end{array}$$

19
$$\begin{array}{l}(-x^2-y^2-yz)\\ +(-3x^2+5xy-6y^2-7yz)\\ \hline -4x^2+5xy-7y^2-8yz\end{array}$$

20
$$\begin{array}{l}(x^3+x^2y^2+a^2bx-2bxy^2-2xyab)\\ +(5x^3-13x^2y^2+a^2bx-5bxy^2-7xyab)\\ \hline 6x^3-12x^2y^2+2a^2bx-7bxy^2-9xyab\end{array}$$

21
$$\begin{array}{l}(3x^4+5x^2y^2-12y^4)\\ +(-5x^4+6x^3y-7x^2y^2+8xy^3+19y^4)\\ \hline -2x^4+6x^3y-2x^2y^2+8xy^3+7y^4\end{array}$$

22
$$\begin{array}{l}(-7x^5-4x^3y^2+13x^2y^3+29y^5)\\ +(5x^5+3x^4y-6x^3y^2-17x^2y^3-13xy^4+21y^5)\\ \hline -2x^5+3x^4y-10x^3y^2-4x^2y^3-13xy^4+50y^5\end{array}$$

23
$$\begin{array}{l}(2a^2+5ab-6b^2)\\ +(-a^2-2b^2)\\ \hline a^2+5ab-8b^2\end{array}$$

24
$$\begin{array}{l}(5x^2-6xy+4y^2-8x-10y+15)\\ +(-x^2-2xy-3y^2-4x-5y-6)\\ \hline 4x^2-8xy+y^2-12x-15y+9\end{array}$$

25
$$\begin{array}{l}(3a^3-4a^2b+5ab^2-8b^3)\\ +(-a^3+2ab^2-7b^3)\\ \hline 2a^3-4a^2b+7ab^2-15b^3\end{array}$$

26
$$\begin{array}{l}(-8x^3y+4x^2y^2-11xy^3+12x^2-13y+27)\\ +(-4x^3y+3x^2y^2+11xy^3-20x^2+30y-56)\\ \hline -12x^3y+7x^2y^2-8x^2+17y-29\end{array}$$

27
$$\begin{array}{l}(3a^2-7ab-8bc+9b^2)\\ +(2a^2+3ab+3bc+2b^2)\\ \hline 5a^2-4ab-5bc+11b^2\end{array}$$

28
$$\begin{array}{l}(-3x^3+5y^2-7xy+8x-9)\\ +(x^3-8y^2+2xy-11x+7)\\ \hline -2x^3-3y^2-5xy-3x-2\end{array}$$

29
$$
\begin{array}{l}
(-7a^3-8b^2c-13ac^2+3b^3)\\
\underline{+(4a^3-3b^2c+7ac^2-8b^3)}\\
-3a^3-11b^2c-6ac^2-5b^3
\end{array}
$$

30
$$
\begin{array}{l}
(21x^3-37xy^2+42y^3-18x^2+19xy-39)\\
\underline{+(-25x^3+15xy^2-87y^3+7x^2-43xy+24)}\\
-4x^3-22xy^2-45y^3-11x^2-24xy-15
\end{array}
$$

Exercise 13

1 The given expression

$$= 2a-3b-4a+6b-2a+5b=\underline{-4a+8b}$$

2 The given expression

$$= x-y+4x+2x-3y=\underline{7x-4y}$$

3 The given expression

$$=-5x+y-3x+y-2y+6x=\underline{-2x}$$

4 The given expression $=3a-6a+(2b-a)$

$$=3a-6a+2b-a=\underline{-4a+2b}$$

5 The given expression $=-a-2b+(6a+4b)$

$$=-a-2b+6a+4b=\underline{5a+2b}$$

6 The given expression $=2a-\{5b-7b+2a\}$

$$=2a-5b+7b-2a=\underline{2b}$$

7 The given expression $=3-\{5-(6-7+9)\}$

$$
\begin{aligned}
&=3-\{5-6+7-9\}\\
&=3-5+6-7+9\\
&=18-12=6
\end{aligned}
$$

8 The given expression

$$
\begin{aligned}
&=-2+3+\{-4-(-5-6)\}\\
&=-2+3-4-(-5-6)\\
&=-2+3-4+5+6\\
&=-6+14=8
\end{aligned}
$$

9 The given expression

$$= -a+3b+\{-2a-(-a-4b)\}$$
$$= -a+3b-2a+a+4b$$
$$= \underline{-2a+7b}$$

10 The given expression

$$=a-[2b-\{3c-(a-2b+3c)\}]$$
$$=a-[2b-\{3c-a+2b-3c\}]$$
$$=a-[2b-3c+a-2b+3c]$$
$$=a-a=0$$

11 The given expression

$$=3x-[5y-\{10z-(5x-10y+3z)\}]$$
$$=3x-[5y-\{10z-5x+10y-3z\}]$$
$$=3x-[5y-10z+5x-10y+3z]$$
$$=3x-5y+10z-5x+10y-3z$$
$$=\underline{-2x+5y+7z}$$

12 The given expression

$$=-a-[-b-\{-c-(-\overline{a-b-c})\}]$$
$$=-a-[-b-\{-c-(-a+b+c)\}]$$
$$=-a-[-b-\{-c+a-b-c\}]$$
$$=-a-[-b+c-a+b+c]$$
$$=-a+b-c+a-b-c$$
$$=\underline{-2c}$$

13 The given expression

$$=2x-5y+\{9x-(10y-4x)\}$$
$$=2x-5y+9x-(10y-4x)$$
$$=2x-5y+9x-10y+4x$$
$$=\underline{15x-15y}$$

14. The given expression

$$=-5a-3b+\{6a-(5b-7a)\}$$
$$=-5a-3b+6a-(5b-7a)$$
$$=-5a-3b+6a-5b+7a$$
$$=\underline{8a-8b}$$

15 The given expression

$$= -7m - 3n + \{8m - (4n - 10m)\}$$
$$= -7m - 3n + 8m - (4n - 10m)$$
$$= m - 3n - 4n + 10m$$
$$= \underline{11m - 7n}$$

16 The given expression

$$= -2a + 4b + \{-6c - (-8a - \overline{-10b - 12c})\}$$
$$= -2a + 4b - 6c - (-8a - \overline{-10b - 12c})$$
$$= -2a + 4b - 6c + 8a + \overline{-10b - 12c}$$
$$= -2a + 4b - 6c + 8a - 10b - 12c$$
$$= \underline{6a - 6b - 18c}$$

17 The given expression

$$= -3x + 5y + \{-7z - (-9x - \overline{-11y - 13z})\}$$
$$= -3x + 5y - 7z - (-9x - \overline{-11y - 13z})$$
$$= -3x + 5y - 7z + 9x + \overline{-11y - 13z}$$
$$= -3x + 5y - 7z + 9x - 11y - 13z$$
$$= \underline{6x - 6y - 20z}$$

18 The given expression

$$= -2x + 4y + \{-6z - (-3x - \overline{-5y - 7z})\}$$
$$= -2x + 4y - 6z - (-3x - \overline{-5y - 7z})$$
$$= -2x + 4y - 6z + 3x + \overline{-5y - 7z}$$
$$= -2x + 4y - 6z + 3x - 5y - 7z$$
$$= \underline{x - y - 13z}$$

19 The given expression

$$= -x + 3y - \{-5z - (-2x + \overline{-4y - 6z})\}$$
$$= -x + 3y + 5z + (-2x + \overline{-4y - 6z})$$
$$= -x + 3y + 5z - 2x + \overline{-4y - 6z}$$
$$= -x + 3y + 5z - 2x - 4y - 6z$$
$$= \underline{-3x - y - z}$$

20 The given expression

$$=-2a+[-5b-\{-8c+(-3a-\overline{-6b+9c})\}]$$
$$=-2a-5b-\{-8c+(-3a-\overline{-6b-9c})\}$$
$$=-2a-5b+8c-(-3a-\overline{-6b+9c})$$
$$=-2a-5b+8c+3a+\overline{-6b+9c}$$
$$=-2a-5b+8c+3a-6b-9c$$
$$=\underline{a-11b-17c}$$

21 The given expression

$$=-x-5y-\{-9z+(-3x-\overline{-7y+11z})\}$$
$$=-x-5y+9z-(-3x-\overline{-7y-11z})$$
$$=-x-5y+9z+3x+\overline{-7y-11z}$$
$$=-x-5y+9z+3x-7y+11z$$
$$=\underline{2x-12y+20z}$$

22 The given expression

$$=\{2a-3b-5c\}-a+2b-(c-4a)+7c$$
$$=2a-3b-5c-a-2b-c-4a+7c$$
$$=\underline{5a-b+11c}$$

23 The given expression

$$=x-y-(z-x)-y-z-z+x-(y-z)$$
$$=2x-2y+z-x-y-z$$
$$=\underline{x-3y-2z}$$

24 The given expression

$$=2a-(b-c)-3b+(2a-c)-2a-(c-4b)+3b+(2a-4c)-\{6c-(2b-3a)\}-5c-(6a-7b)\}$$
$$=4a-b+c-3b+2c-c-c-4b+3b+2a-4c-6c+2b-3a-5c+6a-7b$$
$$=\underline{11a-2b-16c}$$

25 The given expression

$$=a-(-b-c-d)+(-m+n-x)-y-z$$
$$=a-(b-c-d)+(-m-n-x)-y-z$$

26 The given expression

$$=a-(b+c-d+m-n+x-y+z)$$
$$=a-\{b+c-d+m+(-n+x-y+z)\}$$

27 The given expression

$$=(a-b-c+d-m)-(-n+x-y+z)$$
$$=\{a-b-(c-d+m)\}-\{-n-(-x+y-z)\}$$

28 The given expression

$$=-(-a+b+c)-(-d+m-n)-(x-y+z)$$
$$=-\{-a-(-b-c)\}-\{-d-(-m+n)\}-\{x-(y-z)\}$$

Miscellaneous Exercises (I).

I

1 (i) 5 hours $=10\times\frac{1}{2}$ hr (unit of time), and
10 is the required number

(ii) 5 hours $=\frac{1}{2}\times 10$ hrs (unit of time), and
$\frac{1}{2}$ is the required number

2 $x\sim y=17\sim 25=25-17=8$

3 See Art 10, Page 9

The respective coefficients are 15, $2a$, $7ab^2$ and $16m^2pq$

4 $\sqrt{ab}$ stands for "the square root of ab," while $b\sqrt{a}$ stands for "b multiplied by the square root of a"

$$\sqrt{ab}\sim b\sqrt{a}=\sqrt{9\cdot 4}\sim 4\sqrt{9}=3\cdot 2\sim 3\cdot 4=6\sim 12=12-6=6$$

5 A distance of 880 yds to the north is represented by 40, a distance of 1 yd to the north must be represented by $\frac{40}{880}$, or $\frac{1}{22}$, 1 yard to the south must be represented by $-\frac{1}{22}$, and a distance of 11 yds to the south must be represented by $-\frac{1}{22}\times 11$, or $-\frac{1}{2}$

6 See Art 4, Page 22

7 See Art I, Page 30

8 9, 7, 5, 2, −1, −3, −4, 8, −12

II

1. (i) $ab - a \times b = ab - ab = 0$

 (ii) $45 - ab = 45 - 4 \cdot 5 = 45 - 20 = 25$

 (iii) $74 - 7a = 74 - 7 \cdot 4 = 74 - 28 = 46$

 (iv) $85 - 8b = 85 - 8 \cdot 5 = 85 - 40 = 45.$

2. a^n means $a \times a \times a \times a \times$ .... to n factors,
 whereas n^a means $n \times n \times n \times n \times$ .... a factors

The given expression

$= 2401 - 4 \cdot 343 \cdot 5 + 6 \cdot 49 \cdot 25 - 4 \cdot 7 \cdot 125 + 625$

$= 2401 - 6860 + 7350 - 3500 + 625$

$= (2401 + 7350 + 625) - (6860 + 3500)$

$= 10376 - 10360 = 16$

3. $\sqrt[3]{a}$ = a number whose cube is "a"
 $\sqrt[5]{a}$ = ... 5th power is "a"
 $\sqrt[8]{a}$ = ... 8th power is "a"
 $\sqrt[n]{a}$ = ... nth power is "a"

The given expression

$= \sqrt{64 - 15} \times \sqrt[3]{343 - 216 - 2}$

$= \sqrt{49} \times \sqrt[3]{125} = 7 \times 5 = 35$

4. See Note 2 to Art 3, Page 19

5. The required sum

$= (3x^2y - 8x^2y - 19x^2y + 17x^2y)$

$= (3x^2y + 17x^2y) - (8x^2y + 19x^2y)$

$= 20x^2y - 27x^2y = \underline{-7x^2y}$

And the numerical value $= -7 \cdot 4^2 \cdot 5 = -560$

6. The required sum

$= 16x^4 - 8xy^3 + 24x^2y^2 + y^4 - 32x^3y$, and its numerical value

$= 16 \cdot 256 - 8 \cdot 4 \cdot 125 + 24 \cdot 16 \cdot 25 + 625 - 32 \cdot 64 \cdot 5$

$= 4096 - 4000 + 9600 + 625 - 10240$

$= (4096 + 9600 + 625) - (4000 + 10240)$

$= 14321 - 14240 = 81$

7 $$\begin{array}{r} (17b-12c-19a) \\ +(13b+25c-\ 4a) \\ \hline 30b+13c-23a \end{array}$$

8 The given expression

$$\begin{aligned}
&= 3x-[4y+\{2z-(x-5y+3z)\}]-(3x-7y) \\
&= 3x-4y-\{2z-(x-5y+3z)\}-3x+7y \\
&= 3x-4y-3x+7y-2z+(x-5y+3z) \\
&= 3x-4y-3x+7y-2z+x-5y+3z \\
&= \underline{x-2y+z}
\end{aligned}$$

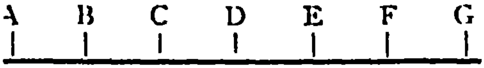
III

1 (i) $(a+b)c = x-yz$

(ii) $(x+y)^2 = x^2+y^2+2xy$

(iii) $\sqrt[3]{(m-n)}-m^3n^3 < \sqrt{x}+\sqrt{y}$

(iv) $a>b \qquad 3a>3b$

2 

The length of CD is 6 inches, $\therefore$ the length of DC must be -6 inches. Since DC is represented by 3, the unit of length must be $(-6 \text{ in} \div 3)$, or -2 in.

$DB = DC + CB = (-CD)+(-BC) = (-4)+(-6) = -10$,

$\therefore$ DB will be represented by $(-10 \text{ in} \div -2 \text{ in})$ *i e* by 5,

$DF = (DE+EF) = 5+8 = 13$ in, $\therefore$ it will be represented by $(13 \text{ in} \div -2 \text{ in})$ *i e* by $-\frac{13}{2}$,

$DA = (-CD)+(-BC)+(-AB) = -13$ in, $\therefore$ it will be represented by $\dfrac{-13 \text{ in}}{-2 \text{ in}}$ *i e* by $\frac{13}{2}$

$DG = (DE+EF+FG) = 20$ in, $\therefore$ it will be represented by $\dfrac{20 \text{ in}}{-2 \text{ in}}$ *i e* by -10

3 See Art 3, page 20

The required sum $= (-a^3)+(-3a^2b)+(-3ab^2)+(-b^3)$

$$\begin{aligned}
&= -(a^3+3a^2b+3ab^2+b^3) \\
&= -(216+3\cdot 36\cdot 4+3\cdot 6\cdot 16+64) \\
&= -(216+432+288+64) = -1000
\end{aligned}$$

4 See Art 7, Page 6

5 The given expression

$$=\sqrt[3]{5(2+3)^2}+\sqrt[3]{(2+6)(5-4)}+\sqrt[3]{2(5-3)^2}$$
$$=\sqrt[3]{5\cdot 5^2}+\sqrt[3]{8\cdot 1}+\sqrt[3]{2\cdot 2^2}=5+2+2=9$$

6 A receives x pounds, B receives $(x+a)$ pounds and C $(x+a+b)$ pounds

Hence the whole sum $=x+(x+a)+(x+a+b)=\underline{3x+2a+b}$

7
$$\begin{array}{rrrrr} a^2 & -3ab & -1\frac{1}{4}b^2 & & \\ & & +2\frac{1}{2}b^2 & -\frac{1}{3}b^3 & +c^2 \\ & +ab & -\frac{1}{4}b^2 & +b^3 & \\ & +2ab & & -\frac{2}{3}b^3 & \\ \hline a^2 & & +b^2 & & +c^2 \end{array}$$

8 $\{2x^2-(y^2-xy)\}-\{y^2-(4x^2-y^2)\}+\{2y^2-(3xy-x^2)\}$
$$=2x^2-y^2+xy-y^2+(4x^2-y^2)+2y^2-3xy+x^2$$
$$=2x^2-y^2+xy-y^2+4x^2-y^2+2y^2-3xy+x^2$$
$$=\underline{7x^2-2xy-y^2}$$

V

1 See Arts 16 & 17, Pages 15, 16

2 Follow the directions given in Art 6, (Page 6), in finding separately the values of (*i*) $a\times b$, (*ii*) $c-d\times e$ and (*iii*) $f+gh$

Now subtract (*ii*) from (*i*), and to the remainder add (*iii*)

3 See Art 9, Page 9

The required factors are 2, a, b and $(a+b)$

4 See Art 6, Page 23

5 The given expression

$$=\frac{2\cdot 4\cdot 4}{(4-2)^2}-\frac{6\sqrt[3]{4\cdot 2}}{4^2\sqrt{2\cdot 4+4\cdot 2}}-\frac{29\cdot 4}{64\cdot 4}$$
$$=\frac{2\cdot 4\cdot 4}{4}-\frac{6\cdot 2}{4\cdot 4\cdot 4\sqrt{16}}-\frac{29}{64}$$
$$=8-\frac{6\cdot 2}{4\cdot 4\cdot 4\cdot 4}-\frac{29}{64}$$
$$=8-\tfrac{3}{64}-\tfrac{29}{64}$$
$$=8-\tfrac{1}{2}=7\tfrac{1}{2}$$

6
$$\begin{array}{r} x^3-7x^2+6x+\ 5 \\ 2x^3+5x^2-3x+\ 4 \\ -4x^3-7x^2+2x-11 \\ 5x^3+9x^2-4x+\ 2 \\ \hline 4x^3 \qquad\quad +x \end{array}$$

the required value
$=4\cdot 5^3+5=4\cdot 125+5$
$=500+5=$ **505**

7 See Art 4, Page 32

8 The given expression
$=2x-[3x-9y-\{2x-3y-x-5y\}]$
$=2x-[3x-9y-2x+3y+x+5y]$
$=2x-3x+9y+2x-3y-x-5y=y$

VI

1 See Art 22, Page 9

2 $(16-1)\cdot\sqrt{24\cdot 10\cdot 5+25}+\sqrt{(16-5)(10+1)}$
$=15\sqrt{1225}+\sqrt{11\cdot 11}$
$=15\cdot 35+11=525+11=$ **536**

3 (i) The left-hand expression
$=3^3+4^3+5^3-3\cdot 3\cdot 4\cdot 5$
$=27+64+125-180=216-180=36$

and the right-hand expression
$=(3+4+5)(9+16+25-12-15-20)=12\cdot 3=36$

Therefore the two expressions are equal

(ii) The left hand expression

$$=\left(\frac{3}{2}\right)^3+\left(\frac{5}{2}\right)^3+\left(\frac{7}{2}\right)^3-3\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}$$

$$=\frac{27}{8}+\frac{125}{8}+\frac{343}{8}-\frac{315}{8}=\frac{495-315}{8}=\frac{180}{8}=22\frac{1}{2},$$

and the right-hand expression

$$=\left(\frac{3}{2}+\frac{5}{2}-\frac{7}{2}\right)\left(\frac{9}{4}+\frac{25}{4}+\frac{49}{4}-\frac{15}{4}-\frac{21}{4}-\frac{35}{4}\right)$$

$$=\frac{15}{2}\ \frac{12}{4}=\frac{45}{2}=22\frac{1}{2}$$

Therefore the two expressions are equal

4 See Cor 2, Art 6, Page 25

5 See Art 3, Page 31

6
$$\begin{array}{r} 7y^2-25\sqrt{yz}+z^2 \\ -12y^2+19\sqrt{yz}-3z^2 \\ 5y^2+7\sqrt{yz}+2z^2 \\ \hline \sqrt{yz} \end{array}$$

Hence the required value $=\sqrt{16}\ 15=4\ 15=60$

7 Subtract $7c$ from $2d$, then subtract the remainder from $3c$, then subtract the remainder from $4b$ and lastly subtract the remainder thus obtained from $5a$

8 The given expression

$$=[(64+27+8+1)\{4+3-(2-4)\}+16\ 3+4\ 1]$$
$$\times\{16-(9+4)+1\}$$
$$=[100\{4+3+2\}+48+4]\times\{16-13+1\}$$
$$=[900+48+4]\times 4=952\times 4=3808$$

VII

1 (i) $a \div bc$ means that a is to be divided by the product of b and c, whereas $a \div b \times c$ means that c is to be multiplied by the quotient of a by b

(ii) a^4 stands for $a\times a\times a\times a$, while $4a$ stands for $4\times a$

(iii) $3\sqrt{a}$ means that the square root of a is to be multiplied by 3, while $\sqrt[3]{a}$ means the quantity whose cube is a

(iv) $\sqrt{a+b}$ means the square root of the sum of a and b, while $\sqrt{a}+b$ means that b is to be added to the square root of a

(v) $\sqrt{ab}$ stands for "the square root of ab," while $\sqrt{a}b$ stands for "b multiplied by the square root of a"

2 (i) The given expression

$$=\frac{1\cdot 2+4\cdot 3+9\cdot 0+0\cdot 1}{(1+2)(3+0)-\{(1-0)+(3-2)\}}$$

$$=\frac{2+12}{3\cdot 3-1\cdot 2}=\frac{14}{9-2}=\frac{14}{7}=2$$

(ii) The given expression

$$=\sqrt[3]{2-1}+\sqrt[3]{4(3-1)}-\sqrt[4]{3(8\cdot 1+5\cdot 2+3\cdot 3-2\cdot 0)}$$

$$=\sqrt[3]{1}+\sqrt[3]{4\cdot 2}-\sqrt[4]{3(8+10+9)}=1+\sqrt[3]{8}-\sqrt[4]{3\cdot 27}$$

$$=1+2-3=0$$

3 (i) The first expression $=(2+3+4)^3+2^3+3^3+4^3$

$$=9^3+8+27+64=729+8+27+64=828,$$

the second expression

$$=(2+3)^3+(3+4)^3+(4+2)^3+6\cdot 2\cdot 3\cdot 4$$

$$=125+343+216+144=828,$$

and the third expression

$$=2\cdot 2^3+3\cdot 3^2(2+4)+2\cdot 3^3+3\cdot 4^2(2+3)$$
$$+2\cdot 4^3+3\cdot 2^2(3+4)+6\cdot 2\cdot 3\cdot 4$$

$$=2\cdot 8+3\cdot 9\cdot 6+2\cdot 27+3\cdot 16\cdot 5+2\cdot 64+3\cdot 4\cdot 7+6\cdot 2\cdot 3\cdot 4$$

$$=16+162+54+240+128+84+144=828$$

$\therefore$ the three expressions are equal to one another

(ii) The first expression $=(7+4+1)^3+7^3+4^3+1^3$

$$=(12)^3+343+64+1=1728+343+64+1=2136,$$

the second expression $=(7+4)^3+(4+1)^3+(1+7)^3+6\cdot 7\cdot 4\cdot 1$

$$=11^3+5^3+8^3+168=1331+125+512+168=2136,$$

and the third expression

$$=2\cdot 7^3+3\cdot 4^2(7+1)+2\cdot 4^3+3\cdot 1^2(7+4)+2\cdot 1^3$$
$$+3\cdot 7^2(4+1)+6\cdot 7\cdot 4\cdot 1=2\cdot 343+3\cdot 16\cdot 8+2\cdot 64$$
$$+3\cdot 11+2+3\cdot 49\cdot 5+6\cdot 7\cdot 4=686+384+128$$
$$+33+2+735+168=2136$$

$\therefore$ the three expressions are equal to one another

4 (i) $1-[1-\{1-(1-\overline{1+x})\}]$

$$=1-[1-\{1-(1-1-x)\}]$$
$$=1-[1-\{1-1+1+x\}]$$
$$=1-[1-1+1-1-x]$$
$$=1-1+1-1+1+x=\underline{1+x}$$

(ii) The given expression

$$=3a-b+2c-\{a+c-3a+b+2c\}-2a+3b-4c$$
$$=3a-b+2c-a-c+3a-b-2c-2a+3b-4c$$
$$=\underline{3a+b-5c}$$

5 (i) $(x+y)(x-y)=x^2-y^2$ (ii) $(x+y)^2=x^2+y^2+2xy$

6 The given expression

$$=17a-5b-[7a-3b-\{4a-4b-2a-3b\}]$$
$$=17a-5b-[7a-3b-4a+4b+2a+3b]$$
$$=17a-5b-7a+3b+4a-4b-2a-3b$$
$$=12a-9b$$

Hence the required value $=12 \cdot 39-9 \cdot 52=468-468=0$

7 $V-(X+Y)+Z$

$$=(5a+4b-6c)-(-3a-9b+7c+20a+7b-5c)$$
$$+13a-5b+9c$$
$$=5a+4b-6c+3a+9b-7c-20a-7b+5c+13a-5b+9c$$
$$=\underline{a+b+c}$$

8

$$\begin{array}{rrrr} a-\frac{1}{3}b & +\frac{1}{4}c & -\frac{1}{5}d \\ \frac{1}{4}a-\frac{1}{4}b & -\frac{1}{2}c & +d \\ -a-\frac{1}{10}b & +c & +\frac{1}{4}d \\ \frac{1}{4}a+b & -\frac{5}{8}c & -\frac{1}{10}d \\ 8a-6b & +3c & -4d \\ \hline \frac{17}{2}a-\frac{341}{60}b & +\frac{25}{8}c & -\frac{61}{20}d \end{array}$$

$$\begin{array}{r} (\frac{17}{2}a-\frac{341}{60}b+\frac{25}{8}c-\frac{61}{20}d) \\ +(-\frac{13}{2}a+\frac{163}{30}b-\frac{9}{4}c+\frac{11}{4}d) \\ \hline 2a-\frac{1}{4}b+\frac{7}{8}c-\frac{3}{10}d, \end{array}$$

which is the required difference

Exercise 14

1 $6\times5=5+5+5+5+5+5=30$

2 $7\times4=4+4+4+4+4+4+4=28$

3 $3\times12=12+12+12=36$

4 $-8\times4=(-8)+(-8)+(-8)+(-8)=32$

5 $5\times(-9)=(-9)+(-9)+(-9)+(-9)+(-9)=-45$

6 $6\times(-13)=(-13)+(-13)+(-13)+(-13)+(-13)+(-13)$
$$=-78$$

7 $(-3)\times 8=-8-8-8=-24$

8 $(-5)\times 14=-14-14-14-14-14=-70$

9 $(-3)\times 15=-15-15-15=-45$

10 $(-4)\times(-9)=-(-9)-(-9)-(-9)-(-9)$
$=9+9+9+9=36$

11 $(-5)\times(-12)=-(-12)-(-12)-(-12)-(-12)-(-12)$
$=12+12+12=12+12=60$

12 $(-4)\times(-16)=-(-16)-(-16)-(-16)-(-16)$
$=16+16+16+16=64$

Exercise 15

1 $ab-cd=(-2)(-3)-(-8)6=6+48=54$

2 $(a^2-b^2)c=(25-36)(-9)=(-11)(-9)=99$

3 $abc+b^2c-c^2d=3(-5)(-7)+25(-7)-49(-4)$
$=105-175+196=126$

4 $(-a)b^2-cd^2+b(-c)^2=(-5)49-4\cdot 9-7\cdot 16$
$=-245-36-112=-393$

5 $\{a^2+(-b)^2-(-c)^2\}(bc-ad)=\{9+16-36\}(24-27)$
$=(-11)(-3)=33$

6 $a^2(b-c)+b^2(c-a)+c^2(a-b)$
$=4\{(-5)-(-7)\}+25\{(-7)-(-2)\}+49\{(-2)-(-5)\}$
$=4(-5+7)+25(-7+2)+49(-2+5)$
$=4\cdot 2+25(-5)+49\cdot 3=8-125+147=30$

7 $x^3(y-z)+y^3(z-x)+z^3(x-y)$
$=-27\{8-(-5)\}+512\{(-5)-(-3)\}+(-125)\{(3)-(8)\}$
$=-27\cdot 13+512(-2)+(-125)(-11)$
$=-351-1024+1375=0$

8 $p^3(q^2-r^2)+q^3(r^2-p^2)+r^3(p^2-q^2)$
$=(-27)(25-49)+(-125)(49-9)+(-343)(9-2$
$=(-27)(-24)+(-125)40+(-343)(-16)$
$=648-5000+5488=1136$

9 $a^3+b^3+c^3-3abc=(-1728)+(-2197)$
$$+(-3375)-3(-12)(-13)(-15)$$
$$=-(1728+2197+3375)+39\,180$$
$$=-7300+7020=-280$$

10 $(a+b)^5=(-5+3)^5=-32$

Again the right-hand expression

$=(-5)^5+5(-5)^4\times 3+10(-5)^3\times 9+10(-5)^2\times 27+5(-5)81+243$

$=-3125+9375-11250+6750-2025+243=-32$,

$(a+b)^5=a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

Exercise 16

1 $4\times 5=5+5+5+5=20$,
also $5\times 4=4+4+4+4+4=20$, $4\times 5=5\times 4$

2 $6\times 3=3+3+3+3+3+3=18$,
also $3\times 6=6+6+6=18$, $6\times 3=3\times 6$

3 $7\times 5=5+5+5+5+5+5+5=35$,
also $5\times 7=7+7+7+7+7=35$, $7\times 5=5\times 7$

4 $4\times 8=8+8+8+8=32$,
also $8\times 4=4+4+4+4+4+4+4+4=32$, $4\times 8=8\times 4$

5 $9\times 5=5+5+5+5+5+5+5+5+5=45$,
also $5\times 9=9+9+9+9+9=45$, $9\times 5=5\times 9$

Exercise 17.

1 $(-a)\times 3b=(-1)\times a\times 3\times b=(-1)\times 3\times a\times b=-3ab$

2 $(2a)\times(-4b)=2\times a\times(-1)\times 4\times b$
$$=(-1)\times 2\times 4\times a\times b=-8ab$$

3 $(-3a)\times(-6a^2)=(3a)\times(6a^2)=3\times a\times 6\times a\times a$
$$=3\times 6\times a\times a\times a=18a^3$$

4 $(-4b)\times(-5a)=(4b)\times(5a)=4\times b\times 5\times a=4\times 5\times a\times b=20ab$

5 $(-7c)\times(-3ab)=(7c)\times(3ab)=7\times c\times 3\times a\times b$
$$=7\times 3\times a\times b\times c=21abc$$

6 $10\times35=5\times2\times5\times7=5\times5\times2\times7=25\times14$

7 $15\times75=5\times3\times5\times5\times3=5\times5\times5\times3\times3=5^3\times3^2$

8 $(-a)^3=(-1)\,a\times(-1)\,a\times(-1)\,a=(-1)^3\times a\times a\times a=-a^3$

9 $(ab)^3=a\times b\times a\times b\times a\times b=a\times a\times a\times b\times b\times b=a^3b^3$

10 $(-ab)^3=(-ab)\times(-ab)\times(-ab)$
$=(-1)\times a\times b\times(-1)\times a\times b\times(-1)\times a\times b$
$=(-1)^3\times a\times a\times a\times b\times b\times b=(-1)^3\times a^3\times b^3=-a^3b^3$

11 $(-a^3b^5)^2=(-a^3b^5)\times(-a^3b^5)=a^3b^5\times a^3b^5$
$=a\times a\times a\times b\times b\times b\times b\times b\times a\times a\times a\times b\times b\times b\times b\times b$
$=a\times a\times a\times a\times a\times a\times b\times b\times b\times b\times b\times b\times b\times b\times b\times b$
$=a^6b^{10}$

12 $(-x)^5=(-x)\times(-x)\times(-x)\times(-x)\times(-x)$
$=(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times x\times x\times x\times x\times x$
$=-x^5$

13 $(-xy)^5=(-xy)\times(-xy)\times(-xy)\times(-xy)\times(-xy)$
$=(-1)\times x\times y\times(-1)\times x\times y\times(-1)\times x\times y\times(-1)\times x\times y$
$\times(-1)\times x\times y$
$=(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times x\times x\times x$
$\times x\times x\times y\times y\times y\times y\times y=(-1)^5\times x^5\times y^5=-x^5y^5$

14 $6x^2y\times(-7x^5y^4)=-\{6\times7\times x^2\times x^5\times y\times y^4\}=\underline{-42x^7y^5}$

15 $(-3x^4y^3)\times(8x^7y^8)=-(3\times8\times x^4\times x^7\times y^3\times y^8)=\underline{-24x^{11}y^{11}}$

16 $(-5x^{12}y^3)\times(-8x^5y^{13})=5\times8\times x^{12}\times x^5\times y^3\times y^{13}=\underline{40x^{17}y^{16}}$

17 $(-12x^3y^3z^2)\times(13x^7y^6z^4)$
$=-\{12\times13\times x^3\times x^7\times y^3\times y^6\times z^2\times z^4\}=\underline{-156x^{10}y^9z^6}$

18 $(-14xy^5z^8)\times(-10x^5y^2z^{12})=(14xy^5z^8)\times(10x^5y^2z^{12})$
$=14\times10\times x\times x^5\times y^5\times y^2\times z^8\times z^{12}=\underline{140x^6y^7z^{20}}$

19 $(-3ab)\times(4a^2b^2)\times(-2a^3b^3)$
$=(-3)\times4\times(-2)\times a\times a^2\times a^3\times b\times b^2\times b^3=\underline{24a^6b^6}$

20 $(-2a^2)\times(-7a^4b^7)\times(-5a^9b^3)$

$=(-2)\times(-7)\times(-5)\times a^2\times a^4\times a^9\times b^7\times b^3$

$=\underline{-70a^{15}b^{10}}$

21 $(-6x^5y^2z)\times 2z^4x^7y^5\times(-4y^3z^2x^8)$

$=-6\times 2\times -4\times x^5\times x^7\times x^8\times y^2\times y^5\times y^3\times z\times z^4\times z^2$

$=\underline{48x^{20}y^{10}z^7}$

22 $(-3x^2y)\times 4zy^2x\times(-x^7z^5y^4)\times 2zxy$

$=(-3)\times 4\times(-1)\times 2\times x^2\times x\times x^7\times x\times y\times y^2\times y^4$
$\times y\times z\times z^5\times z$

$=\underline{24x^{11}y^8z^7}$

Exercise 18

The examples should be worked out mentally by the method illustrated in Art 6, Chap X

Exercise 19

1
$$\begin{array}{r} 2a-3b \\ -ab \\ \hline -2a^2b+3ab^2 \end{array}$$

2
$$\begin{array}{r} a-2b+3c \\ -5a \\ \hline -5a^2+10ab-15ac \end{array}$$

3
$$\begin{array}{r} 2a^2-3b^2-c^2 \\ abc \\ \hline 2a^3bc-3ab^3c-abc^3 \end{array}$$

4
$$\begin{array}{r} x^2y-2xy^2-y^3 \\ -3xy \\ \hline -3x^3y^2+6x^2y^3+3xy^4 \end{array}$$

5
$$\begin{array}{l} (-a^2b+ab^2-3a^2b^2+5a^3) \\ -7b^2 \\ \hline 7a^2b^3-7ab^4+21a^2b^4-35a^3b^2 \end{array}$$

6
$$\begin{array}{l} (2ab-3bc+4ac-5abc) \\ -8a^2bc \\ \hline -16a^3b^2c+24a^2b^2c^2-32a^3bc^2+40a^3b^2c^2 \end{array}$$

7
$$\begin{array}{r} 3a^2x-4ax^2+5ax \\ -2a^2 \\ \hline -6a^4x+8a^3x^2-10a^3x \end{array}$$

8
$$\begin{array}{r} -2m^3+3m^2n-5mn^2 \\ 4mn \\ \hline -8m^4n+12m^3n^2-20m^2n^3 \end{array}$$

9 $$\begin{array}{l} 5x^4-6x^3+7x^2-8x \\ -4x^3 \\ \hline -20x^7+24x^6-28x^5+32x^4 \end{array}$$

10 $$\begin{array}{l} -2c^2d+3d^3c-5cd^2-4c^2d^2 \\ -6c^2d^4 \\ \hline 12c^4d^5-18c^3d^7+30c^3d^6+24c^4d^6 \end{array}$$

11 $$\begin{array}{l} 8a^4-6a^3b+5a^2b^2-4ab^3 \\ -2a^3b^3 \\ \hline -16a^7b^3+12a^6b^4-10a^5b^5+8a^4b^6 \end{array}$$

12 $$\begin{array}{l} -a^2bc+ab^2c-abc^2+a^2b^2-b^2c^2 \\ -a^3b^3c^3 \\ \hline a^5b^4c^4-a^4b^5c^4+a^4b^4c^5-a^5b^5c^3+a^3b^5c^5 \end{array}$$

13 The given expression

$$\begin{aligned} &=7x^4-14x^3-2x^3+6x^2-8x^2+16x^3 \\ &=7x^4+(-14x^3-2x^3+16x^3)+(6x^2-8x^2) \\ &=\underline{7x^4-2x^2} \end{aligned}$$

14 The given expression

$$\begin{aligned} &=-5x^4-35x^3-21x^2+35x^3+21x^2-12x+17x \\ &=-5x^4+(-35x^3+35x^3)+(-21x^2+21x^2) \\ &\qquad\qquad +(-12x+17x) \\ &=\underline{-5x^4+5x} \end{aligned}$$

15 The given expression

$$\begin{aligned} &=9x^6-18x^3y^2+15x^3y^2+5y^4+3x^3y^2-30y^4 \\ &=9x^6+(-18x^3y^2+15x^3y^2+3x^3y^2)+(5y^4-30y^4) \\ &=\underline{9x^6-25y^4} \end{aligned}$$

16 The given expression

$$\begin{aligned} &=x^6+2x^5+2x^4-2x^5-4x^4-4x^3+2x^4+4x^3+4x^2 \\ &=\underline{x^6+4x^2} \end{aligned}$$

17 The given expression

$$\begin{aligned} &=a^{12}b^6-2a^{10}b^5+2a^8b^4+2a^{10}b^5-4a^8b^4 \\ &\qquad +4a^6b^3+2a^8b^4-4a^6b^3+4a^4b^2 \\ &=\underline{a^{12}b^6+4a^4b^2} \end{aligned}$$

18 The given expression

$$= 4a^{18}b^{12} + 12a^{15}b^{10} + 18a^{12}b^8 - 12a^{15}b^{10}$$
$$- 36a^{12}b^8 - 54a^9b^6 + 18a^{12}b^8 + 54a^9b^6 + 81a^6b^4$$
$$= 4a^{18}b^{12} + 81a^6b^4$$

19

$$a^2(2x - 3y) + a^2(3x + 4y) - a^2(5x - 2y)$$
$$= a^2\{(2x - 3y) + (3x + 4y) - (5x - 2y)\}$$
$$= a^2\{2x - 3y + 3x + 4y - 5x + 2y\}$$
$$= a^2(3y) = 3a^2y$$

20 The given expression

$$= 2ab\{(3a^2 - 4b^2) - (2a^2 - 3b^2) - (2a^2 - b^2)\}$$
$$= 2ab\{3a^2 - 4b^2 - 2a^2 + 3b^2 - 2a^2 + b^2\}$$
$$= 2ab(-a^2) = \underline{-2a^3b}$$

21 The given expression

$$= x^2\{(2ax + 3bx + 4cx) - (ax + 2bx + 3cx) - (bx + cx)\}$$
$$= x^2\{2ax + 3bx + 4cx - ax - 2bx - 3cx - bx - cx\}$$
$$= x^2(ax) = \underline{ax^3}$$

Exercise 20

1 $(2a + 3b)(a + b) = 2a \cdot a + 3b \cdot a + 2a \cdot b + 3b \cdot b$

$$= 2a^2 + 3ab + 2ab + 3b^2 = 2a^2 + 5ab + 3b^2$$

2 $(2m - 3n)(m - n) = 2m \cdot m - 3n \cdot m - 2m \cdot n + (-3n)(-n)$

$$= 2m^2 - 3mn - 2mn + 3n^2 = 2m^2 - 5mn + 3n^2$$

3 $(a + b + c)(a + b + c)$

$$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$$
$$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

4 $(a - b + c)(a - b + c)$

$$= a^2 - ab + ac - ab + b^2 - bc + ac - bc + c^2$$
$$= a^2 + b^2 - c^2 - 2ab + 2ac - 2bc$$

5 $(a - b - c)(a - b - c)$

$$= a^2 - ab - ac - ab + b^2 + bc - ac + bc + c^2$$
$$= a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$$

6 $(a-2b-3c)(2a-b-c)$
$=2a^2-4ab-6ac-ab+2b^2+3bc-ac+2bc+3c^2$
$=2a^2+2b^2+3c^2-5ab-7ac+5bc$

7 $(2x-3y-4z)(x-y-z)$
$=2x^2-3xy-4xz-2xy+3y^2+4yz-2xz+3yz+4z^2$
$=2x^2+3y^2+4z^2-5xy-6xz+7yz$

8 $(-5x+2a-3b)(-x-a+b)$
$=5x^2-2ax+3bx+5ax-2a^2+3ab-5bx+2ab-3b^2$
$=5x^2-2a^2-3b^2+3ax-2bx+5ab$

9 $(x^2+y^2+z^2)(x-y-z)$
$=x^3+xy^2+xz^2-x^2y-y^3-yz^2-x^2z-y^2z-z^3$
$=x^3-y^3-z^3-x^2y-x^2z+xy^2-y^2z+xz^2-yz^2$

10 $(xy+yz+zx)(xy-yz-zx)$
$=x^2y^2+xy^2z+x^2yz-xy^2z-y^2z^2-xyz^2-x^2yz-xyz^2-x^2z^2$
$=x^2y^2-y^2z^2-x^2z^2-2xyz^2$

Exercise 21

1
$$\begin{array}{rrr}
x^2 & -x & +1 \\
x^2 & +x & +1 \\
\hline
x^4-x^3+x^2 & & \\
+x^3-x^2+x & & \\
+x^2-x+1 & & \\
\hline
x^4 \quad +x^2 & & +1
\end{array}$$

2
$$\begin{array}{l}
a^2-ab+b^2 \\
a+b \\
\hline
a^3-a^2b+ab^2 \\
\quad +a^2b-ab^2+b^3 \\
\hline
a^3 \qquad\qquad +b^3
\end{array}$$

3
$$\begin{array}{l}
a^2+ab+b^2 \\
a-b \\
\hline
a^3+a^2b+ab^2 \\
\quad -a^2b-ab^2-b^3 \\
\hline
a^3 \qquad\qquad -b^3
\end{array}$$

4
$$\begin{array}{l}
a^2-2ab+b^2 \\
a^2+2ab+b^2 \\
\hline
a^4-2a^3b+a^2b^2 \\
\quad +2a^3b-4a^2b^2+2ab^3 \\
\qquad +a^2b^2-2ab^3+b^4 \\
\hline
a^4 \qquad -2a^2b^2 \qquad +b^4
\end{array}$$

5
$$\begin{array}{l}
x^4+x^2+1 \\
x^4-x^2+1 \\
\hline
x^8+x^6+x^4 \\
\quad -x^6-x^4-x^2 \\
\qquad +x^4+x^2+1 \\
\hline
x^8 \quad +x^4 \quad +1
\end{array}$$

6
$$\begin{array}{l}
x^3-x^2y^2+y^3 \\
x^3+x^2y^2+y^3 \\
\hline
x^6-x^5y^2 \qquad +x^3y^3 \\
\quad -x^5y^2-x^4y^4 \qquad +x^2y^5 \\
\qquad +x^3y^3-x^2y^5+y^6 \\
\hline
x^6 \qquad -x^4y^4+2x^3y^3 \qquad +y^6.
\end{array}$$

7
$$\begin{array}{l}
m^4-m^2n^2+n^4\\
\underline{m^2+n^2}\\
m^6-m^4n^2+m^2n^4\\
\underline{\quad +m^4n^2-m^2n^4+n^6}\\
m^6 \qquad\qquad\qquad +n^6
\end{array}$$

8
$$\begin{array}{l}
p^4+p^2q^2+q^4\\
\underline{p^2-q^2}\\
p^6+p^4q^2+p^2q^4\\
\underline{\quad -p^4q^2-p^2q^4-q^6}\\
p^6 \qquad\qquad\qquad -q^6
\end{array}$$

9
$$\begin{array}{l}
9a^2+30ab+25b^2\\
\underline{3a-5b}\\
27a^3+90a^2b+75ab^2\\
\underline{\quad -45a^2b-150ab^2-125b^3}\\
27a^3+45a^2b-75ab^2-125b^3
\end{array}$$

10
$$\begin{array}{l}
a^3-6a^2b+5ab^2\\
\underline{a^2+6ab+5b^2}\\
a^5-6a^4b+5a^3b^2\\
\quad +6a^4b-36a^3b^2+30a^2b^3\\
\underline{\qquad +5a^3b^2-30a^2b^3+25ab^4}\\
a^5 \qquad -26a^3b^2 \qquad +25ab^4
\end{array}$$

11
$$\begin{array}{l}
2a-3b+4c\\
\underline{2a+3b-4c}\\
4a^2-6ab+8ac\\
\quad +6ab \qquad -9b^2+12bc\\
\underline{\qquad -8ac \qquad +12bc-16c^2}\\
4a^2 \qquad -9b^2+24bc-16c^2
\end{array}$$

12
$$\begin{array}{l}
x^3-3x^2+3x-1\\
\underline{x^2+3x+1}\\
x^5-3x^4+3x^3-x^2\\
\quad +3x^4-9x^3+9x^2-3x\\
\underline{\qquad +x^3-3x^2+3x-1}\\
x^5 \qquad -5x^3+5x^2 \qquad -1
\end{array}$$

13
$$\begin{array}{l}
x^4+2ax^3+3a^2x^2+2a^3x+a^4\\
\underline{x^2-2ax+a^2}\\
x^6+2ax^5+3a^2x^4+2a^3x^3+x^2a^4\\
\quad -2ax^5-4a^2x^4-6a^3x^3-4x^2a^4-2a^5x\\
\underline{\qquad +a^2x^4+2a^3x^3+3x^2a^4+2a^5x+a^6}\\
x^6 \qquad -2a^3x^3 \qquad +a^6
\end{array}$$

14

$$\begin{array}{l}
a^3+3a^2b+3ab^2+b^3 \\
a^3-3a^2b+3ab^2-b^3 \\
\hline
a^6+3a^5b+3a^4b^2+a^3b^3 \\
\quad -3a^5b-9a^4b^2-9a^3b^3-3a^2b^4 \\
\qquad +3a^4b^2+9a^3b^3+9a^2b^4+3ab^5 \\
\qquad\quad -a^3b^3-3a^2b^4-3ab^5-b^6 \\
\hline
a^6 \quad -3a^4b^2 \quad +3a^2b^4 \quad -b^6
\end{array}$$

15

$$\begin{array}{l}
x^4+2x^3+x^2-4x-11 \\
x^2-2x+3 \\
\hline
x^6+2x^5+x^4-4x^3-11x^2 \\
\quad -2x^5-4x^4-2x^3+8x^2+22x \\
\qquad +3x^4+6x^3+3x^2-12x-33 \\
\hline
x^6 \qquad\qquad +10x-33
\end{array}$$

16

$$\begin{array}{l}
x^4+2x^3+3x^2+2x+1 \\
x^2-2x+1 \\
\hline
x^6+2x^5+3x^4+2x^3+x^2 \\
\quad -2x^5-4x^4-6x^3-4x^2-2x \\
\qquad +x^4+2x^3+3x^2+2x+1 \\
\hline
x^6 \qquad -2x^3 \qquad +1
\end{array}$$

17

$$\begin{array}{l}
a^4+a^3b+a^2b^2+ab^3+b^4 \\
a^4-a^3b+a^2b^2-ab^3+b^4 \\
\hline
a^8+a^7b+a^6b^2+a^5b^3+a^4b^4 \\
\quad -a^7b-a^6b^2-a^5b^3-a^4b^4-a^3b^5 \\
\qquad +a^6b^2+a^5b^3+a^4b^4+a^3b^5+a^2b^6 \\
\qquad\quad -a^5b^3-a^4b^4-a^3b^5-a^2b^6-ab^7 \\
\qquad\qquad +a^4b^4+a^3b^5+a^2b^6+ab^7+b^8 \\
\hline
a^8 \quad +a^6b^2 \quad +a^4b^4 \quad +a^2b^6 \quad +b^8
\end{array}$$

18

$$\begin{array}{l}
x^2-xy-xz+y^2-yz+z^2 \\
x+y+z \\
\hline
x^3-x^2y-x^2z+xy^2-xyz+xz^2 \\
\quad +x^2y \qquad -xy^2-xyz \qquad +y^3-y^2z+yz^2 \\
\qquad +x^2z \qquad -xyz-xz^2 \qquad +y^2z-yz^2+z^3 \\
\hline
x^3 \qquad\qquad -3xyz \quad +y^3 \qquad +z^3
\end{array}$$

19

$$\begin{array}{llllllllll}
a^2 & -ab & -ac & +b^2 & -bc & +c^2 & & & & \\
a & +b & +c & & & & & & & \\
\hline
a^3 & -a^2b & -a^2c & +ab^2 & -abc & +ac^2 & & & & \\
 & +a^2b & & -ab^2 & -abc & & +b^3 & -b^2c & +bc^2 & \\
 & & +a^2c & & -abc & -ac^2 & & +b^2c & -bc^2 & +c^3 \\
\hline
a^3 & & & & -3abc & & +b^3 & & & +c^3
\end{array}$$

20

$$\begin{array}{lllllll}
2a^3 & +5a^2b & -3ab^2 & +4b^3 & & & \\
a^3 & -3a^2b & +2ab^2 & -5b^3 & & & \\
\hline
2a^6 & +5a^5b & -3a^4b^2 & +4a^3b^3 & & & \\
 & -6a^5b & -15a^4b^2 & +9a^3b^3 & -12a^2b^4 & & \\
 & & +4a^4b^2 & +10a^3b^3 & -6a^2b^4 & +8ab^5 & \\
 & & & -10a^3b^3 & -25a^2b^4 & +15ab^5 & -20b^6 \\
\hline
2a^6 & -a^5b & -14a^4b^2 & +13a^3b^3 & -43a^2b^4 & +23ab^5 & -20b^6.
\end{array}$$

21

$$\begin{array}{lll}
a & +b & \\
a & -b & \\
a^2 & +ab & \\
 & -ab & -b^2 \\
\hline
a^2 & & -b^2
\end{array}
\qquad
\begin{array}{lll}
a^2 & -b^2 & \\
a^2 & +b^2 & \\
\hline
a^4 & -a^2b^2 & \\
 & +a^2b^2 & -b^4 \\
\hline
a^4 & & -b^4
\end{array}$$

22

$$\begin{array}{lll}
a & +b & \\
a & -b & \\
\hline
a^2 & +ab & \\
 & -ab & -b^2 \\
\hline
a^2 & & -b^2
\end{array}
\qquad
\begin{array}{llll}
a^4 & +a^2b^2 & +b^4 & \\
a^2 & -b^2 & & \\
\hline
a^6 & +a^4b^2 & +a^2b^4 & \\
 & -a^4b^2 & -a^2b^4 & -b^6 \\
\hline
a^6 & & & -b^6
\end{array}$$

23

$$\begin{array}{lll}
x & -y & \\
x & +y & \\
\hline
x^2 & -xy & \\
 & +xy & -y^2 \\
\hline
x^2 & & -y^2 \\
x^2 & & +y^2 \\
\hline
x^4 & -x^2y^2 & \\
 & +x^2y^2 & -y^4 \\
\hline
x^4 & & -y^4
\end{array}
\qquad
\begin{array}{llll}
x^8 & +x^4y^4 & +y^8 & \\
x^4 & -y^4 & & \\
\hline
x^{12} & +x^8y^4 & +x^4y^8 & \\
 & -x^8y^4 & -x^4y^8 & -y^{12} \\
\hline
x^{12} & & & -y^{12}
\end{array}$$

24

$$\begin{array}{l}
x^2+3xy+5y^2 \\
\underline{x^2-3xy+5y^2} \\
x^4+3x^3y+5x^2y^2 \\
\quad -3x^3y-9x^2y^2-15xy^3 \\
\quad\quad +5x^2y^2+15xy^3+25y^4 \\
\hline
x^4 \quad +x^2y^2 \quad +25y^4
\end{array}
\qquad
\begin{array}{l}
x^4+x^2y^2+25y^4 \\
\underline{x^4-x^2y^2+y^4} \\
x^8+x^6y^2+25x^4y^4 \\
\quad -x^6y^2-x^4y^4 \quad -25x^2y^6 \\
\quad\quad +x^4y^4 \quad +x^2y^6 \quad +25y^8 \\
\hline
x^8 \quad +25x^4y^4 \quad -24x^2y^6+25y^8
\end{array}$$

25

$$\begin{array}{l}
a-b \\
\underline{a+b} \\
a^2-ab \\
\quad +ab-b^2 \\
\hline
a^2 \quad -b^2
\end{array}
\qquad
\begin{array}{l}
a^4+a^2b^2+b^4 \\
\underline{a^2- \quad b^2} \\
a^6+a^4b^2+a^2b^4 \\
\quad -a^4b^2-a^2b^4-b^6 \\
\hline
a^6 \quad\quad -b^6
\end{array}
\qquad
\begin{array}{l}
a^{12}+a^6b^6+b^{12} \\
\underline{a^6 - \quad b^6} \\
a^{18}+a^{12}b^6+a^6b^{12} \\
\quad -a^{12}b^6-a^6b^{12}-b^{18} \\
\hline
a^{18} \quad\quad -b^{18}
\end{array}$$

26

$$\begin{array}{l}
ax^2+bx-c \\
\underline{px \quad -q} \\
apx^3+bpx^2-cpx \\
\quad -aqx^2-bqx+cq \\
\hline
apx^3+(bp-aq)x^2-(cp+bq)x+cq
\end{array}$$

27

$$\begin{array}{l}
mx^2-nx-r \\
\underline{nx \quad -r} \\
mnx^3- \quad n^2x^2-nrx \\
\quad -mrx^2+nrx+r^2 \\
\hline
mnx^3-(n^2+mr)x^2+r^2
\end{array}$$

28

$$\begin{array}{l}
ax^2-bx+c \\
\underline{x^2-bx-c} \\
ax^4- \quad bx^3+ \quad cx^2 \\
\quad -abx^3+b^2x^2-bcx \\
\quad\quad -acx^2+bcx-c^2 \\
\hline
ax^4-(b+ab)x^3+(c+b^2-ac)x^2-c^2 \\
\quad\quad = ax^4-(1+a)bx^3+(c+b^2-ac)x^2-c^2
\end{array}$$

29

$$\begin{array}{l}
ax^3-bx^2+cx-d \\
\underline{bx^2-cx \quad +d} \\
abx^5-b^2x^4+bcx^3 \quad -bdx^2 \\
\quad -acx^4+bcx^3 \quad -c^2x^2 \quad +cdx \\
\quad\quad +adx^3-bdx^2+cdx-d^2 \\
\hline
abx^5-(b^2+ac)x^4+(2bc+ad)x^3-(2bd+c^2)x^2+2cdx-d^2
\end{array}$$

30
$$\begin{array}{l}
px^2-(q-r)x+s \\
mx^2-nx-s \\
\hline
mpx^4-m(q-r)x^3+msx^2 \\
\qquad -npx^3 \qquad +n(q-r)x^2-nsx \\
\qquad\qquad\qquad -psx^2 \qquad +(q-r)sx-s^2 \\
\hline
mpx^4-(mq-mr+np)x^3+(ms+nq-nr-ps)x^2+(q-r-n)sx-s^2
\end{array}$$

31
$$\begin{array}{l}
x+a \\
x+b \\
\hline
x^2+ax \\
\quad +bx+ab \\
\hline
x^2+(a+b)x+ab
\end{array}
\qquad
\begin{array}{l}
x^2+(a+b)x+ab \\
x+c \\
\hline
x^3+(a+b)x^2+abx \\
\quad +cx^2+(a+b)cx+abc \\
\hline
x^3+(a+b+c)x^2+(ab+ac+bc)x+abc
\end{array}$$

32
$$\begin{array}{l}
x-a \\
x-b \\
\hline
x^2-ax \\
\quad -bx+ab \\
\hline
x^2-(a+b)x+ab
\end{array}
\qquad
\begin{array}{l}
x^2-(a+b)x+ab \\
x-c \\
\hline
x^3-(a+b)x^2+abx \\
\quad -cx^2 \qquad +(a+b)cx-abc \\
\hline
x^3-(a+b+c)x^2+(ab+ac+bc)x-abc
\end{array}$$

33 Worked out in the book

34 $x^{\frac13}\times x^{\frac23}=x^{\frac13+\frac23}=\underline{x}$

35 and 36 worked out in the book

37 $(x^{\frac23})^3=x^{\frac23}\times x^{\frac23}\times x^{\frac23}=x^{\frac23+\frac23+\frac23}=x^2$, $\qquad x^{\frac23}=\underline{\sqrt[3]{x^2}}$

38 $(y^{\frac45})^5=y^{\frac45}\times y^{\frac45}\times y^{\frac45}\times y^{\frac45}\times y^{\frac45}=y^{\frac45+\frac45+\frac45+\frac45+\frac45}=y^4$,

$y^{\frac45}=\underline{\sqrt[5]{y^4}}$

39 $(z^{\frac34})^4=z^{\frac34}\times z^{\frac34}\times z^{\frac34}\times z^{\frac34}=z^{\frac34+\frac34+\frac34+\frac34}=z^3$, $\qquad z^{\frac34}=\underline{\sqrt[4]{z^3}}$

40 $b^{\frac12}\times b^{\frac52}=b^{\frac12+\frac52}=\underline{b^3}$

41 $c^{\frac33}\times c^{\frac43}\times c^{\frac83}=c^{\frac{3+4+8}{3}}=c^{\frac{15}{3}}=\underline{c^5}$

42 $y^2\times y^{\frac32}\times y^{\frac72}=y^{2+\frac32+\frac72}=y^{\frac{4+3+7}{2}}=y^{\frac{14}{2}}=\underline{y^7}$

43 Worked out in the book

44 $y^4 \times y^{-7} = y^{4-7} = \underline{y^{-3}}$

45 $z^{\frac{3}{2}} \times z^{-\frac{1}{2}} = z^{\frac{3}{2}-\frac{1}{2}} = z^{\frac{3-1}{2}} = z^{\frac{2}{2}} = \underline{z}$

46 Worked out in the book

47 $\left(b^{-\frac{5}{3}}\right)^3 = b^{-\frac{5}{3}} \times b^{-\frac{5}{3}} \times b^{-\frac{5}{3}} = b^{-\frac{5}{3}-\frac{5}{3}-\frac{5}{3}} = b^{-\frac{15}{3}} b^{-5}$,

$b^{-\frac{5}{3}} = \underline{\sqrt[3]{b^{-5}}}$

48 $\left(c^{-\frac{2}{5}}\right)^5 = c^{-\frac{2}{5}} \times c^{-\frac{2}{5}} \times c^{-\frac{2}{5}} \times c^{-\frac{2}{5}} = c^{-\frac{2}{5}}$

$= c^{-\frac{2}{5}-\frac{2}{5}-\frac{2}{5}-\frac{2}{5}-\frac{2}{5}} = c^{-\frac{10}{5}} = c^{-2}$, $\quad c^{-\frac{2}{5}} = \underline{\sqrt[5]{c^{-2}}}$

49 $a^{-\frac{3}{4}} \times a^{-\frac{1}{4}} = a^{-\frac{3}{4}-\frac{1}{4}} = a^{-\frac{4}{4}} = \underline{a^{-1}}$

50 $x^{-\frac{5}{3}} \times x^{-\frac{4}{3}} = x^{-\frac{5}{3}-\frac{4}{3}} = x^{-\frac{9}{3}} = \underline{x^{-3}}$

51 $-3x^{\frac{1}{2}} \times 2x^{\frac{1}{2}} = -6x^{\frac{1}{2}+\frac{1}{2}} = -6x^{\frac{2}{2}} = \underline{-6x}$

52 $5y^{\frac{3}{2}} \times \left(-\frac{2}{5}y^{\frac{5}{2}}\right) = -2y^{\frac{3}{2}+\frac{5}{2}} = -2y^{\frac{8}{2}} = \underline{-2y^4}$

53 $2x^{\frac{1}{2}}y^{\frac{1}{2}} \times 3x^{\frac{3}{2}}y^{\frac{1}{2}} = 6x^{\frac{1}{2}+\frac{3}{2}}y^{\frac{1}{2}+\frac{1}{2}} = 6x^{\frac{4}{2}}y^{\frac{2}{2}} = \underline{6x^2y}$

54 $\left(-5xy^{\frac{3}{4}}\right) \times \left(-3x^{\frac{2}{3}}y^{\frac{1}{2}}\right) = 15x^{1+\frac{2}{3}}y^{\frac{3}{4}+\frac{1}{2}} = \underline{15x^{\frac{5}{3}}y^{\frac{5}{4}}}$

55 $4a^{-2}b^3 \times \left(-\frac{3}{4}a^3b^{-5}\right) = -3a^{3-2}b^{3-5} = \underline{-3ab^{-2}}$

56 $\frac{1}{5}a^{\frac{2}{5}}y^3 \times \left(-\frac{5}{1}a^{\frac{3}{5}}y^{-4}\right) = -a^{\frac{2}{5}+\frac{3}{5}}y^{3-4} = -a^{\frac{5}{5}}y^{-1} = \underline{-ay^{-1}}$

57 $\left(-4a^{\frac{1}{2}}b^{\frac{2}{3}}c^{\frac{3}{4}}\right) \times \left(-3a^{\frac{3}{2}}b^{\frac{4}{3}}c^{\frac{5}{4}}\right) = 12a^{\frac{1}{2}+\frac{3}{2}}b^{\frac{2}{3}+\frac{4}{3}}c^{\frac{3}{4}+\frac{5}{4}} = 12a^{\frac{4}{2}}b^{\frac{6}{3}}c^{\frac{8}{4}}$

$= \underline{12a^2b^2c^2}$

58 $\left(-5x^{\frac{2}{3}}y^{\frac{3}{5}}z^{\frac{4}{3}}\right) \times \left(-3x^{\frac{1}{3}}y^{\frac{2}{5}}z^{-\frac{1}{3}}\right)$

$= 15x^{\frac{2}{3}+\frac{1}{3}}y^{\frac{3}{5}+\frac{2}{5}}z^{\frac{4}{3}-\frac{1}{3}} = 15x^{\frac{3}{3}}y^{\frac{5}{5}}z^{\frac{3}{3}}$

$\underline{= 15xyz}$

59 $\left(-6a^{\frac{5}{3}}b^{-\frac{1}{4}}c^{-\frac{4}{5}}\right)\times\left(5a^{\frac{1}{3}}b^{\frac{5}{4}}c^{-\frac{1}{5}}\right)$

$$= -30a^{\frac{5}{3}+\frac{1}{3}}b^{-\frac{1}{4}+\frac{5}{4}}c^{-\frac{4}{5}-\frac{1}{5}}$$

$$= -30a^{\frac{6}{3}}b^{\frac{4}{4}}c^{-\frac{5}{5}} = -30a^{2}b^{+1}c^{-1}$$

60 $\left(-4a^{\frac{5}{6}}x^{-\frac{5}{6}}y^{-\frac{4}{5}}\right)\times\left(-19a^{\frac{1}{2}}x^{-\frac{1}{3}}y^{-\frac{6}{5}}\right)$

$$=76a^{\frac{5}{6}+\frac{1}{2}}x^{-\frac{5}{6}-\frac{1}{3}}y^{-\frac{4}{5}-\frac{6}{5}}$$

$$=76a^{\frac{8}{6}}x^{-\frac{7}{6}}y^{-\frac{10}{5}}$$

$$=\underline{76a^{\frac{4}{3}}x^{-\frac{7}{6}}y^{-2}}$$

61
$$\begin{array}{l} a^{\frac{1}{2}}+b^{\frac{1}{2}} \\ \underline{a^{\frac{1}{2}}+b^{\frac{1}{2}}} \\ a+a^{\frac{1}{2}}b^{\frac{1}{2}} \\ \underline{\quad +a^{\frac{1}{2}}b^{\frac{1}{2}}+b} \\ a+2a^{\frac{1}{2}}b^{\frac{1}{2}}+b \end{array}$$

62
$$\begin{array}{l} a^{\frac{1}{2}}-b^{\frac{1}{2}} \\ \underline{a^{\frac{1}{2}}-b^{\frac{1}{2}}} \\ a-a^{\frac{1}{2}}b^{\frac{1}{2}} \\ \underline{\quad -a^{\frac{1}{2}}b^{\frac{1}{2}}+b} \\ a-2a^{\frac{1}{2}}b^{\frac{1}{2}}+b \end{array}$$

63
$$\begin{array}{l} 3x^{\frac{2}{3}}-4y^{\frac{1}{3}} \\ \underline{3x^{\frac{2}{3}}+4y^{\frac{1}{3}}} \\ 9x^{\frac{4}{3}}-12x^{\frac{2}{3}}y^{\frac{1}{3}} \\ \underline{\quad +12x^{\frac{2}{3}}y^{\frac{1}{3}}-16y^{\frac{2}{3}}} \\ 9x^{\frac{4}{3}} \qquad -16y^{\frac{2}{3}} \end{array}$$

64
$$\begin{array}{l} a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}} \\ \underline{a^{\frac{1}{3}}+b^{\frac{1}{3}}} \\ a-a^{\frac{2}{3}}b^{\frac{1}{3}}+a^{\frac{1}{3}}b^{\frac{2}{3}} \\ \underline{\quad +a^{\frac{2}{3}}b^{\frac{1}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b} \\ a \qquad +b \end{array}$$

65
$$\begin{array}{l} x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}} \\ \underline{x^{\frac{1}{3}}-y^{\frac{1}{3}}} \\ x+x^{\frac{2}{3}}y^{\frac{1}{3}}+x^{\frac{1}{3}}y^{\frac{2}{3}} \\ \underline{\quad -x^{\frac{2}{3}}y^{\frac{1}{3}}-x^{\frac{1}{3}}y^{\frac{2}{3}}-y} \\ x \qquad -y \end{array}$$

66
$$\begin{array}{l} a^{\frac{3}{2}}-a^{\frac{3}{4}}b^{\frac{3}{4}}+b^{\frac{3}{2}} \\ \underline{a^{\frac{3}{2}}+a^{\frac{3}{4}}b^{\frac{3}{4}}+b^{\frac{3}{2}}} \\ a^{3}-a^{\frac{9}{4}}b^{\frac{3}{4}}+a^{\frac{3}{2}}b^{\frac{3}{2}} \\ \quad +a^{\frac{9}{4}}b^{\frac{3}{4}}-a^{\frac{3}{2}}b^{\frac{3}{2}}+a^{\frac{3}{4}}b^{\frac{9}{4}} \\ \underline{\qquad +a^{\frac{3}{2}}b^{\frac{3}{2}}-a^{\frac{3}{4}}b^{\frac{9}{4}}+b^{3}} \\ a^{3} \qquad +a^{\frac{3}{2}}b^{\frac{3}{2}} \qquad +b^{3} \end{array}$$

67 $$2x^{\frac43} - 5x^{\frac23}y^{\frac23} - 3y^{\frac43}$$
$$2x^{\frac43} + 5x^{\frac23}y^{\frac23} - 3y^{\frac43}$$
$$4x^{\frac83} - 10x^2y^{\frac23} - 6x^{\frac43}y^{\frac43}$$
$$+10x^2y^{\frac23} - 25x^{\frac43}y^{\frac43} - 15x^{\frac23}y^2$$
$$-6x^{\frac43}y^{\frac43} + 15x^{\frac23}y^2 + 9y^{\frac83}$$
$$4x^{\frac83} \quad - 37x^{\frac43}y^{\frac43} \quad + 9y^{\frac83}$$

68 $$a^{\frac52} + a^2b^{\frac13} + a^{\frac32}b^{\frac23} + ab + a^{\frac12}b^{\frac43} + b^{\frac53}$$
$$a^{\frac12} - b^{\frac13}$$
$$a^3 + a^{\frac52}b^{\frac13} + a^2b^{\frac23} + a^{\frac32}b + ab^{\frac43} + a^{\frac12}b^{\frac53}$$
$$-a^{\frac52}b^{\frac13} - a^2b^{\frac23} - a^{\frac32}b - ab^{\frac43} - a^{\frac12}b^{\frac53} - b^2$$
$$a^3 \quad - b^2$$

69 $$x^{\frac32} - xy^{\frac12} + x^{\frac12}y - y^{\frac32}$$
$$x^{\frac12} + y^{\frac12}$$
$$x^2 - x^{\frac32}y^{\frac12} + xy - x^{\frac12}y^{\frac32}$$
$$-x^{\frac32}y^{\frac12} - xy + x^{\frac12}y^{\frac32} - y^2$$
$$x^2 \quad - y^2$$

70 $$a^{\frac34} + a^{\frac12}b^{\frac12} + a^{\frac14}b + b^{\frac32}$$
$$a^{\frac14} - b^{\frac12}$$
$$a + a^{\frac34}b^{\frac12} + a^{\frac12}b + a^{\frac14}b^{\frac32}$$
$$-a^{\frac34}b^{\frac12} - a^{\frac12}b - a^{\frac14}b^{\frac32} - b^2$$
$$a \quad - b^2$$

71 $$x^{\frac23} - x^{\frac13}y^{\frac13} - x^{\frac13}z^{\frac13} + y^{\frac23} - y^{\frac13}z^{\frac13} + z^{\frac23}$$
$$x^{\frac13} + y^{\frac13} + z^{\frac13}$$
$$x - x^{\frac23}y^{\frac13} - x^{\frac23}z^{\frac13} + x^{\frac13}y^{\frac23} - x^{\frac13}y^{\frac13}z^{\frac13} + x^{\frac13}z^{\frac23}$$
$$-x^{\frac23}y^{\frac13} \quad - x^{\frac13}y^{\frac23} - x^{\frac13}y^{\frac13}z^{\frac13} \quad + y - y^{\frac23}z^{\frac13} + y^{\frac13}z^{\frac23}$$
$$+x^{\frac23}z^{\frac13} \quad - x^{\frac13}y^{\frac13}z^{\frac13} - x^{\frac13}z^{\frac23} \quad + y^{\frac23}z^{\frac13} - y^{\frac13}z^{\frac23} + z$$
$$x \quad - 3x^{\frac13}y^{\frac13}z^{\frac13} \quad + y \quad + z$$

72
$$\begin{array}{l}
a^{2n} - a^n x^n + x^{2n} \\
\underline{a^n + x^n} \\
a^{3n} - a^{2n}x^n + a^n x^{2n} \\
\qquad + a^{2n}x^n - a^n x^{2n} + x^{3n} \\
\hline
a^{3n} \qquad\qquad\qquad + x^{3n}
\end{array}$$

73
$$\begin{array}{l}
a^{-3} - 4a^{-2}b + 4a^{-1}b^2 - b^3 \\
\underline{a^{-2} - 2a^{-1}b + b^2} \\
a^{-5} - 4a^{-4}b + 4a^{-3}b^2 - a^{-2}b^3 \\
\qquad - 2a^{-4}b + 8a^{-3}b^2 - 8a^{-2}b^3 + 2a^{-1}b^4 \\
\qquad\qquad + a^{-3}b^2 - 4a^{-2}b^3 + 4a^{-1}b^4 - b^5 \\
\hline
a^{-5} - 6a^{-4}b + 13a^{-3}b^2 - 13a^{-2}b^3 + 6a^{-1}b^4 - b^5
\end{array}$$

74
$$\begin{array}{l}
x^{-3} + 3x^{-\frac{3}{2}}y^{\frac{3}{2}} + 2y^3 \\
\underline{x^{-3} - 3x^{-\frac{3}{2}}y^{\frac{3}{2}} + 2y^3} \\
x^{-6} + 3x^{-\frac{9}{2}}y^{\frac{3}{2}} + 2x^{-3}y^3 \\
\qquad - 3x^{-\frac{9}{2}}y^{\frac{3}{2}} - 9x^{-3}y^3 - 6x^{-\frac{3}{2}}y^{\frac{9}{2}} \\
\qquad\qquad + 2x^{-3}y^3 + 6x^{-\frac{3}{2}}y^{\frac{9}{2}} + 4y^6 \\
\hline
x^{-6} \qquad - 5x^{-3}y^3 \qquad + 4y^6
\end{array}$$

75
$$\begin{array}{l}
2a^{-5} + 3a^{-\frac{5}{2}}b^{-\frac{3}{2}} - 5b^{-3} \\
\underline{2a^{-5} + 3a^{-\frac{5}{2}}b^{-\frac{3}{2}} + 5b^{-3}} \\
4a^{-10} + 6a^{-\frac{15}{2}}b^{-\frac{3}{2}} - 10a^{-5}b^{-3} \\
\qquad + 6a^{-\frac{15}{2}}b^{-\frac{3}{2}} + 9a^{-5}b^{-3} - 15a^{-\frac{5}{2}}b^{-\frac{9}{2}} \\
\qquad\qquad + 10a^{-5}b^{-3} + 15a^{-\frac{5}{2}}b^{-\frac{9}{2}} - 25b^{-6} \\
\hline
4a^{-10} + 12a^{-\frac{15}{2}}b^{-\frac{3}{2}} + 9a^{-5}b^{-3} \qquad - 25b^{-6}
\end{array}$$

76
$$\begin{array}{l}
3x^{-\frac{5}{4}} - 5x^{-\frac{5}{8}}y^{-\frac{3}{8}} + 7y^{-\frac{3}{4}} \\
\underline{3x^{-\frac{5}{4}} + 5x^{-\frac{5}{8}}y^{-\frac{3}{8}} - 7y^{-\frac{3}{4}}} \\
9x^{-\frac{5}{2}} - 15x^{-\frac{15}{8}}y^{-\frac{3}{8}} + 21x^{-\frac{5}{4}}y^{-\frac{3}{4}} \\
\qquad + 15x^{-\frac{15}{8}}y^{-\frac{3}{8}} - 25x^{-\frac{5}{4}}y^{-\frac{3}{4}} + 35x^{-\frac{5}{8}}y^{-\frac{9}{8}} \\
\qquad\qquad - 21x^{-\frac{5}{4}}y^{-\frac{3}{4}} + 35x^{-\frac{5}{8}}y^{-\frac{9}{8}} - 49y^{-\frac{3}{2}} \\
\hline
9x^{-\frac{5}{2}} \qquad - 25x^{-\frac{5}{4}}y^{-\frac{3}{4}} + 70x^{-\frac{5}{8}}y^{-\frac{9}{8}} - 49y^{-\frac{3}{2}}
\end{array}$$

77 The given expression

$$=2x^5-3x^{\frac{7}{2}}a+4x^2a^2-5x^{\frac{1}{2}}a^3+6x^{-1}a^4-7x^{-\frac{5}{2}}a^5$$
$$+8x^{-4}a^6$$

78 The given expression

$$=-3a^{-\frac{5}{6}}-2a^{-4}b^{-2}+5a^{-\frac{2}{1}}b^{-4}-7a^{-\frac{5}{8}}b^{-7}+6a^{\frac{5}{4}}b^{-4}$$
$$-5a^{\frac{1}{2}}b^{-4}+8a^{\frac{5}{4}}b^{-2}$$

Exercise 22

1 $(3x+4)^2=(3x)^2+2(3x)4+4^2$
$= 9x^2+24x+16$

2 $(7x+8)^2=(7x)^2+2(7x)8+8^2$
$=49x^2+112x+64$

3 $(a+5b)^2=a^2+2(a)(5b)+(5b)^2$
$=a^2+10ab+25b^2$

4 $(2b+7b)^2=(2a)^2+2(2a)(7b)+(7b)^2$
$=4a^2+28ab+49b^2$

5 $(3x+8y)^2=(3x)^2+2(3x)(8y)+(8y)^2$
$=9x^2+48xy+64y^2$

6 $(5m+8n)^2=(5m)^2+2(5m)(8n)+(8n)^2$
$=25m^2+80mn+64n^2$

7 $(ax+3by)^2=(ax)^2+2(ax)(3by)+(3by)^2$
$=a^2x^2+6abxy+9b^2y^2$

8 $(4ab+c^2)^2=(4ab)^2+2(4ab)c^2+(c^2)^2$
$=16a^2b^2+8abc^2+c^4$

9 $(2a^2+5b^2)^2=(2a^2)^2+2(2a^2)(5b^2)+(5b^2)^2$
$=4a^4+20a^2b^2+25b^4$

10 $(4m^3+n^2)^2=(4m^3)^2+2(4m^3)(n^2)+(n^2)^2$
$=16m^6+8m^3n^2+n^4$

11 $(a+2b+3c)^2=\{(a+2b)+3c\}^2$
$$=(a+2b)^2+2\cdot 3c\,(a+2b)+(3c)^2$$
$$=a^2+2\cdot a\cdot 2b+4b^2+6ac+12bc+9c^2$$
$$=a^2+4b^2+c^2+4ab+6ac+12bc$$

12 $(ab+bc+ca)^2=\{(ab+bc)+ca\}^2$
$$=(ab+bc)^2+2(ab+bc)ca+(ca)^2$$
$$=a^2b^2+2ab^2c+b^2c^2+2a^2bc+2abc^2+c^2a^2$$
$$=a^2b^2+b^2c^2+c^2a^2+2a^2bc+2ab^2c+2abc^2$$

13 $(2p+3q+4r)^2=\{(2p+3q)+4r\}^2$
$$=(2p+3q)^2+2\,(2p+3q)\,4r+(4r)^2$$
$$=4p^2+12pq+9q^2+16pr+24qr+16r^2$$
$$=4p^2+9q^2+16r^2+12pq+16pr+24qr$$

14 $(x^2+y^2+z^2)^2=\{(x^2+y^2)+z^2\}^2$
$$=(x^2+y^2)^2+2(x^2+y^2)z^2+(z^2)^2$$
$$=x^4+2x^2y^2+y^4+2x^2z^2+2y^2z^2+z^4$$
$$=x^4+y^4+z^4+2x^2y^2+2x^2z^2+2y^2z^2$$

15 $(2x+3y+4z)^2=\{(2x+3y)+4z\}^2$
$$=(2x+3y)^2+2\,(2x+3y)\,4z+(4z)^2$$
$$=4x^2+12xy+9y^2+16xz+24yz+16z^2$$
$$=4x^2+9y^2+16z^2+12xy+16xz+24yz$$

16 $(x^2+y^3+z^4)^2=\{(x^2+y^3)+z^4\}^2$
$$=(x^2+y^3)^2+2(x^2+y^3)z^4+(z^4)^2$$
$$=x^4+2x^2y^3+y^6+2x^2z^4+2y^3z^4+z^8$$
$$=x^4+y^6+z^8+2x^2y^3+2x^2z^4+2y^3z^4$$

17 $(x+y+2a+3b)^2=\{(x+y)+(2a+3b)\}^2$
$$=(x+y)^2+2(x+y)(2a+3b)+(2a+3b)^2$$
$$=x^2+y^2+2xy+2(2ax+2ay+3bx+3by)+4a^2+9b^2+12ab$$
$$=x^2+y^2+2xy+4ax+4ay+6bx+6by+4a^2+9b^2+12ab$$
$$=x^2+y^2+4a^2+9b^2+2xy+4ax+6bx+4ay+6by+12ab$$

18 $(3a+4b+c+2d)^2=\{(3a+4b)+(c+2d)\}^2$
$$=(3a+4b)^2+2(3a+4b)(c+2d)+(c+2d)^2$$
$$=9a^2+24ab+16b^2+2(3ac+4bc+6ad+8bd)+c^2+4cd+4d^2$$
$$=9a^2+16b^2+c^2+4d^2+24ab+6ac+12ad+8bc+16bd+4cd$$

19 $(2a+x+4y+3z)^2=\{(2a+x)+(4y+3z)\}^2$
$=(2a+x)^2+2(2a+x)(4y+3z)+(4y+3z)^2$
$=4a^2+4ax+x^2+2(8ay+6az+4xy+3xz)+16y^2+24yz+9z^2$
$=4a^2+4ax+x^2+16ay+12az+8xy+6xz+16y^2+24yz+9z^2$
$=4a^2+x^2+16y^2+9z^2+4ax+16ay+12az+8xy+6xz+24yz$

20 $(4m+3n+3p+2q)^2$
$=\{(4m+3n)+(3p+2q)\}^2$
$=(4m+3n)^2+2(4m+3n)(3p+2q)+(3p+2q)^2$
$=16m^2+24mn+9n^2+2(12pm+9pn+8qm+6qn)$
$\qquad+9p^2+12pq+4q^2$
$=16m^2+24mn+9n^2+24pm+18pn+16qm+12qn$
$\qquad+9p^2+12pq+4q^2$
$=16m^2+9n^2+9p^2+4q^2+24mn+24pm+16qm+18np$
$\qquad+12nq+12pq$

21 Putting a for $(x+y)$ and b for $(x-y)$, we have the given expression $=a^2+2ab+b^2=(a+b)^2$
$=\{(x+y)+(x-y)\}^2=(2x)^2=\underline{4x^2}$

22 Putting a for $(x-y+z)$ and b for $(y+z-x)$, we have the given expression $=a^2+b^2+2ab=(a+b)^2$
$=\{(x-y+z)+(y+z-x)\}^2$
$=(2z)^2=\underline{4z^2}$

23 Putting x for $(2a-3b+4c)$ and y for $(2a+3b-4c)$, we have the given expression $=x^2+y^2+2xy=(x+y)^2$
$=\{(2a-3b+4c)+(2a+3b-4c)\}^2$
$=(4a)^2=\underline{16a^2}$

24 Putting x for $(5a-7b)$ and y for $(9b-4a)$ we have the given expression $=x^2+2xy+y^2=(x+y)^2$
$=\{(5a-7b)+(9b-4a)\}^2=(a+2b)^2$
$=\underline{a^2+4ab+4b^2}$

25 Putting a for $(2x-5y-3z)$ and b for $(6y+3z-x)$ we have the given expression $=a^2+b^2+2ab=(a+b)^2$
$=\{2x-5y-3z+6y+3z-x\}^2$
$=(x+y)^2=\underline{x^2+2xy+y^2}$

23 The given expression $=(2x)^2+2(2x)7+7^2$
$=(2x+7)^2=(-16+7)^2=(-9)^2=81$

27 The given expression
$=(5a)^2+2(5a)(4b)+(4b)^2$
$=(5a+4b)^2=(-90+92)^2=(2)^2=4$

28 The given expression
$=(9x)^2+2(9x)(5y)+(5y)^2$
$=(9x+5y)^2=(135-135)^2=0$

29 The given expression
$=(4m)^2+2(4m)(7n)+(7n)^2$
$=(4m+7n)^2=(-52+49)^2=(3)^2=9$

30 The given expression
$=(8a)^2+2(8a)c+c^2$
$=(8a+c)^2=(48-49)^2=(-1)^2=1$

31 The given expression
$=(9x)^2+2(9x)z+z^2$
$=(9x+z)^2=(63-67)^2=(-4)^2=16$

32 The given expression
$=6p+2(6p)(11q)+(11q)^2$
$=(6p+11q)^2=(72-77)^2=(-5)^2=25$

33 $m^2+\left(\frac{1}{m}\right)^2=\left(m+\frac{1}{m}\right)^2-2m\frac{1}{m}=\left(m+\frac{1}{m}\right)^2-2$
$=(4)^2-2=16-2=14$

Exercise 23

1 $(2x-7)^2=(2x)^2-2(2x)7+(7)^2$
$=4x^2-28x+49$

2 $(8x-5)^2=(8x)^2-2(8x)5+(5)^2$
$=64x^2-80x+25$

3 $(ax-by)^2=(ax)^2-2(ax)(by)+(by)^2$
$=a^2x^2-2abxy+b^2y^2$

4 $(-5x - 3y)^2 = \{(-5x) - 3y\}^2$
$= (-5x)^2 - 2(-5x)(3y) + (3y)^2$
$= 25x^2 + 30xy + 9y^2$

5 $(3m - 8n)^2 = (3m)^2 - 2(3m)(8n) + (8n)^2$
$= 9m^2 - 48mn + 64n^2$

6 $(-ab - cd)^2 = \{(-ab) - cd\}^2$
$= (-ab)^2 - 2(-ab)cd + (cd)^2$
$= a^2b^2 + 2abcd + c^2d^2$

7 $(a^2b - c^2d)^2 = (a^2b)^2 - 2(a^2b)(c^2d) + (c^2d)^2 = a^4b^2 - 2a^2bc^2d + c^4d^2$

8 $(x^3 - 2yz)^2 = (x^3)^2 - 2(x^3)(2yz) + (2yz)^2 = x^6 - 4x^3yz + 4y^2z^2$

9 $(-mnp - qr)^2 = \{(-mnp) - qr\}^2$
$= (-mnp)^2 - 2(-mnp)(qr) + (qr)^2$
$= m^2n^2p^2 + 2mnpqr + q^2r^2$

10 $(2a^3 - 5b^3)^2 = (2a^3)^2 - 2(2a^3)(5b^3) + (5b^3)^2$
$= 4a^6 - 20a^3b^3 + 25b^6$

11 $(a - 2b - 3c)^2 = \{a - (2b + 3c)\}^2$
$= a^2 - 2a(2b + 3c) + (2b + 3c)^2$
$= a^2 - 4ab - 6ac + 4b^2 + 12bc + 9c^2$
$= a^2 + 4b^2 + 9c^2 - 4ab - 6ac + 12bc$

12 $(2x - 3y - 4z)^2 = \{2x - (3y + 4z)\}^2$
$= (2x)^2 - 2(2x)(3y + 4z) + (3y + 4z)^2$
$= 4x^2 - 12xy - 16xz + 9y^2 + 24yz + 16z^2$
$= 4x^2 + 9y^2 + 16z^2 - 12xy - 16xz + 24yz$

13 $(3m - 4n - 5q)^2 = \{3m - (4n + 5q)\}^2$
$= (3m)^2 - 2(3m)(4n + 5q) + (4n + 5q)^2$
$= 9m^2 - 24mn - 30mq + 16n^2 + 40nq + 25q^2$
$= 9m^2 + 16n^2 + 25q^2 - 24mn - 30mq + 40nq$

14 $(a^2 - 3b^2 - 5c^2)^2 = \{a^2 - (3b^2 + 5c^2)\}^2$
$= (a^2)^2 - 2a^2(3b^2 + 5c^2) + (3b^2 + 5c^2)^2$
$= a^4 - 6a^2b^2 - 10a^2c^2 + 9b^4 + 30b^2c^2 + 25c^4$
$= a^4 + 9b^4 + 25c^4 - 6a^2b^2 - 10a^2c^2 + 30b^2c^2$

15 $(x-y-a-b)^2=\{(x-y)-(a+b)\}^2$
$$=(x-y)^2-2(x-y)(a+b)+(a+b)^2$$
$$=x^2-2xy+y^2-2(ax+bx-ay-by)$$
$$+a^2+2ab+b^2$$
$$=x^2+y^2+a^2+b^2-2xy-2ax-2bx+2ay$$
$$+2by+2ab$$

16 $(a-2x-3b-4y)^2=\{(a-2x)-(3b+4y)\}^2$
$$=(a-2x)^2-2(a-2x)(3b+4y)+(3b+4y)^2$$
$$=a^2-4ax+4x^2-2(3ab+4ay-6bx-8xy)$$
$$+9b^2+24by+16y^2$$
$$=a^2+4x^2+9b^2+16y^2-4ax-6ab-8ay$$
$$+12bx+16xy+24by$$

17 $(90-1)^2=(90)^2-2\cdot 90+1=8100-180+1=7921$

18 $(120-3)^2=(120)^2-2(120)3+(3)^2=14400-720+9=13689$

19 $(500-2)^2=(500)^2-2(500)(2)+(2)^2=250000-2000+4$
$$=248004$$

20 $(1000-7)^2=(1000-7)^2=(1000)^2-2(1000)7+(7)^2$
$$=1000000-14000+49=986049$$

21 Putting x for $(a+3b)$ and y for $(a-3b)$ we have the given expression $=x^2-2xy+y^2=(x-y)^2=\{(a+3b)-(a-3b)\}^2$
$$=(6b)^2=\underline{36b^2}$$

22 Putting x for $(2a-4b+5c)$ and y for $(2a+4b+5c)$, we have the given expression $=x^2+y^2-2xy=(x-y)^2$
$$=\{(2a-4b+5c)-(2a+4b+5c)\}^2=(-8b)^2=\underline{64b^2}$$

23 Putting x for $(3a+5b+7c)$ and y for $(7c-4a+5b)$, we have the given expression $=x^2+y^2-2xy=(x-y)^2$
$$=\{(3a+5b+7c)-(7c-4a+5b)\}$$
$$=(7a)^2=\underline{49a^2}$$

24 Putting a for $(2x^2-y^2-5z^2)$ and b for $(6z^2+2x^2-y^2)$, we have the given expression $=a^2-2ab+b^2$
$$=(a-b)^2=\{(2x^2-y^2-5z^2)-(6z^2+2x^2-y^2)\}^2$$
$$=(-11z)^2=\underline{121z^4}$$

25 Putting x for $(ab-bc+ca)$ and y for $(ab+4bc+2ca)$, we have the given expression $=x^2+y^2-2xy=(x-y)^2$

$$=\{(ab-bc+ca)-(ab+4bc+2ac)\}^2$$
$$=(-5bc-ac)^2=(-5bc)^2-2(-5bc)(ac)+(ac)^2$$
$$=\underline{25b^2c^2-10abc^2+a^2c^2}$$

26 The given expression

$$=(ab)^2-2(ab)(bc)+(bc)^2$$
$$=(ab-bc)^2=(28-30)^2=(-2)^2=4$$

27 The given expression

$$=(xy)^2-2(xy)(12z)+(12z)^2$$
$$=(xy-12z)^2=(63-72)^2=(-9)^2=81$$

28 The given expression

$$=\{5(x+y)\}^2+z^2-2(z)\{5(x+y)\}$$
$$=\{5(x+y)-z\}^2=\{5(47-22)-129\}^2$$
$$=(5\cdot 25-129\}^2=\{125-129\}^2=(-4)^2=16.$$

29 The given expression

$$=(3c)^2-2(3c)\{7(a+b)\}+\{7(a+b)\}^2$$
$$=\{3c-7(a+b)\}^2=\{135-7(-37+57)\}^2$$
$$=\{135-7\cdot 20\}^2=\{135-140\}^2=(-5)^2=25$$

30 The given expression

$$=\{8(7p-5q)\}^2-2\{8(7p-5q)\}(6r)+(6r)^2$$
$$=\{8(7p-5q)-6r\}^2=\{8(196-160)-276\}^2$$
$$=\{8\cdot 36-276\}=(288-276)^2=(12)^2=144$$

31 We have $c^2+\frac{1}{c^2}=\left(c-\frac{1}{c}\right)^2+2c\cdot\frac{1}{c}$

$$=\left(c-\frac{1}{c}\right)^2+2=(4)^2+2=16+2=18$$

Exercise 24

1 $(ax+by)(ax-by)=(ax)^2-(by)^2=a^2x^2-b^2y^2$

2 $(5x+8)(5x-8)=(5x)^2-(8)^2=25x^2-64$

3 $(cx+d^2)(cx-d^2)=(cx)^2-(d^2)^2=c^2x^2-d^4$

4 $(ab+bc)(ab-bc)=(ab)^2-(bc)^2=a^2b^2-b^2c^2$

5 $(7a+3b)(7a-3b)=(7a)^2-(3b)^2=49a^2-9b^2$

6 $(4a^2-5b^2)(4a^2+5b^2)=(4a^2)^2-(5b^2)^2=16a^4-25b^4$

7 $(m^2-3n)(m^2+3n)=(m^2)^2-(3n)^2=m^4-9n^2$

8 $(2ab-7bc)(2ab+7bc)=(2ab)^2-(7bc)^2=4a^2b^2-49b^2c^2$

9 $(x+1)(x-1)(x^2+1)=(x^2-1)(x^2+1)=x^4-1$

10 $(a^2+b^2)(a^2-b^2)(a^4+b^4)=(a^4-b^4)(a^4+b^4)=a^8-b^8$

11 $(a+b+c)(a+b-c)$
$=\{(a+b)+c\}\{(a+b)-c\}$
$=(a+b)^2-c^2=a^2+2ab+b^2-c^2$

12 $(a+b+c)(a-b-c)$
$=\{a+(b+c)\}\{a-(b+c)\}$
$=a^2-(b+c)^2=a^2-(b^2+2bc+c^2)$
$=a^2-b^2-2bc-c^2$

13 $(m^2+mn+n^2)(m^2-mn+n^2)$
$=\{(m^2+n^2)+mn\}\{(m^2+n^2)-mn\}$
$=m^4+2m^2n^2+n^4-m^2n^2=m^4+m^2n^2+n^4$

14 $(ax+by-cz)(ax-by+cz)$
$=\{ax+(by-cz)\}\{ax-(by-cz)\}$
$=a^2x^2-(by-cz)^2=a^2x^2-(b^2y^2-2bcyz+c^2z^2)$
$=a^2x^2-b^2y^2+2bcyz-c^2z^2$

15 $(a-2b+3c)(a+2b-3c)$
$=\{a-(2b-3c)\}\{a+(2b-3c)\}$
$=a^2-(2b-3c)^2=a^2-(4b^2-12bc+9c^2)$
$=a^2-4b^2+12bc-9c^2$

16 $(x^2-x+1)(x^2+x+1)$
$=\{(x^2+1)-x\}\{(x^2+1)+x\}$
$=(x^2+1)^2-x^2=x^4+2x^2+1-x^2=x^4+x^2+1$

17 $(x^4-x^2+1)(x^4+x^2+1)$
$=\{(x^4+1)-x^2\}\{(x^4+1)+x^2\}$
$=(x^4+1)^2-x^4=x^8+2x^4+1-x^4=x^8+x^4+1$

18 $(x^2+2xy+2y^2)(x^2-2xy+2y^2)$
$=(x^2+2y^2)^2-(2xy)^2$
$=x^4+4x^2y^2+4y^4-4x^2y^2=x^4+4y^4$

19 $(a^2+ab\sqrt{2}+b^2)(a^2-ab\sqrt{2}+b^2)$
$=\{(a^2+b^2)+ab\sqrt{2}\}\{(a^2+b^2)-ab\sqrt{2}\}$
$=(a^2+b^2)^2-(ab\sqrt{2})^2$
$=a^4+2a^2b^2+b^4-2a^2b^2=a^4+b^4$

20 $(x^2-2x+1)(x^2+2x+1)$
$=\{(x^2+1)^2-2x\}\{(x^2+1)+2x\}$
$=(x^2+1)^2-4x^2=(x^4+1)-2x^2$

Hence the given expression $=\{(x^4+1)-2x^2\}\{(x^4+1)+2x^2\}$
$=(x^4+1)^2-4x^4=x^8-2x^4+1$

21 $(a+b-c)^2-(a-b+c)^2$
$=(a+b-c+a-b+c)\{(a+b-c)-(a-b+c)\}$
$=2a\{a+b-c-a+b-c\}=2a(2b-2c)=4a(b-c)$

22 $(a-2b+3c)^2-(a+2b-3c)^2$
$=\{(a-2b+3c)+(a+2b-3c)\}\times\{(a-2b+3c)-(a+2b-3c)\}$
$=\{2a\}\{-4b+6c\}=2a\cdot 2(3c-2b)=4a(3c-2b)$

23 $(x^2+xy+y^2)^2-(x^2-xy+y^2)^2$
$=\{(x^2+xy+y^2)+(x^2-xy+y^2)\}\times\{(x^2+xy+y^2)-(x^2-xy+y^2)\}$
$=\{2x^2+2y^2\}\{2xy\}=2(x^2+y^2)\cdot 2xy=4xy(x^2+y^2)$

24 $(x+y-a+b)^2-(x-y+a-b)^2$
$=\{(x+y-a+b)+(x-y-a-b)\}\times\{(x+y-a+b)-(x-y+a-b)\}$
$=\{2x\}\times\{2y-2a+2b\}=2x\cdot 2\{y-a+b\}=4x(y-a+b)$

25 $(2a+3b-5c+7d)^2-(2a-3b+5c-7d)^2$
$=\{(2a+3b-5c+7d)+(2a-3b+5c-7d)\}$
$\times\{(2a+3b-5c+7d)-(2a-3b+5c-7d)\}$
$=\{4a\}\{6b-10c+14d\}=4a\cdot 2(3b-5c+7d)=8a(3b-5c+7d)$

26 $2345\times2345-2343\times2343=(2345)^2-(2343)^2=\{2345+2343\}$
$\times\{2345-2343\}=4688\times2=$ **9376**

27 $(53497)^2-(53487)^2=(53497+53487)(53497-53487)$
$=106984\times 10=\mathbf{1069840}$

28 $498567\times 498567-498562\times 498562=(498567)^2-(498562)^2$
$=(498567+498562)(498567-498562)$
$=997129\times 5=\mathbf{4985645}$

29 $25x^2-36=(5x)^2-(6)^2=(5x+6)(5x-6)$

30 $9a^2-16c^2=(3a)^2-(4c)^2=(3a+4c)(3a-4c)$

31 $16m^2-49n^2=(4m)^2-(7n)^2=(4m+7n)\times(4m-7n)$

32 $4p^2-81q^2=(2p)^2-(9q)^2=(2p+9q)(2p-9q)$

33 $a^2x^2-64b^2=(ax)^2-(8b)^2=(ax+8b)(ax-8b)$

34 $36x^2-121y^4=(6x)^2-(11y^2)^2=(6x+11y^2)(6x-11y^2)$

35 $49-64d^2=(7)^2-(8d)^2=(7+8d)(7-8d)$

36 $144c^2-25d^2=(12c)^2-(5d)^2=(12c+5d)(12c-5d)$

37 $(a+b)^2-c^2=(a+b+c)(a+b-c)$

38 $(a+2b)^2-25c^2=(a+2b)^2-(5c)^2=(a+2b+5c)(a+2b-5c)$

39 $4x^2-(3a-4b)^2=(2x)^2-(3a-4b)^2=(2x+3a-4b)(2x-3a+4b)$

40 $a^2-(2b-3c)^2=\{a+(2b-3c)\}\{a-(2b-3c)\}$
$=(a+2b-3c)(a-2b+3c)$

41 $a^4-81b^4=(a^2)^2-(9b^2)^2=(a^2+9b^2)(a^2-9b^2)$
$=(a^2+9b^2)\{a^2-(3b)^2\}=(a^2+9b^2)$
$(a+3b)(a-3b)$

42 $(x-y)^2-(a-b)^2$
$=\{(x-y)+(a-b)\}\{(x-y)-(a-b)\}$
$=(x-y+a-b)(x-y-a+b)$

43 $81x^4-625y^4=(9x^2)^2-(25y^2)^2$
$=(9x^2+25y^2)(9x^2-25y^2)$
$=(9x^2+25y^2)\{(3x)^2-(5y)^2\}$
$=(9x^2+25y^2)(3x+5y)(3x-5y)$

44 $(2a-5x)^2-36x^2=(2a-5x)^2-(6x)^2$
$=(2a-5x+6x)(2a-5x-6x)=(2a+x)(2a-11x)$

45 $25x^2-(4x+1)^2=(5x)^2-(4x+1)^2$
$=(5x+4x+1)\{5x-(4x+1)\}=(9x+1)(x-1)$

46 $(4a+7b)^2-(3a-8b)^2$
$=\{(4a+7b)+(3a-8b)\}\{(4a+7b)-(3a-8b)\}$
$=(7a-b)(a+15b)$

47 $(3x+5y)^2-(2x-7y)^2$
$=\{(3x+5y)+(2x-7y)\}\{(3x+5y)-(2x-7y)\}$
$=(5x-2y)(x+12y)$

48 $(a+2b-3c)^2-(a+b-c)^2$
$=\{(a+2b-3c)+(a+b-c)\}\{(a+2b-3c)-(a+b-c)\}$
$=(2a+3b-4c)(b-2c)$

49 $(2m+3n-5p)^2-(2n+3p)^2$
$=\{(2m+3n-5p)+(2n+3p)\}$
$\times\{(2m+3n-5p)-(2n+3p)\}$
$=(2m+5n-2p)(2m+n-8p)$

50 $(3x-4y+7z)^2-(2x-3y+5z)^2$
$=\{(3x-4y+7z)+(2x-3y+5z)\}$
$\times\{(3x-4y+7z)-(2x-3y+5z)\}$
$=(5x-7y+12z)(x-y+2z)$

Exercise 25

1 $(2x+1)^3=(2x)^3+3(2x)^2 1+3(2x)1^2+1^3$
$=8x^3+12x^2+6x+1$

2 $(3x+y)^3=(3x)^3+3(3x)^2y+3(3x)y^2+y^3$
$=27x^3+27x^2y+9xy^2-y^3$

3 $(2x+3a)^3=(2x)^3+3(2x)^2(3a)+3\,2x\,(3a)^2+(3a)^3$
$=8x^3+36x^2a+54xa^2+27a^3$

4 $(a^2+2b)^3=(a^2)^3+3(a^2)^2\,2b+3\,a^2\,(2b)^2+(2b)^3$
$=a^6+6a^4b+12a^2b^2+8b^3$

5 $(ab+bc)^3=(ab)^3+3(ab)^2(bc)+3(ab)(bc)^2+(bc)^3$
$=a^3b^3+3a^2b^3c+3ab^3c^2+b^3c^3$

6 $(a+2x+b)^3=\{a+(2x+b)\}^3$

$$=a^3+3a^2(2x+b)+3a(2x+b)^2+(2x+b)^3$$

$$=a^3+6a^2x+3a^2b+3a(4x^2+4xb+b^2)+(8x^3+12x^2b+6xb^2+b^3)$$

$$=a^3+8x^3+b^3+6a^2x+3a^2b+12ax^2+3ab^2$$

$$+12x^2b+6xb^2+12abx$$

7 $(2m+3n+p)^3=\{2m+(3n+p)\}^3$

$$=(2m)^3+3(2m)^2(3n+p)+3(2m)(3n+p)^2+(3n+p)^3$$

$$=8m^3+3\,4m^2(3n+p)+6m(9n^2+6np+p^2)$$

$$+(27n^3+27n^2p+9np^2+p^3)$$

$$=8m^3+36m^2n+12m^2p+54mn^2+36mnp$$

$$+6mp^2+27n^3+27n^2p+9np^2+p^3$$

$$=8m^3+27n^3+p^3+12m^2p+6mp^2$$

$$+36m^2n+54mn^2+9np^2+27n^2p+36mnp$$

8 $(xy+yz+zx)^3=\{(xy+yz)+zx\}^3$

$$=(xy+yz)^3+3(xy+yz)^2(zx)+3(xy+yz)(zx)^2+(zx)^3$$

$$=(x^3y^3+3x^2y^3z+3xy^3z^2+y^3z^3)+$$

$$3(x^2y^2+2xy^2z+y^2z^2)(zx)+3(xy+yz)z^2x^2+z^3x^3$$

$$=x^3y^3+3x^2y^3z+3xy^3z^2+y^3z^3+3x^3y^2z$$

$$+6x^2y^2z^2+3xy^2z^3+3x^3yz^2+3x^2yz^3+z^3x^3$$

$$=x^3y^3+y^3z^3+z^3x^3+3x^2y^3z+3xy^3z^2$$

$$+3x^3y^2z+3xy^2z^3+3x^3yz^2+3x^2yz^3+6x^2y^2z^2$$

9 Putting x for $(3m+5n)$ and y for $(2m-5n)$, we have the given expression $=x^3+3x^2y+3xy^2+y^3$

$$=(x+y)^3=\{(3m+5n)+(2m-5n)\}^3$$

$$=(5m)^3=\underline{125m^3}$$

10 Putting a for $(3x-8y)$ and b for $(9y-2x)$,

we have $a+b=(3x-8y)+(9y-2x)=x+y$

We have the given expression

$$=a^3+b^3+3ab(a+b)=(a+b)^3=(x+y)^3$$

$$=\underline{x^3+3x^2y+3xy^2+y^3}$$

11 Putting x for $(3a-7b)$ and y for $(10b-3a)$,

we have $x+y=(3a-7b)+(10b-3a)=3b$

$9b=3(x+y)$, and therefore we have the given expression

$$=x^3+y^3+3(x+y)xy=(x+y)^3=(3b)^3=\underline{27b^3}$$

12 Putting a for $(5x-2)$ and b for $(3-4x)$,

we have $a+b=(5x-2)+(3-4x)=x+1$

We have the given expression

$$=a^3+b^3+3ab(a+b)=(a+b)^3=(x+1)^3$$
$$=\underline{x^3+3x^2+3x+1}.$$

13 Putting a for $(3-7x)$ and b for $(8x-1)$,

we have $a+b=(3-7x)+(8x-1)=x+2$

We have the given expression

$$=a^3+b^3+3ab(a+b)=(a+b)^3$$
$$=(x+2)^3=\underline{x^3+6x^2+12x+8}$$

14 Putting x for $(a-b+c)$ and y for $(a+b-c)$,

we have $x+y=(a-b+c)+(a+b-c)=2a.$

$$3(x+y)=6a$$

Also $xy=(a-b+c)(a+b-c)=\{a-(b-c)\}\{a+(b-c)\}$
$$=\{a^2-(b-c)^2\}$$

We have the given expression

$$=x^3+y^3+3(x+y)xy=(x+y)^3=(2a)^3=\underline{8a^3}$$

15 $a^3+b^3=(a+b)^3-3ab(a+b)=(6)^3-3\cdot 7\cdot 6=216-126=90$

16 $a^3+b^3=(a+b)^3-3ab(a+b)=7^3-3\cdot 8\cdot 7=343-168=175$

17 $a^3+\left(\frac{1}{a}\right)^3=\left(a+\frac{1}{a}\right)^3-3a\cdot\frac{1}{a}\left(a+\frac{1}{a}\right)=\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$
$$=(3)^3-3\cdot 3=27-9=18$$

18 $z^3+\frac{1}{z^3}=\left(z+\frac{1}{z}\right)^3-3z\cdot\frac{1}{z}\left(z+\frac{1}{z}\right)=\left(z+\frac{1}{z}\right)^3-3\left(z+\frac{1}{z}\right)$
$$=4^3-3\cdot 4=64-12=52$$

19 The given expression

$$=(4)^3+3\cdot 4^2\cdot x+3\cdot 4\cdot x^2+x^3=(4+x)^3$$
$$=(4-8)^3=(-4)^3=-64.$$

20 The given expression

$$=(2m)^3+3\,(2m)^2\cdot 3n+3\,(2m)\,(3n)^2+(3n)^3+64$$
$$=(2m+3n)^3+64=(-3)^3+64=-27+64=37.$$

21 The given expression

$$= x^3 + 3(x^2)(5) + 3(x)(5)^2 + 125 + 64$$
$$= x^3 + 3(x^2)(5) + 3(x)(5)^2 + (5)^3 + 64$$
$$= (x+5)^3 + 64 = (-9+5)^3 + 64$$
$$= (-4)^3 + 64 = -64 + 64 = 0$$

22 The given expression

$$= x^3 + 3x^2 \cdot 6 + 3x \cdot 36 + 216 + 135$$
$$= x^3 + 3(x)^2(6) + 3(x)(6)^2 + (6)^3 + 135$$
$$= (x+6)^3 + 135 = (-11+6)^3 + 135$$
$$= (-5)^3 + 135 = -125 + 135 = 10.$$

23 $x^3 + y^3 + 15xy = x^3 + y^3 + 3 \cdot 5 \cdot xy$
$$= x^3 + y^3 + 3(x+y)xy = (x+y)^3 = (5)^3 = 125$$

24 $a^6 + b^6 + 3a^2b^2c^2 = (a^2)^3 + (b^2)^3 + 3a^2b^2c^2$
$$= (a^2)^3 + (b^2)^3 + 3a^2b^2(a^2+b^2)$$
$$= (a^2+b^2)^3 = (c^2)^3 = \underline{c^6}$$

25 $p^3 + q^3 + 6pq = p^3 + q^3 + 3 \cdot 2pq$
$$= p^3 + q^3 + 3(p+q)pq = (p+q)^3 = (2)^3 = 8$$

Exercise 26

1 $(1-2a)^3 = (1)^3 - 3(1)^2\, 2a + 3(1)(2a)^2 - (2a)^3$
$$= 1 - 6a + 12a^2 - 8a^3$$

2 $(2-3x)^3 = (2)^3 - 3(2)^2\, 3x + 3(2)(3x)^2 - (3x)^3$
$$= 8 - 3 \cdot 4 \cdot 3x + 3 \cdot 2 \cdot 9x^2 - 27x^3$$
$$= 8 - 36x + 54x^2 - 27x^3$$

3 $(3-4x)^3 = (3)^3 - 3(3)^2\, 4x + 3(3)(4x)^2 - (4x)^3$
$$= 27 - 3 \cdot 9 \cdot 4x + 3 \cdot 3 \cdot 16x^2 - 64x^3$$
$$= 27 - 108x + 144x^2 - 64x^3$$

4 $(5m-4n)^3 = (5m)^3 - 3(5m)^2\, 4n + 3(5m)(4n)^2 - (4n)^3$
$$= 125m^3 - 3 \cdot 25m^2 \cdot 4n + 3 \cdot 5m \cdot 16n^2 - 64n^3$$
$$= 125m^3 - 300m^2n + 240mn^2 - 64n^3$$

5 $(2p-5q)^3 = (2p)^3 - 3(2p)^2\, 5q + 3(2p)(5q)^2 - (5q)^3$
$$= 8p^3 - 3 \cdot 4p^2 \cdot 5q + 3 \cdot 2p \cdot 25q^2 - 125q^3$$
$$= 8p^3 - 60p^2q + 150pq^2 - 125q^3$$

6 $(2x-y-z)^3=\{2x-(y+z)\}^3$

$$=(2x)^3-3(2x)^2(y+z)+3(2x)(y+z)^2-(y+z)^3$$
$$=8x^3-3\cdot 4x^2(y+z)+6x(y^2+2yz+z^2)-y^3-3y^2z-3yz^2-z^3$$
$$=8x^3-12x^2y-12x^2y+6xy^2+12xyz+6xz^2-y^3-3y^2z-3yz^2-z^3$$
$$=8x^3-y^3-z^3-12x^2y-12x^2z+6xy^2-3y^2z-3yz^2+6xz^2+12xyz$$

7 $(2m-3n-p)^3=\{2m-(3n+p)\}^3$

$$=(2m)^3-3(2m)^2(3n+p)+3(2m)(3n+p)^2-(3n+p)^3$$
$$=8m^3-3\cdot 4m^2(3n+p)+6m(9n^2+6np+p^2)-(27n^3+27n^2p+9np^2+p^3)$$
$$=8m^3-36m^2n-12m^2p+54mn^2+36mnp+6mp^2-27n^3-27n^2p-9np^2-p^3$$
$$=8m^3-27n^3-p^3-36m^2n+54mn^2-12m^2p+6mp^2-27n^2p-9np^2+36mnp$$

8 $(l^2-m^2-n^2)^3=\{l^2-(m^2+n^2)\}^3$

$$=(l^2)^3-3(l^2)^2(m^2+n^2)+3l^2(m^2+n^2)^2-(m^2+n^2)^3$$
$$=l^6-3l^4(m^2+n^2)+3l^2(m^4+2m^2n^2+n^4)-(m^6+3m^4n^2+3m^2n^4+n^6)$$
$$=l^6-3l^4m^2-3l^4n^2+3l^2m^4+6l^2m^2n^2+3l^2n^4-m^6-3m^4n^2-3m^2n^4-n^6$$
$$=l^6-m^6-n^6-3l^4m^2+3l^2m^4-3l^4n^2+3l^2n^4-3m^4n^2-3m^2n^4+6l^2m^2n^2$$

9 Putting x for $(a+2b)$ and y for $(a-2b)$,

we have the given expression

$$=x^3-3x^2y+3xy^2-y^3$$
$$=(x-y)^3=\{(a+2b)-(a-2b)\}^3=(4b)^3=\underline{64b^3}$$

10 Putting a for $(3x-8y)$ and b for $(2x-7y)$,

we have $a-b=(3x-8y)-(2x-7y)=x-y$

We have the given expression

$$=a^3-b^3-3ab(a-b)$$
$$=(a-b)^3=(x-y)^3=x^3-3x^2y+3xy^2-y^3$$

11 Putting a for $(5x-8)$ and b for $(3x-8)$ we have,

$a-b=(5x-8)-(3x-8)=2x$ we have the

given expression $=a^3-b^3-3(a-b)ab$

$=(a-b)^3=(2x)^3=8x^3$

12 The given expression

$=(m)^3-3(m)^2(4n)+3(m)(4n)^2-(4n)^3$

$=(m-4n)^3=(12-12)^3=0$

13 The given expression

$=(3a)^3-3(3a)^2(5)+3(3a)(5)^2-(5)^3$

$=(3a-5)^3=(12-5)^3=7^3=343$

14. The given expression

$=7+1-9a+27a^2-27a^3$

$=7+(1)^3-3(1)^2\,3a+3(1)(3a)^2-(3a)^3$

$=7+(1-3a)^3=7+(1-9)^3=7+(-8)^3$

$=7-512=-505$

15 $216-144x+108x^2-27x^3$

$=(216-64)+(64-144x+108x^2-27x^3)$

$=152+(4)^3-3(4)^2\,3x+3(4)(3x)^2-(3x)^3$

$=152+\{4-3x\}^3=152+\{4-9\}^3=152+(-5)^3$

$=152-125=27$

16. $a^3-\left(\frac{1}{a}\right)^3=\left(a-\frac{1}{a}\right)^3+3a\frac{1}{a}\left(a-\frac{1}{a}\right)$

$=\left(a-\frac{1}{a}\right)^3+3a\left(a-\frac{1}{a}\right)$

$=(3)^3+3\cdot 3=27+9=36$

17. $c^3-\left(\frac{1}{c}\right)^3=\left(c-\frac{1}{c}\right)^3+3c\frac{1}{c}\left(c-\frac{1}{c}\right)$

$=\left(c-\frac{1}{c}\right)^3+3\left(c-\frac{1}{c}\right)$

$=(5)^3+3\cdot 5=125+15=140.$

18 x^3-y^3-9xy

$=x^3-y^3-3xy\cdot 3$

$=x^3-y^3-3xy(x-y)$

$=(x-y)^3=(3)^3=27$

19 p^3-8q^3-24pq

$=(p)^3-(2q)^3-3\cdot 4(p)(2q)$

$=p^3-(2q)^3-3(p-2q)p\,2q$

$=(p-2q)^3=(4)^3=64$

20 $8a^3-27b^3-90ab$
$=(2a)^3-(3b)^3-3\cdot 2a\cdot 3b\cdot 5$
$=(2a)^3-(3b)^3-3(2a)(3b)(2a-3b)$
$=(2a-3b)^3=(5)^3=125$

Exercise 27

1 Putting b for 1, we have,
$$a^2-a+1=a^2-a\cdot 1+(1)^2=a^2-ab+b^2$$
Hence $(a+1)(a^2-a+1)=(a+b)(a^2-ab+b^2)$
$$=(a^3+b^3)=a^3+1$$

2 Putting a for $(2x)$ and b for 1, we have,
$$4x^2-2x+1=(2x)^2-(2x)\cdot 1+1^2=a^2-ab+b^2$$
$$(2x+1)(4x^2-2x+1)=(a+b)(a^2-ab+b^2)$$
$$=a^3+b^3=\{(2x)^3+1^3\}=8x^3+1$$

3 Putting a for $5m$ and b for 1, we have,
$$25m^2-5m+1=(5m)^2-5m\cdot 1+1^2=a^2-ab+b^2$$
Hence, $(5m+1)(25m^2-5m+1)$
$$=(a+b)(a^2-ab+b^2)$$
$$=a^3+b^3=\{(5m)^3+1^3\}$$
$$=125m^3+1$$

4 Putting a for $4x$ and b for $7y$, we have,
$$16x^2-28xy+49y^2=(4x)^2-(4x)(7y)+(7y)^2=a^2-ab+b^2$$
Hence $(4x+7y)(16x^2-28xy+49y^2)$
$$=(a+b)(a^2-ab+b^2)$$
$$=a^3+b^3=\{(4x)^3+(7y)^3\}$$
$$=64x^3+343y^3$$

5 Putting a for $3m$ and b for $8n$, we have $9m^2-24mn+64n^2$
$$=(3m)^2-(3m)(8n)+(8n)^2=a^2-ab+b^2$$
$$(3m+8n)(9m^2-24mn+64n^2)$$
$$=(a+b)(a^2-ab+b^2)=(a^3+b^3)=(3m)^3+(8n)^3$$
$$=27m^3+512n^3$$

6 Putting x for ab and y for $4c$, we have,

$$(ab+4c)(a^2b^2-4abc+16c^2)=(x+y)(x^2-xy+y^2)$$
$$=x^3+y^3=(ab)^3+(4c)^3$$
$$=a^3b^3+64c^3$$

7 Putting m for ax and n for $5b$, we have,

$$a^2x^2-5abx+25b^2=(ax)^2-(ax)(5b)+(5b)^2=m^2-mn+n^2$$
$$(ax+5b)(a^2x^2-5abx+25b^2)=(m+n)(m^2-mn+n^2)$$
$$=m^3+n^3=(ax)^3+(5b)^3$$
$$=a^3x^3+125b^3$$

8 Putting x for $5a$ and y for $9b$, we have,

$$25a^2-45ab+81b^2=(5a)^2-(5a)(9b)+(9b)^2=x^2-xy+y^2$$

Hence $(5a+9b)(25a^2-45ab+81b^2)$
$$=(x+y)(x^2-xy+y^2)=x^3+y^3$$
$$=(5a)^3+(9b)^3=125a^3+729b^3$$

9 $(x^3+8)=x^3+(2)^3=(x+2)\{x^2-2x+(2)^2\}$
$$=(x+2)(x^2-2x+4)$$

10 $8a^3+1=(2a)^3+1=(2a+1)\{(2a)^2-2a+1\}$
$$=(2a+1)(4a^2-2a+1)$$

11 $m^3+27=m^3+(3)^3=(m+3)\{m^2-m\,3+(3)^2\}$
$$=(m+3)(m^2-3m+9)$$

12 $27x^3+1=(3x)^3+1=(3x+1)\{(3x)^2+(3x)+1\}$
$$=(3x+1)(9x^2-3x+1)$$

13 $z^3+64=z^3+(4)^3=(z+4)\{(z^2-z\,4+(4)^2\}$
$$=(z+4)(z^2-4z+16)$$

14 $125m^3+1=(5m)^3+1=\{(5m)+1\}\{(5m)^2-(5m)+1\}$
$$=(5m+1)(25m^2-5m+1)$$

15 $8a^3+343x^3=(2a)^3+(7x)^3$
$$=(2a+7x)(4a^2-14ax+49x^2)$$

16 $64a^3x^6+27y^3=(4ax^2)^3+(3y)^3$
$$=(4ax^2+3y)\{(4ax^2)^2-(4ax^2)(3y)+(3y)^2\}$$
$$=(4ax^2+3y)(16a^2x^4-12ax^2y+9y^2)$$

Exercise 28

1 $(1-3a)(1+3a+9a^2)$
$=(1-3a)\{1^2+1\cdot(3a)+(3a)^2\}$
$=1^3-(3a)^3=1-27a^3$

2 $(4x-1)(16x^2+4x+1)$
$=(4x-1)\{(4x)^2+(4x)\cdot 1+1^2\}$
$=(4x)^3-1=64x^3-1$

3. $(5m-3n)(25m^2+15mn+9n^2)$
$=(5m-3n)\{(5m)^2+(5m)(3n)+(3n)^2\}$
$=(5m)^3-(3n)^3=125m^3-27n^3$

4 $(x-2yz)(x^2+2xyz+4y^2z^2)$
$=(x-2yz)\{x^2+x(2yz)+(2yz)^2\}$
$=(x)^3-(2yz)^3=x^3-8y^3z^3$

5 $(3a-bc)(9a^2+3abc+b^2c^2)$
$=(3a-bc)\{(3a)^2+(3a)(bc)+(bc)^2\}$
$=(3a)^3-(bc)^3=27a^3-b^3c^3$

6 $125a^3-1=(5a)^3-1$
$=\{(5a)-1\}\{(5a)^2+(5a)+1\}$
$=(5a-1)(25a^2+5a+1)$

7 $343x^3-8y^6=(7x)^3-(2y^2)^3$
$=(7x-2y^2)\{(7x)^2+(7x)(2y^2)+(2y^2)^2\}$
$=(7x-2y^2)(49x^2+14xy^2+4y^4)$

8 $216k^3-125l^3=(6k)^3-(5l)^3$
$=(6k-5l)\{(6k)^2+(6k)(5l)+(5l)^2\}$
$=(6k-5l)(36k^2+30kl+25l^2)$

9 $1-512k^3=1-(8k)^3=(1-8k)\{1^2+1\cdot(8k)+(8k)^2\}$
$=(1-8k)(1+8k+64k^2)$

10 $729m^3-64a^3n^6=(9m)^3-(4an^2)^3$
$=(9m-4an^2)\{(9m)^2+(9m)(4an^2)+(4an^2)^2\}$
$=(9m-4an^2)(81m^2+36amn^2+16a^2n^4)$

Exercise 29.

1 $\left.\begin{aligned} 2+8&=10 \\ \text{and } 2\times 8&=16 \end{aligned}\right\}$, the required product $=x^2+10x+16$

2 $\left.\begin{aligned} 3+5&=8 \\ \text{and } 3\times 5&=15 \end{aligned}\right\}$, the required product $=x^2+8x+15$

3 $\left.\begin{aligned} 6+11&=17 \\ \text{and } 6\times 11&=66 \end{aligned}\right\}$, the required product $=x^2+17x+66$

4 $\left.\begin{aligned} 7+9&=16 \\ \text{and } 7\times 9&=63 \end{aligned}\right\}$, the required product $=x^2-4x-12$

5 $\left.\begin{aligned} -6+2&=-4 \\ \text{and } (-6)\times 2&=-12 \end{aligned}\right\}$, the required product $=x^2-4x-12$

6 $\left.\begin{aligned} -2+8&=6 \\ \text{and } (-2)\times 8&=-16 \end{aligned}\right\}$, the required product $=m^2+6m-16$

7 $\left.\begin{aligned} -3-4&=-7 \\ \text{and } (-3)\times(-4)&=12 \end{aligned}\right\}$, the required product $=a^2-7a+12$

8 $\left.\begin{aligned} 5-3&=2 \\ \text{and } 5\times(-3)&=-15 \end{aligned}\right\}$, the required product $=x^2+2x-15$

9 $\left.\begin{aligned} -4+9&=5 \\ \text{and } (-4)\times 9&=-36 \end{aligned}\right\}$, the required product $=x^2+5x-36$

10. $\left.\begin{aligned} -5-10&=-15 \\ \text{and } (-5)\times(-10)&=50 \end{aligned}\right\}$, the required product $=x^2-15x+50$

11 $\left.\begin{aligned} -12+5&=-7 \\ \text{and } (-12)\times 5&=-60 \end{aligned}\right\}$, the required product $=x^2-7x-60$

12 $\left.\begin{aligned} -13+2&=-11 \\ \text{and } (-13)\times 2&=-26 \end{aligned}\right\}$, the required product $=k^2-11k-26$

13 $\left.\begin{aligned} 5+14&=19 \\ \text{and } 5\times 14&=70 \end{aligned}\right\}$, the required product $=a^2+19a+70$

14 $\left.\begin{aligned} -14+6&=-8 \\ \text{and } -14\times 6&=-84 \end{aligned}\right\}$, the required product $=m^2-8m-84$

15 $\left.\begin{aligned} -5-13&=-18 \\ \text{and } (-5)\times(-13)&=65 \end{aligned}\right\}$, the required product $=x^2-18x+65$

16 $\left.\begin{aligned} 7+12&=19 \\ \text{and } 7\times 12&=84 \end{aligned}\right\}$, the required product $=x^2+19x+84$

17 $\left.\begin{aligned} -3-11&=-14 \\ \text{and } (-3)\times(-11)&=33 \end{aligned}\right\}$, the required product $=a^2-14a+33$

18 $\left.\begin{array}{r} 4-13=-9 \\ \text{and } 4\times(-13)=-52 \end{array}\right\}$, the required product $=x^2-9x-52$

19 $\left.\begin{array}{r} 5-16=-11 \\ \text{and } 5\times(-16)=-80 \end{array}\right\}$, the required product $=m^2-11m-80$

20 $\left.\begin{array}{r} -8-10=-18 \\ \text{and } (-8)\times(-10)=80 \end{array}\right\}$, the required product $=x^2-18x+80$

21 $\left.\begin{array}{r} 6-12=-6 \\ \text{and } 6\times(-12)=-72 \end{array}\right\}$, the required product $=a^2-6a-72$

22 $\left.\begin{array}{r} -7+13=6 \\ \text{and } (-7)\times(13)=-91 \end{array}\right\}$, the required product $=m^2+6m-91$

23 $\left.\begin{array}{r} -10-16=-26 \\ \text{and } (-10)\times(-16)=160 \end{array}\right\}$, the required product $=x^2-26x+160$

24 $\left.\begin{array}{r} 5-18=-13 \\ \text{and } (5)\times(-18)=-90 \end{array}\right\}$, the required product $=x^2-13x-90$

25 $\left.\begin{array}{r} -16+10=-6 \\ \text{and } (-16)\times 10=-160 \end{array}\right\}$, the required product $=x^2-6x-160$

Exercise 30

1 $1+2+3=6$,
$1\times 2+1\times 3+2\times 3=2+3+6=11$,
$1\times 2\times 3=6$

The required product $=x^3+6x^2+11x+6$

2 $2+5+7=14$,
$2\times 5+2\times 7+5\times 7=10+14+35=59$,
$2\times 5\times 7=70$

The required product $=x^3+14x^2+59x+70$

3 $3-6+2=-1$,
$(3)\times(-6)+(3)\times(2)+(-6)\times(2)=-18+6-12=-24$,
$(3)\times(-6)\times(2)=-36$

The required product $=x^3-x^2-24x-36$

4 $-4+5-10=-1$,
$(4\times 5)+(4)\times(-10)+(5)\times(-10)=20-40-50=-70$,
$(4)\times(5)\times(-10)=-200$

The required product $=x^3-x^2-70x-200$

5 $-8+3+1=-4$,
$(-8)\times(3)+(-8)\times(1)+(3)\times(1)=-24-8+3=-29$,
$(-8)\times(3)\times(1)=-24$

The required product $=x^3-4x^2-29x-24$

6 $-5-2+8=1$, $(-5)\times(-2)+(-5)\times(8)+(-2)\times(8)$
$=10-40-16=-46$, $(-5)\times(-2)\times 8=80$

The required product $=x^3+x^2-46x+80$

7 $-3+7-4=0$, $(-3)\times(7)+(-3)\times(-4)+(7)\times(-4)$
$=-21+12-28=-37$, $(-3)\times(7)\times(-4)=84$

The required product $=x^3+0.x^2-37x+84$
$=x^3-37x+84$

8 $6-5-7=-6$, $(6)(-5)+(6)(-7)+(-5)(-7)$
$=-30-42+35=-37$, $(6)(-5)(-7)=210$

The required product $=x^3-6x^2-37x+210$

9 $-5-7-11=-23$, $(-5)(-7)+(-5)(-11)+(-7)(-11)$
$=35+55+77=167$, $(-5)(-7)(-11)=-385$

The required product $=x^3-23x^2+167x-385$

10 $-3-6-9=-18$, $(-3)(-6)+(-6)(-9)+(-3)(-9)$
$=18+54+27=99$, $(-3)(-6)(-9)=-162$

The required product $=x^3-18x^2+99x-162$

11 $4-5-12=-13$,
$(4)(-5)+(4)(-12)+(-5)(-12)=-20-48+60=-8$,
$(4)(-5)(-12)=240$

The required product $=x^3-13x^2-8x+240$

12 $5+9+11=25$, $5\times 9+5\times 11+9\times 11=45+55+99=199$
$5\times 9\times 11=495$

The required product $=x^3+25x^2+199x+495$

13 $-6+8-2=0$, $(-6)(8)+(-6)(-2)+(8)(-2)$
$=-48+12-16=-52$, $(-6)\times 8\times(-2)=96$

The required product $=x^3-0.x^2-52x+96$
$=x^3-52x+96$

14 $-3-7-13=-23$, $(-3)(-7)+(-3)(-13)+(-7)(-13)$
$=21+39+91=151$, $(-3)\times(-7)\times(-13)=-273$

The required product $=x^3-23x^2+151x-273$

15 $-3+12+4=13$, $(-3)(12)+(-3)(4)+(12)(4)$
$=-36-12+48=0$, $(-3)(12)(4)=-144$

The required product $=x^3+13x^2-0.x-144$
$=x^3+13x^2-144$

16 $-9-10+12=-7$, $(-9)(-10)+(-9)(12)+(-10)(12)$
$=90-108-120=-138$, $(-9)(-10)(12)=1080$

The required product $=x^3-7x^2-138x+1080$

17 $9-5-7=-3$, $(9)(-5)+(9)(-7)+(-5)(-7)$
$=-45-63+35=-73$, $(9)(-5)(-7)=315$

The required product $=x^3-3x^2-73x+315$

18 $8+12+15=35$, $8\times 12+8\times 15+12\times 15$
$=96+120+180=396$, $8\times 12\times 15=1440$

The required product $=x^3+35x^2+396x+1440$

19 $-14+8+6=0$, $(-14)(8)+(-14)(6)+(8)(6)$
$=-112-84+48=-148$, $(-14)(8)(6)=-672$

The required product $=x^3-0.x^2-148x-672$
$=x^3-148x-672$

20 $-5-10-16=-31,$

$(-5)(-10)+(-5)(-16)+(-10)(-16)$

$=50+80+160=290,\ (-5)(-10)(-16)=-800$

The required product $=x^3-31x^2+290x-800$

Exercise 31.

1 $\{x+y-z\}^2=x^2+y^2+z^2+2x(y-z)+2y(-z)$

$=x^2+y^2+z^2+2xy-2xz-2yz$

2 $(x-y+z)^2=x^2+y^2+z^2+2x(-y+z)+2(-y)(z)$

$=x^2+y^2+z^2-2xy+2xz-2yz$

3 $(-x+y+z)^2=x^2+y^2+z^2+2(-x)(y+z)+2yz$

$=x^2+y^2+z^2-2xy-2xz+2yz$

4 $(-x-y+z)^2=x^2+y^2+z^2+2(-x)(-y+z)+2(-y)(z)$

$=x^2+y^2+z^2+2xy-2xz-2yz$

5 $(x-y-z)^2=x^2+y^2+z^2+2x(-y-z)+2(-y)(-z)$

$=x^2+y^2+z^2-2xy-2xz+2yz$

6 $(a-x+y-z)^2=a^2+x^2+y^2+z^2+2a(-x+y-z)$

$+2(-x)(y-z)+2y(-z)$

$=a^2+x^2+y^2+z^2-2ax+2ay-2az-2xy$

$+2xz-2yz$

7 $(a-x-y-z)^2=a^2+x^2+y^2+z^2+2a(-x-y-z)$

$+2(-x)(-y-z)+2(-y)(-z)$

$=a^2+x^2+y^2+z^2-2ax-2ay-2az+2xy$

$+2xz+2yz$

8 $(m+n+p+q+r)^2=m^2+n^2+p^2+q^2+r^2+2m(n+p+q+r)$

$+2n(p+q+r)+2p(q+r)+2qr$

$=m^2+n^2+p^2+q^2+r^2+2mn+2mp+2mq+2mr$

$+2np+2nq+2nr+2pq+2pr+2qr$

9 $(p-q+r-x-y)^2=p^2+q^2+r^2+x^2+y^2+2p(-q+r-x-y)+2(-q)(r-x-y)+2r(-x-y)+2(-x)(-y)$

$=p^2+q^2+r^2+x^2+y^2-2pq+2pr-2px-2py-2qr+2qx+2qy-2rx-2ry+2xy$

10 $(-a+b-c+x-y-z)^2$

$=a^2+b^2+c^2+x^2+y^2+z^2+2(-a)(b-c+x-y-z)+2b(-c+x-y-z)+2(-c)(x-y-z)+2x(-y-z)+2(-y)(-z)$

$=a^2+b^2+c^2+x^2+y^2+z^2-2ab+2ac-2ax+2ay+2az-2bc+2bx-2by-2bz-2cx+2cy+2cz-2xy-2xz+2yz$

11 $(a-2x-3y-4z)^2$

$=a^2+(-2x)^2+(-3y)^2+(-4z)^2+2a(-2x-3y-4z)+2(-2x)(-3y-4z)+2(-3y)(-4z)$

$=a^2+4x^2+9y^2+16z^2-4ax-6ay-8az+12xy+16xz+24yz$

12 $(2a-b+2c-d)^2=(2a)^2+(-b)^2+(2c)^2+(-d)^2+2(2a)(-b+2c-d)+2(-b)(2c-d)+2(2c)(-d)$

$=4a^2+b^2+4c^2+d^2-4ab+8ac-4ad-4bc+2bd-4cd$

13 The given expression

$=l^2+(-m)^2+n^2+2l(-m+n)+2(-m)(n)$

$=(l-m+n)^2=(17-23+13)^2=(7)^2=49$

14 The given expression

$=p^2+q^2+(-r)^2+2p(q-r)+2q(-r)$

$=(p+q-r)^2=(16+12-25)^2=(3)^2=9$

15 The given expression

$=(-a)^2+b^2+c^2+2(-a)(b+c)+2bc$

$=(-a+b+c)^2=(-28+13+15)^2=0$

16 The given expression

$=x^2+y^2+(-1)^2+2x(y-1)+2y(-1)$

$=(x+y-1)^2=(6+7-1)=(12)^2=144$

17. The given expression

$$=x^2+y^2+1+2xy-2x-2y+35$$
$$=x^2+y^2+(-1)^2+2x(y-1)+2y(-1)+35$$
$$=(x+y-1)^2+35=(23+18-1)^2+35$$
$$=(40)^2+35=1600+35=1635$$

18. The given expression

$$=x^2+(-2y)^2+(-1)^2+2x(-2y-1)+2(-2y)(-1)$$
$$=(x-2y-1)^2=(26-2\cdot12-1)^2$$
$$=(26-24-1)^2=(1)^2=1$$

19. The given expression

$$=x^2+9y^2+1-6xy-2x+6y+63$$
$$=x^2+(-3y)^2+(-1)^2+2x(-3y-1)+2(-3y)(-1)+63$$
$$=(x-3y-1)^2+63=(49-48-1)^2+63=0+63=63$$

20. The given expression

$$=9x^2+y^2+1-6xy+6x-2y-25$$
$$=(3x)^2+(-y)^2+1^2+2(3x)(-y+1)+2\times(-y)(1)-25$$
$$=(3x-y+1)^2-25=(42-38+1)^2-25$$
$$=(5)^2-25=25-25=0$$

21. We have $2(ab+ac+bc)=(a+b+c)^2-(a^2+b^2+c^2)$

$$=(12)^2-50=144-50=94$$
$$ab+ac+bc=94\div2=47$$

22. We have $a^2+b^2+c^2=(a+b+c)^2-2(ab+ac+bc)$

$$=(13)^2-2\cdot50=169-100=69$$

Exercise 32.

1. The 1st term $=x^5$, 2nd term $=5x^4\cdot1=5x^4$, 3rd term $=\frac{5\times4}{2}x^3\cdot1^2=10x^3$, 4th term $=\frac{10\times3}{3}x^2\cdot1^3=10x^2$, 5th term $=\frac{10\times2}{4}x\cdot1^4=5x$, 6th term $=\frac{5}{5}\cdot1^5=1$

$$(x+1)^5=x^5+5x^4+10x^3+10x^2+5x+1$$

2 The 1st term $=x^6$, 2nd term $=6x^5$, 3rd term $=\frac{6\times 5}{2}x^4=15x^4$, 4th term $=\frac{15\times 4}{3}x^3=20x^3$

The remaining terms are respectively $15x^2$, $6x$ and 1

$(x+1)^6=x^6+6x^5+15x^4+20x^3+15x^2+6x+1$

3 The 1st term $=a^8$, 2nd term $=8a^7b$, 3rd term $=\frac{8\times 7}{2}a^6b^2$ $=28a^6b^2$, 4th term $=\frac{28\times 6}{3}a^5b^3=56a^5b^3$, 5th term $=\frac{56\times 5}{4}a^4b^4=70a^4b^4$

The remaining terms are respectively $56a^3b^5$, $28a^2b^6$, $8ab^7$ and b^8,

$$(a+b)^8=a^8+8a^7b+28a^6b^2+56a^5b^3+70a^4b^4+56a^3b^5+28a^2b^6+8ab^7+b^8$$

4 The 1st term $=a^9$, 2nd term $=9a^8b$, 3rd term $=\frac{9\times 8}{2}a^7b^2$ $=36a^7b^2$, 4th term $=\frac{36\times 7}{3}a^6b^3=84a^6b^3$, 5th term $=\frac{84\times 6}{4}a^5b^4=126a^5b^4$

The remaining 5 terms are respectively $126a^4b^5$, $84a^3b^6$, $36a^2b^7$, $9ab^8$ and b^9

$$(a+b)^9=a^9+9a^8b+36a^7b^2+84a^6b^3+126a^5b^4+126a^4b^5+84a^3b^6+36a^2b^7+9ab^8+b^9$$

5 The 1st term $=x^5$, 2nd term $=-5x^4y$, 3rd term $=\frac{5\times 4}{2}x^3y^2=10x^3y^2$

The remaining 3 terms are respectively $-10x^2y^3$, $5xy^4$ and $-y^5$

$(x-y)^5=x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$

6 The 1st term $=m^7$, 2nd term $=-7m^6n$, 3rd term $=\frac{7\times 6}{2}m^5n^2=21m^5n^2$, 4th term $=-\frac{21\times 5}{3}m^4n^3=-35m^4n^3$

The remaining 4 terms are respectively $35m^3n^4$, $-21m^2n^5$, $7mn^6$ and $-n^7$

$$(m-n)^7=m^7-7m^6n+21m^5n^2-35m^4n^3+35m^3n^4-21m^2n^5+7mn^6-n^7$$

7 The 1st term $= x^4$, 2nd term $= 4x^3 \cdot 2$, 3rd term $= \frac{4\times 3}{2} x^2\, 2^2 = 6x^2 \cdot 4$

The remaining 2 terms are respectively $4x \cdot 2^3$ and 2^4

$$(x+2)^4 = x^4 + (4x^3 \cdot 2) + (6x^2 \cdot 4) + (4x \cdot 2^3) + (2)^4$$
$$= x^4 + 8x^3 + 24x^2 + 32x + 16$$

8 The 1st term $= x^5$, 2nd term $= 5x^4 \cdot 2$, 3rd term $= \frac{5\times 4}{2} x^3\, (2)^2 = 10x^3 \cdot 4$

The remaining 3 terms are respectively $10x^2 (2)^3$, $5x(2)^4$ and $(2)^5$

$$(x+2)^5 = x^5 + 5x^4 \cdot 2 + 10x^3 \cdot 4 + 10x^2 (2)^3 + 5x(2)^4 + (2)^5$$
$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

9 The 1st term $= x^8$, 2nd term $= 8x^7 \cdot 1 = 8x^7$, 3rd term $= \frac{8\times 7}{2} x^6\, 1^2 = 28x^6$, 4th term $= \frac{28\times 6}{3} x^5\, 1^3 = 56x^5$, 5th term $= \frac{56\times 5}{4} x^4\, 1^4 = 70x^4$

The remaining 4 terms are respectively $56x^3$, $28x^2$, $8x$ and 1

$$(x+1)^8 = x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 56x^3 + 28x^2 + 8x + 1$$

10. The 1st term $= x^4$, 2nd term $= 4x^3 \cdot 3$, 3rd term $= \frac{4\times 3}{2} x^2 (3)^2 = 6x^2 \cdot 9$

The remaining two terms are respectively $4x(3)^3$ and $(3)^4$

$$(x+3)^4 = x^4 + 4x^3 \cdot 3 + 6x^2 \cdot 9 + 4x(3)^3 + (3)^4$$
$$= x^4 + 12x^3 + 54x^2 + 108x + 81$$

11 The 1st term $= x^5$, 2nd term $= -5x^4$, 3rd term $= 10x^3$

The remaining 3 terms are respectively $-10x^2$, $5x$ and -1

$$(x-1)^5 = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$$

12 The 1st term $= 2^6$, 2nd term $= -6 \cdot 2^5 z$, 3rd term $= \frac{6\times 5}{2} 2^4 z^2 = 15 \cdot 2^4 z^2$, 4th term $= -\frac{15\times 4}{3} 2^3 z^3 = -20 \cdot 2^3 z^3$

The remaining 3 terms are respectively $15 \cdot 2^2 z^4$, $-6 \cdot 2 z^5$ and z^6

$$(2-z)^6 = 2^6 - 6 \cdot 2^5 z + 15 \cdot 2^4 z^2 - 20 \cdot 2^3 z^3 + 15 \cdot 2^2 z^4 - 6 \cdot 2 z^5 + z^6$$
$$= 64 - 192z + 240z^2 - 160z^3 + 60z^4 - 12z^5 + z^6$$

13 The 1st term $=(2x)^4$, 2nd term $=-4(2x)^3$, 3rd term $=\frac{4\times3}{2}(2x)^2=6\,(2x)^2$

The remaining two terms are respectively $-4(2x)$ and 1

$$(2x-1)^4=(2x)^4-4(2x)^3+6(2x)^2-4(2x)+1$$
$$=16x^4-32x^3+24x^2-8x+1$$

14 The 1st term $=x^9$, 2nd term $=-9x^8y$, 3rd term $=\frac{9\times8}{2}x^7y^2=36x^7y^2$, 4th term $=-\frac{36\times7}{3}x^6y^3=-84x^6y^3$, 5th term $=\frac{84\times6}{4}x^5y^4=126x^5y^4$

The remaining 5 terms are respectively $-126x^4y^5$, $84x^3y^6$, $-36x^2y^7$, $9xy^8$ and $-y^9$

$$(x-y)^9=x^9-9x^8y+36x^7y^2-84x^6y^3+126x^5y^4-126x^4y^5$$
$$+84x^3y^6-36x^2y^7+9xy^8-y^9$$

15 The 1st term $=(3x)^5$, 2nd term $=-5(3x)^4\,2$, 3rd term $=\frac{5\times4}{2}(3x)^3(2)^2=10(3x)^3(2)^2$

The remaining 3 terms are respectively

$-10(3x)^2(2)^3$, $5(3x)(2)^4$ and $-(2)^5$

$$(3x-2)^5=(3x)^5-5\,(3x)^4\,2+10\,(3x)^3(2)^2-10(3x)^2(2^3)$$
$$+5(3x)(2)^4-(2)^5$$
$$=243x^5-810x^4+1080x^3-720x^2+240x-32$$

16 The 1st term $=1^8=1$, 2nd term $=-8\ 1^7\,a=-8a$, 3rd term $=\frac{8\times7}{2}\ 1^6\,a^2=28a^2$, 4th term $=-\frac{28\times6}{3}\ 1^5\,a^3=-56a^3$,

5th term $=\frac{56\times5}{4}\ 1^4a^4=70\ 1^4\ a^4=70a^4$

The remaining 4 terms are respectively

$-56a^5$, $28a^6$, $-8a^7$ and a^8

$$(1-a)^8=1-8a+28a^2-56a^3+70a^4-56a^5+28a^6-8a^7+a^8$$

17 The 1st term $=1^7$, 2nd term $=-7\ 1^6\ c=-7c$, 3rd term $=\frac{7\times6}{2}\ 1^5\ c^2=21c^2$, 4th term $=-\frac{21\times5}{3}\ 1^4\ c^3=-35\ 1^4\ c^3$
$=-35c^3$

The remaining 4 terms are respectively

$35c^4$, $-21c^5$, $7c^6$ and $-c^7$

$$(1-c)^7=1-7c+21c^2-35c^3+35c^4-21c^5+7c^6-c^7$$

18. The 1st term $=1^6$, the 2nd term $=-6\cdot 1^5\cdot 3x$, 3rd term
$=\frac{6\times 5}{2}\,1^4(3x)^2=15\cdot 1^4(3x)^2$, 4th term
$=-\frac{15\times 4}{3}\,1^3(3x)^3=-20\cdot 1^3(3x)^3$

The remaining 3 terms are respectively
$15\cdot 1^2(3x)^4$, $-6\cdot 1\,(3x)^5$ and $(3x)^6$

$$(1-3x)^6=1-6(3x)+15(3x)^2-20(3x)^3+15(3x)^4-6(3x)^5+(3x)^6$$
$$=1-18x+135x^2-540x^3+1215x^4-1458x^5+729x^6.$$

19 The 1st term $=1^7$, 2nd term $=-7\cdot 1^6(2x)$, 3rd term
$=\frac{7\times 6}{2}\,1^5(2x)^2=21\cdot 1^5(2x)^2$, 4th term
$=-\frac{21\times 5}{3}\,1^4(2x)^3=-35\cdot 1^4(2x)^3$

The remaining 4 terms are respectively
$35\cdot 1^3(2x)^4$, $-21\cdot 1^2(2x)^5$, $7\cdot 1\,(2x)^6$ and $-(2x)^7$.

$$(1-2x)^7=1-7(2x)+21(2x)^2-35(2x)^3+35(2x)^4-21(2x)^5+7(2x)^6-(2x)^7$$
$$=1-14x+84x^2-280x^3+560x^4-672x^5+448x^6-128x^7$$

20 The 1st term $=(2x)^8$, 2nd term $=-8(2x)^7a$, 3rd term
$=\frac{8\times 7}{2}(2x)^6a^2=28(2x)^6a^2$, 4th term
$=-\frac{28\times 6}{3}(2x)^5a^3=-56(2x)^5a^3$, 5th term
$=\frac{56\times 5}{4}(2x)^4a^4=70(2x)^4a^4$

The remaining 4 terms are respectively $-56(2x)^3a^5$,
$28(2x)^2a^6$, $-8(2x)a^7$ and a^8

$$(2x-a)^8=(2x)^8-8(2x)^7a+28(2x)^6a^2-56(2x)^5a^3+70(2x)^4a^4-56(2x)^3a^5+28(2x)^2a^6-8(2x)a^7+a^8$$
$$=256x^8-1024x^7a+1792x^6a^2-1792x^5a^3+1120x^4a^4-448x^3a^5+112x^2a^6-16xa^7+a^8$$

21 The 1st term $=x^{10}$, 2nd term $=-10x^9a$, 3rd term
$=\frac{10\times 9}{2}x^8a^2=45x^8a^2$, 4th term
$=-\frac{45\times 8}{3}x^7a^3=-120x^7a^3$, 5th term
$=\frac{120\times 7}{4}x^6a^4=210x^6a^4$, 6th term
$=-\frac{210\times 6}{5}x^5a^5=-252x^5a^5$

The remaining 5 terms are $210x^4a^6$, $-120x^3a^7$, $45x^2a^8$
$-10xa^9$ and a^{10}

$$\therefore \quad (x-a)^{10}=x^{10}-10x^9a+45x^8a^2-120x^7a^3+210x^6a^4-252x^5a^5$$
$$+210x^4a^6-120x^3a^7+45x^2a^8-10xa^9+a^{10}$$

22 The 1st term $=(3x)^5$, 2nd term $=-5(3x)^4(2a)$, 3rd term $=\frac{5\times4}{2}(3x)^3(2a)^2=10(3x)^3(2a)^2$

the remaining 3 terms are respectively

$-10(3x)^2(2a)^3$, $5(3x)(2a)^4$ and $-(2a)^5$

$$(3x-2a)^5=(3x)^5-5(3x)^4(2a)+10(3x)^3(2a)^2-10(3x)^2(2a)^3$$
$$+5(3x)(2a)^4-(2a)^5$$
$$=243x^5-810x^4a+1080x^3a^2-720x^2a^3+240xa^4$$
$$-32a^5$$

23 $(x+1)^5-(x-1)^5=(x^5+5x^4+10x^3+10x^2+5x+1)$
$$-(x^5-5x^4+10x^3-10x^2+5x-1)$$
$$=10x^4+20x^2+2$$

24 $(x-1)^6+(x+1)^6=(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)$
$$+(x^6+6x^5+15x^4+20x^3+15x^2+6x+1)$$
$$=2x^6+30x^4+30x^2+2$$

25 $(x+a)^7-(x-a)^7=(x^7+7x^6a+21x^5a^2+35x^4a^3+35x^3a^4+21x^2a^5$
$$+7xa^6+a^7)-(x^7-7x^6a+21x^5a^2-35x^4a^3$$
$$+35x^3a^4-21x^2a^5+7xa^6-a^7)$$
$$=14x^6a+70x^4a^3+42x^2a^5+2a^7$$

26 $(x+a)^4=x^4+4x^3a+6x^2a^2+4xa^3+a^4$, the co efficients are respectively 1, 4, 6, 4 and 1

the required sum $=1+4+6+4+1=16$

27 $(x+a)^5=x^5+5x^4a+10x^3a^2+10x^2a^3+5xa^4+a^5$, the co efficients are respectively 1, 5, 10, 10, 5 and 1

the required sum $=1+5+10+10+5+1=32$

28 $(x+a)^6=x^6+6x^5a+15x^4a^2+20x^3a^3+15x^2a^4+6xa^5+a^6$, the respective co efficients are 1, 6, 15, 20, 15, 6 and 1

the required sum $=1+6+15+20+15+6+1=64$

29 $(x+a)^7=x^7+7x^6a+21x^5a^2+35x^4a^3+35x^3a^4+21x^2a^5+7xa^6+a^7$, the respective co efficients are 1, 7, 21, 35, 35, 21, 7 and 1

the required sum $=1+7+21+35+35+21+7+1=128$

30 $(x+a)^8=x^8+8x^7a+28x^6a^2+56x^5a^3+70x^4a^4+56x^3a^5+28x^2a^6$
$+8xa^7+a^8$, the respective co efficients are
1, 8, 28, 56, 70, 56, 28, 8 and 1
the req sum $=1+8+28+56+70+56+28+8+1=256$

31 The given expression
$=(x^5+5x^4+10x^3+10x^2+5x+1)+31$
$=(x+1)^5+31=(-2+1)^5+31=(-1)^5+31$
$=-1+31=30$

32 The given expression
$=(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)-1$
$=(x-1)^6-1=(\sqrt[3]{2})^6-1=2^2-1=4-1=3$

33 The given expression
$=(2x)^4-4(2x)^3+6(2x)^2-4(2x)+1-81$
$=(2x-1)^4-81=(4-1)^4-81=(3)^4-81$
$=81-81=0$

34 The given expression
$=(x)^4+4(x)^3\ 3+6(x)^2(3)^2+4(x)(3)^3+(3)^4$
$=(x+3)^4=(-5+3)^4=(-2)^4=16$

35 The given expression
$=x^4+4x^3\ 2+6x^2\ (2)^2+6x\ (2)^3+(2)^4-625$
$=(x+2)^4-625=(-7+2)^4-625$
$=(-5)^4-625=625-625=0$

Exercise 33

1 Putting a for x, b for y and c for $(-z)$, we have the given expression $=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$
$=a^3+b^3+c^3-3abc=x^3+y^3-z^3+3xyz$

2 Putting a for p, b for $-2q$ and c for $-r$, we have the given expression $=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$
$=a^3+b^3+c^3-3abc=p^3-8q^3-r^3-6pqr$

3 Putting a for $2x$, b for $-3y$ and c for $-z$, we have the given expression $=(2x-3y-z)\{(2x)^2+(-3y)^2+(-z)^2-(2x)$
$(-3y)-(2x)(-z)-(-3y)(-z)\}$

$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=a^3+b^3+c^3-3abc$$
$$=(2x)^3+(-3y)^3+(-z)^3-3(2x)(-3y)(-z)$$
$$=8x^3-27y^3-z^3-18xyz$$

4 Putting x for a, y for $-2b$ and z for 3, we have the given expression $=(a-2b+3)\{a^2+(-2b)^2+(3)^2-(a)(-2b)$
$$-(a)(3)-(-2b)(3)\}$$
$$=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$$
$$=x^3+y^3+z^3-3xyz$$
$$=(a)^3+(-2b)^3+(3)^3-3(a)(-2b)(3)$$
$$=a^3-8b^3+27+18ab$$

5 Putting x for $3a$, y for $(-5b)$ and z for (-4), we have the given expression $=(3a-5b-4)\{(3a)^2+(-5b)^2+(-4)^2-(3a)$
$$(-5b)-(3a)(-4)-(-5b)(-4)\}$$
$$=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$$
$$=x^3+y^3+z^3-3xyz$$
$$=(3a)^3+(-5b)^3+(-4)^3-3(3a)(-5b)(-4)$$
$$=27a^3-125b^3-64-180ab$$

6 Putting a for x, b for $(-y)$ and c for (-1), we have
$$x^3-y^3-1-3xy=x^3+(-y)^3+(-1)^3-3(x)(-y)(-1)$$
$$=a^3+b^3+c^3-3abc$$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=(x-y-1)(x^2+y^2+1+xy+x-y)$$

7 Putting a for x, b for $(-y)$ and c for 2, we have
$$x^3-y^3+6xy+8=x^3+(-y)^3+(2)^3-3(x)(-y)(2)$$
$$=a^3+b^3+c^3-3abc$$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=(x-y+2)(x^2+y^2+4+xy-2x+2y)$$

8 Putting a for x, b for $(-2y)$ and c for $(-3z)$, we have
$$x^3-8y^3-27z^3-18xyz$$
$$=x^3+(-2y)^3+(-3z)^3-3(x)(-2y)(-3z)$$
$$=a^3+b^3+c^3-3abc$$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=(x-2y-3z)(x^2+4y^2+9z^2+2xy+3xz-6yz)$$

9 $x^3+y^3+18xy-216$

$$=x^3+y^3+(-6)^3-3(x)(y)(-6)$$
$$=(x+y-6)(x^2+y^2+36-xy+6x+6y)$$
$$=(6-6)(x^2+y^2+36-xy+6x+6y)=0$$

10 $a^3-8b^3-24ab-64$

$$=a^3+(-2b)^3+(-4)^3-3\,a\,(-2b)(-4)$$
$$=(a-2b-4)\{a^2+4b^2+16+2ab+4a-8b\}$$
$$=(4-4)(a^2+4b^2+16+2ab+4a-8b)$$
$$=0\times(a^2+4b^2+16+2ab+4a-8b)$$
$$=0$$

11 The given expression

$$=\{(s-a)+(s-b)+(s-c)\}\{(s-a)^2+(s-b)^2+(s-c)^2$$
$$-(s-a)(s-b)-(s-a)(s-c)-(s-b)(s-c)\}$$
$$=\{3s-(a+b+c)\}\{(s-a)^2+(s-b)^2+(s-c)^2-(s-a)(s-b)$$
$$-(s-a)(s-c)-(s-b)(s-c)\}$$
$$=0\times\{(s-a)^2+(s-b)^2+(s-c)^2-(s-a)(s-b)$$
$$-(s-a)(s-c)-(s-b)(s-c)\}$$

12 Putting x for $(a-2b)$, y for $(2b-c)$ and z for $(3c-a)$, we have

$x+y+z=(a-2b)+(2b-c)+(3c-a)=0$

Hence $(a-2b)^3+(2b-3c)^3+(3c-a)^3-3(a-2b)(2b-3c)(3c-a)$

$$=x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$$
$$=0\times(x^2+y^2+z^2-xy-xz-yz)=0$$
$$(a-2b)^3+(2b-3c)^3+(3c-a)^3=3(a-2b)(2b-3c)(3c-a)$$

13 Putting a for $(x+y-2z)$, b for $(y+z-2x)$ and c for $(z+x-2y)$, we have $a+b+c=(x+y-2z)+(y+z-2x)+(z+x-2y)=0$

$$(x+y-2z)^3+(y+z-2x)^3+(z+x-2y)^3$$
$$-3(x+y-2z)(y+z-2x)(z+x-2y)$$
$$=a^3+b^3+c^3-3abc$$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=0\times(a^2+b^2+c^2-ab-ac-bc)=0$$
$$(x+y-2z)^3+(y+z-2x)^3+(z+x-2y)^3$$
$$=3(x+y-2z)(y+z-2x)(z+x-2y)$$

14 Putting x for $(a+2b-3c)$, y for $(b+2c-3a)$ and z for $(c+2a-3b)$, we have $x+y+z=(a+2b-3c)+(b+2c-3a)+(c+2a-3b)=0$

Hence
$$(a+2b-3c)^3+(b+2c-3a)^3+(c+2a-3b)^3-3(a+2b-3c)(b+2c-3a)(c+2a-3b)$$
$$=x^3+y^3+z^3-3xyz$$
$$=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$$
$$=0\times(x^2+y^2+z^2-xy-xz-yz)=0$$

$$(a+2b-3c)^3+(b+2c-3a)^3+(c+2a-3b)^3=3(a+2b-3c)(b+2c-3a)(c+2a-3b)$$

15. Putting x for $(2p-5q+3r)$, y for $(2q-5r+3p)$ and z for $(2r-5p+3q)$ we have $x+y+z$
$$=(2p-5q+3r)+(2q-5r+3p)+(2r-5p+3q)=0$$

Hence
$$(2p-5q+3r)^3+(2q-5r+3p)^3+(2r-5p+3q)^3-3(2p-5q+3r)(2q-5r+3p)(2r-5p+3q)$$
$$=x^3+y^3+z^3-3xyz$$
$$=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$$
$$=0\times(x^2+y^2+z^2-xy-xz-yz)=0$$

$$(2p-5q+3r)^3+(2q-5r+3p)^3+(2r-5p+3q)^3=3(2p-5q+3r)(2q-5r+3p)(2r-5p+3q)$$

Exercise 34

1 Putting a for $(x-y)$, b for $(y-z)$ and c for $(z-x)$, we have $a-b=x-2y+z$, $a-c=2x-y-z$ and $b-c=y-2z+x$

The given expression
$$=(a-b)(a-c)(b-c)=a^2(b-c)+b^2(c-a)+c^2(a-b)$$
$$=(x-y)^2(y-2z+x)+(y-z)^2(z-2x+y)+(z-x)^2(x-2y+z)$$

2 Putting x for $(a+b)$, y for $(b+c)$ and z for $(c+a)$, we have $x-y=a-c$, $y-z=b-a$ and $z-x=c-b$

The given expression
$$=x^2(y-z)+y^2(z-x)+z^2(x-y)+(y-z)(z-x)(x-y)$$
$$=-(y-z)(z-x)(x-y)+(y-z)(z-x)(x-y)=0$$

3 Putting x for $(a-b+c)$, y for $(b-c+a)$ and z for $(c-a+b)$, we have $x-y=2(c-b)$, $y-z=2(a-c)$ and $z-x=2(b-a)$

The given expression

$$=x^2(y-z)+y^2(z-x)+z^2(x-y)$$
$$=(x-y)(x-z)(y-z)$$
$$=2(c-b)\,2(a-b)\,2(a-c)$$
$$=8(c-b)(a-b)(a-c)$$

4 Putting a for $(x+y)$, b for $(y+z)$ and c for $(z+x)$, we have

$$a-b=x-z,\quad b-c=y-x \text{ and } c-a=z-y$$

The given expression

$$=a^2(b-c)+b^2(c-a)+c^2(a-b)$$
$$=(a-b)(a-c)(b-c)=(x-z)(y-z)(y-x)$$

5 Putting x for $(a-b-c)$, y for $(b-c-a)$ and z for $(c-a-b)$, we have $x-y=2(a-b)$, $y-z=2(b-c)$ and $z-x=2(c-a)$

$$(x-y)(y-z)(z-x)=8(a-b)(b-c)(c-a)$$

The given expression

$$=x^2(y-z)+y^2(z-x)+z^2(x-y)+(x-y)(y-z)(z-x)$$
$$=-(x-y)(y-z)(z-x)+(x-y)(y-z)(z-x)=0$$

6 Putting a for $(x-y)$, b for $(y-z)$ and c for $(z-x)$, we have $a-b=x-2y+z$, $b-c=y-2z+x$ and $c-a=z-2x+y$

The given expression

$$=ab(a-b)+bc(b-c)+ac(c-a)+(a-b)(b-c)(c-a)$$
$$=-(a-b)(b-c)(c-a)+(a-b)(b-c)(c-a)=0$$

Exercise 35.

1 $(a^2+ax-x^2)(a^2-ax+x^2)=\{a^2+(ax-x^2)\}\{a^2-(ax-x^2)\}$

$$=a^4-(ax-x^2)^2$$
$$=a^4-\{a^2x^2-2ax^3+x^4\}$$
$$=a^4-a^2x^2+2ax^3-x^4$$

2 $(a^2-ax+x^2)(ax-a^2+x^2)$

$$=\{x^2+(a^2-ax)\}\{x^2-(a^2-ax)\}$$
$$=x^4-(a^2-ax)^2=x^4-\{a^4-2a^3x+a^2x^2\}$$
$$=x^4-a^2x^2+2a^3x-a^4$$

3 $(a+b+c)(a-b-c)+(b+c-a)(a-b+c)$

$$=\{a+(b+c)\}\{a-(b+c)\}+\{c+(b-a)\}\{c-(b-a)\}$$
$$=\{a^2-(b+c)^2\}+\{c^2-(b-a)^2\}$$
$$=\{a^2-b^2-2bc-c^2+c^2-b^2+2ab-a^2\}$$
$$=2ab-2b^2-2bc=2b(a-b-c)$$

4 $2(a^2+b^2+c^2-ab-ac-bc)$

$$=(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)$$
$$=(a-b)^2+(b-c^2)+(c-a)^2$$

5 $\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$

$$=\tfrac{1}{2}(a+b+c)\{a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ac\}$$
$$=\tfrac{1}{2}(a+b+c)(2a^2+2b^2+2c^2-2ab-2bc-2ac)$$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=a^3+b^3+c^3-3abc$$

6 $(a+b)(a+c)(b+c)=\{a^2+(b+c)a+bc\}(b+c)$

$$=a^2(b+c)+a(b+c)^2+bc(b+c)$$
$$=a^2(b+c)+a(b^2+2bc+c^2)+b^2c+bc^2$$
$$=a^2(b+c)+ab^2+2abc+ac^2+b^2c+bc^2$$
$$=a^2(b+c)+b^2(a+c)+c^2(a+b)+2abc$$

7 $2(x^3-x)+3x(x+1)=2x(x^2-1)+3x(x+1)$

$$=2x(x+1)(x-1)+3x(x+1)$$
$$=x(x+1)\{2(x-1)+3\}$$
$$=x(x+1)(2x-2+3)=x(x+1)(2x+1)$$

8 $x^4+x+x(x+1)(2x+1)-2x(x+1)$

$$=x(x^3+1)+x(x+1)(2x+1)-2x(x+1)$$
$$=x(x+1)(x^2-x+1)+x(x+1)(2x+1)-2x(x+1)$$
$$=x(x+1)\{(x^2-x+1)+(2x+1)-2\}$$
$$=x(x+1)(x^2-x+1+2x+1-2)$$
$$=x(x+1)(x^2+x)=x(x+1)\ x(x+1)=x^2(x+1)^2$$

9 $(a^2+b^2)(c^2+d^2)=a^2c^2+b^2c^2+a^2d^2+b^2d^2$

$$=a^2c^2+b^2d^2-2acbd+b^2c^2+a^2d^2+2acbd$$
$$=(ac-bd)^2+(ad+bc)^2$$

10 $(a+b)^2-(c+d)^2+(a+c)^2-(b+d)^2$

$$=(a+b+c+d)(a+b-c-d)+(a+c+b+d)(a+c-b-d)$$

$$=(a+b+c+d)\{(a+b-c-d)+(a+c-b-d)\}$$
$$=(a+b+c+d)(a+b-c-d+a+c-b-d)$$
$$=2(a+b+c+d)(a-d)$$

11. $(9x^2+4y^2+z^2-2yz-3xz-6xy)(3x+2y+z)$
$$=\{(3x)^2+(2y)^2+z^2-(2y)z-(3x)z-(2y)3x\}(3x+2y+z)$$
$$=(3x)^3+(2y)^3+z^3-3(3x)(2y)z$$
$$=27x^3+8y^3+z^3-18xyz$$

12 $(a+b+c-d)(d-a-b+c)=\{c+(a+b-d)\}\{c-(a+b-d)\}$
$$=c^2-(a+b-d)^2$$

13 $\{(b+c)^2-a^2\}\{a^2-b^2-c^2+2bc\}$
$$=\{(b+c)^2-a^2\}\{a^2-(b^2+c^2-2bc)\}$$
$$=\{-a^2+(b+c)^2\}\{a^2-(b-c)^2\}$$
$$=-a^4+a^2\{(b+c)^2+(b-c)^2\}-(b^2-c^2)^2$$
$$=-a^4+a^2(2b^2+2c^2)-b^4-c^4+2b^2c^2$$
$$=-a^4+2a^2b^2+2a^2c^2-b^4-c^4+2b^2c^2$$
$$=2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4$$

14. $(a+b+c)(a+b-c)(a-b+c)(b+c-a)$
$$=\{(a+b)+c\}\{(a+b)-c\}\{c+(a-b)\}\{c-(a-b)\}$$
$$=\{(a+b)^2-c^2\}\{c^2-(a-b)^2\}$$
$$=\{-c^2+(a+b)^2\}\{c^2-(a-b)^2\}$$
$$=-c^4+c^2\{(a+b)^2+(a-b)^2\}-(a^2-b^2)^2$$
$$=-c^4+c^2(2a^2+2b^2)-a^4-b^4+2a^2b^2$$
$$=-c^4+2a^2c^2+2b^2c^2-a^4-b^4+2a^2b^2$$
$$=2a^2c^2+2b^2c^2+2a^2b^2-a^4-b^4-c^4$$

15 We have $(a^2+b^2+c^2)^2=a^4+b^4+c^4+2a^2b^2+2a^2c^2+2b^2c^2$ and $(a+b+c)(a+b-c)(a+c-b)(b+c-a)=2a^2b^2+2b^2c^2+2a^2c^2-a^4-b^4-c^4$ (as in the previous example) ,

The given expression $=4(a^2b^2+a^2c^2+b^2c^2)$

16 It follows evidently from the previous example that the given expression $=2(a^4+b^4+c^4)$

17. $ax=a^3-abc$, $by=b^3-abc$ and $cz=c^3-abc$

Hence $ax+by+cz=a^3+b^3+c^3-3abc$

$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=(a+b+c)\{(a^2-bc)+(b^2-ac)+(c^2-ab)\}$$
$$=(a+b+c)(x+y+z)$$

18 $a^2x=x^3-xyz$, $b^2y=y^3-xyz$ and $c^2z=z^3-xyz$

Hence $a^2x+b^2y+c^2z$

$$=x^3+y^3+z^3-3xyz$$
$$=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$$
$$=(x+y+z)\{(x^2-yz)+(y^2-xz)+(z^2-xy)\}$$
$$=(x+y+z)(a^2+b^2+c^2)$$

19 The given expression

$$=\{(a+b+c)^2-(a+b-c)^2\}+\{(a+c-b)^2-(b+c-a)^2\}$$
$$=\{(a+b+c)+(a+b-c)\}\{(a+b+c)-(a+b-c)\}$$
$$+\{(a+c-b)+(b+c-a)\}\{(a+c-b)-(b+c-a)\}$$
$$=(2a+2b)2c+(2c)(2a-2b)=2c\{(2a+2b)+(2a-2b)\}$$
$$=2c\ 4a=8ac$$

20 The given expression

$$=\{(a^2+b^2+c^2)^2-(b^2+c^2-a^2)^2\}-\{(a^2-b^2+c^2)^2-(a^2+b^2-c^2)^2\}$$
$$=\{(a^2+b^2+c^2)+(b^2+c^2-a^2)\}\{(a^2+b^2+c^2)-(b^2+c^2-a^2)\}$$
$$-\{(a^2-b^2+c^2)+(a^2+b^2-c^2)\}\{(a^2-b^2+c^2)-(a^2+b^2-c^2)\}$$
$$=(2b^2+2c^2)(2a^2)-(2a^2)(2c^2-2b^2)=2a^2\{(2b^2+2c^2-2c^2+2b^2)\}$$
$$=2a^2\ 4b^2=8a^2b^2$$

21 The left-hand expression

$$=\{(a+d)+(b-c)\}\{(a+d)-(b-c)\}+\{(b+c)$$
$$+(a-d)\}\{(b+c)-(a-d)\}$$
$$=\{(a+d)^2-(b-c)^2\}+\{(b+c)^2-(a-d)^2\}$$
$$=\{(a+d)^2-(a-d)^2\}+\{(b+c)^2-(b-c)^2\}$$
$$=4ad+4bc=4(ad+bc)$$

22 $(b+c+a-d)(b+c-a+d)$

$$=\{(b+c)+(a-d)\}\{(b+c)-(a-d)\}$$
$$=(b+c)^2-(a-d)^2=(b^2+c^2+2bc)-(a^2+d^2-2ad)$$
$$=2(ad+bc)-(a^2-b^2-c^2+d^2)$$

23 The given expression

$$=(2ad+2bc)^2-(a^2-b^2-c^2+d^2)^2$$

$$=(2ad+2bc+a^2-b^2-c^2+d^2)(2ad+2bc-a^2+b^2+c^2-d^2)$$

$$=\{(a^2+d^2+2ad)-(b^2+c^2-2bc)\}\{(b^2+c^2+2bc)-(a^2+d^2-2ad)\}$$

$$=\{(a+d)^2-(b-c)^2\}\{(b+c)^2-(a-d)^2\}$$

$$=(a+d+b-c)(a+d-b+c)(b+c+a-d)(b+c-a+d)$$

24 $(x^2+1-2x)(x^2+1+2x)=(x^2+1)^2-(2x)^2=x^4-2x^2+1$

Hence the given expression

$$=(x^4+1+2x^2)(x^4+1-2x^2)=(x^4+1)^2-4x^4$$
$$=x^8-2x^4+1$$

25 $(a^2+2ab+b^2-c^2)(a^2-2ab+b^2+c^2)$

$$=\{(a+b)^2-c^2\}\{(a-b)^2+c^2\}$$
$$=(a^2-b^2)^2+c^2\{(a+b)^2-(a-b)^2\}-c^4$$
$$=a^4-2a^2b^2+b^4+c^2\,4ab-c^4$$
$$=a^4-2a^2b^2+b^4+4abc^2-c^4$$

26 Putting a for $(x-y+z)$, b for $(y-z+x)$ and c for $(z-x+y)$, we have $a+b+c=x+y+z$ and

the given expression $=a^2+b^2+c^2+2ab+2bc+2ca$

$$=(a+b+c)^2=(x+y+z)^2$$

27 The given expression

$$=-\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}-a^2(c-b)+\{c^2(a-b)-b^2(c+a)\}$$
$$=-a^2(b-c)-b^2(c-a)-c^2(a-b)+a^2(b-c)+c^2(a-b)-b^2(c+a)$$
$$=-b^2(c-a+c+a)=-2b^2c$$

28 The given expression

$$=-\{ab(a-b)+bc(b-c)+ca(c-a)\}-ab(b-a)-bc(c-b)+ca(c+a)$$
$$=-ab(a-b)-bc(b-c)-ca(c-a)+ab(a-b)+bc(b-c)+ca(c+a)$$
$$=ca\{(c+a)-(c-a)\}=ca\,2a=2a^2c$$

29 The left-hand expression

$$=(x^2-2ax+a^2)(b-c)+(x^2-2bx+b^2)(c-a)+(x^2-2cx+c^2)(a-b)+(b-c)(c-a)(a-b)$$

$$=x^2\{(b-c)+(c-a)+(a-b)\}-2x\{a(b-c)+b(c-a)$$
$$+c(a-b)\}+\{a^2(b-c)+b^2(c-a)+c^2(a-b)$$
$$+(b-c)(c-a)(a-b)\}$$
$$=-(a-b)(b-c)(c-a)+(b-c)(c-a)(a-b)=0$$

30 The left-hand expression

$$=a^2x^2+a^2y^2+a^2z^2+b^2x^2+b^2y^2+b^2z^2+c^2x^2+c^2y^2+c^2z^2$$
$$-(a^2x^2+b^2y^2+c^2z^2+2abxy+2acxz+2bcyz)$$
$$=(a^2y^2-2abxy+b^2x^2)+(c^2x^2-2acxz+a^2z^2)$$
$$+(b^2z^2-2bcyz+c^2+y^2)$$
$$=(ay-bx)^2+(cx-az)^2+(bz-cy)^2$$

31 $(as+bc)(bs+ca)(cs+ab)$

$$=(a^2+ab+ac+bc)(ab+b^2+bc+ca)(ac+bc+c^2+ab)$$
$$=\{a(a+b)+c(a+b)\}\{b(a+b)+c(a+b)\}\{c(a+c)+b(a+c)\}$$
$$=(a+b)(a+c)(a+b)(b+c)(a+c)(b+c)$$
$$=(a+b)^2(a+c)^2(b+c)^2$$

32 $(a+b)(a+c)(b+c)-2abc$

$$=\{a^2+a(b+c)+bc\}(b+c)-2abc$$
$$=a^2(b+c)+a(b+c)^2+bc(b+c)-2abc$$
$$=a^2(b+c)+a(b^2+c^2+2bc)+b^2c+bc^2-2abc$$
$$=a^2(b+c)+b^2(a+c)+c^2(a+b)$$

33 Putting x for $(a+c)$ and y for $(b+c)$ we have $x-y=(a+c)-(b+c)=a-b$, and the left-hand expression $=x^3-y^3-3xy(x-y)=(x-y)^3=(a-b)^3$

34 Putting a for $(x-2y+3z)$ and b for $(x+2y-3z)$, we have $a+b=x-2y+3z+x+2y-3z=2x$, and the given expression $=a^3+b^3+3(a+b)ab$

$$=(a+b)^3=(2x)^3=8x^3$$

35 $4(a+b+c)^2=(2a+2b+2c)^2$

$$=\{(a+b)+(b+c)+(c+a)\}^2$$
$$=(a+b)^2+(b+c)^2+(c+a)^2+2(a+b)(b+c)$$
$$+2(a+b)(c+a)+2(b+c)(c+a)$$

36 $8(a+b+c)^3=(2a+2b+2c)^3$

$$=\{(a+b)+(b+2c+a)\}^3$$

$$=(a+b)^3+(b+2c+c)^3+3(a+b)(b+2c+a)\{2(a+b+c)\}$$

$$=(a+b)^3+(b+2c+a^3)+6(a+b)(b+2c+a)(a+b+c)$$

37 $27(a+b+c)^3=\{3(a+b+c)\}^3$

$$=(3a+3b+3c)^3$$

$$=\{(a+3b+2c)+(2a+c)\}^3$$

$$=(a+3b+2c)^3+(2a+c)^3+3(a+3b+2c)(2a+c)\{3(a+b+c)\}$$

$$=(a+3b+2c)^3+(2a+c)^3+9(a+3b+2c)(2a+c)(a+b+c)$$

38 $8(a+b+c)^3=(2a+2b+2c)^3=\{(a+b)+(b+c)+(c+a)\}^3$

$$=(a+b)^3+(b+c)^3+(c+a)^3+3\{(a+b)+(b+c)\}\{(b+c)+(c+a)\}\{(c+a)+(a+b)\}$$

(See Ex 6, Page 91)

$$=(a+b)^3+(b+c)^3+(c+a)^3+3(a+2b+c)(a+2c+b)(2a+b+c)$$

$$8(a+b+c)^3-(a+b)^3-(b-c)^3-(c-a)^3$$

$$=3(2a+b+c)(a+2b+c)(a+b+2c)$$

39 $27(a+b+c)^3=(3a+3b+3c)^3$

$$=\{(a+2b)+(b+2c)+(c+2a)\}^3$$

$$=(a+2b)^3+(b+2c)^3+(c+2a)^3+3\{(a+2b)+(b+2c)\}\{(b+2c)+(c+2a)\}\{(c+2a)+(a+2b)\}$$

$$=(a+2b)^3+(b+2c)^3+(c+2a)^3+3(a+3b+2c)(b+3c+2a)(c+3a+2b)$$

$$\therefore 27(a+b+c)^3-(a+2b)^3-(b+2c)^3-(c+2a)^3$$

$$=3(a+3b+2c)(b+3c+2a)(c+3a+2b)$$

40 $2(s-a)(s-b)(s-c)+a(s-b)(s-c)+b(s-c)(s-a)+c(s-a)(s-b)$

$$=(s-b)(s-c)\{2s-2a+a\}+b(s-c)(s-a)+c(s-a)(s-b)$$

$$=(s-b)(s-c)(2s-a)+b(s-c)(s-a)+c(s-a)(s-b)$$

$$=(s-b)(s-c)(b+c)+b(s-c)(s-a)+c(s-a)(s-b)$$

$$=b(s-b)(s-c)+b(s-c)(s-a)+c(s-a)(s-b)+c(s-b)(s-c)$$

$$=b(s-c)(s-b+s-a)+c(s-b)(s-a+s-c)$$

$$=b(s-c)(2s-b-a)+c(s-b)(2s-a-c)$$

$$=b(s-c)c+c(s-b)b$$
$$=bc(s-c)+bc(s-b)=bc(s-c+s-b)$$
$$=bc(2s-b-c)=bc(a)=abc$$

41 $s(s-a)(s-b)+s(s-a)(s-c)+s(s+a)(s-c)+c(s+a)(s+b)$

$$=s(s-a)(s-b+s-c)+s(s+a)(s-c)+c(s+a)(s+b)$$
$$=s(s-a)(s+a)+s(s+a)(s-c)+c(s+a)(s+b)$$
$$[\quad s-b-c=a]$$
$$=s(s+a)(s-a+s-c)+c(s+a)(s+b)$$
$$=s(s+a)(s+b)+c(s+a)(s+b)\qquad[\quad s-a-c=b]$$
$$=(s+a)(s+b)(s+c)$$

42 The left-hand expression

$$=\tfrac{1}{2}\{(s-a)+(s-b)+(s-c)\}[\{(s-a)-(s-b)\}^2$$
$$+\{(s-b)-(s-c)\}^2+\{(s-c)-(s-a)\}^2]$$
$$=\tfrac{1}{2}\{3s-(a+b+c)\}[(b-a)^2+(c-b)^2+(a-c)^2]$$
$$=\tfrac{1}{2}(s)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$
$$=\tfrac{1}{2}\{\tfrac{1}{2}(a+b+c)\}\{(a-b)^2+(b-c)^2+(c-a)^2\}$$
$$=\tfrac{1}{2}\{a^3+b^3+c^3-3abc\}$$

43 $(s-3a)^2+(s-3b)^2+(s-3c)^2$

$$=(s^2-6as+9a^2)+(s^2-6bs+9b^2)+(s^2-6cs+9c^2)$$
$$=3s^2-6s(a+b+c)+9(a^2+b^2+c^2)$$
$$=3\{s^2-2s^2+3(a^2+b^2+c^2)\}$$
$$=3\{3(a^2+b^2+c^2)-s^2\}$$
$$=3\{3(a^2+b^2+c^2)-(a+b+c)^2\}$$
$$=3\{3(a^2+b^2+c^2)-(a^2+b^2+c^2+2ab+2ac+2bc)\}$$
$$=3\{2(a^2+b^2+c^2)-2(ab+bc+ca)\}$$
$$=3\{(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ca)\}$$
$$=3\{(a-b)^2+(b-c)^2+(c-a)^2\}$$

44 We have $2(ab+bc+ca)$

$$=(a+b+c)^2-(a^2+b^2+c^2)$$
$$=(13)^2-69=169-69=100\qquad (ab+bc+ca)=50$$

Hence $a^3+b^3+c^3-3abc=(a+b+c)\{(a^2+b^2+c^2)-(ab+bc+ca)\}$

$$=13\{69-50\}=13\times 19=247$$

45 We have $a^2+b^2+c^2=(a+b+c)^2-2(ab+bc+ca)$

$$=(12)^2-2\cdot 47=144-94=50$$

$$a^3+b^3+c^3-3abc=(a+b+c)\{(a^2+b^2+c^2)$$
$$-(ab+bc+ca)\}=12\{50-47\}=12\times 3=\mathbf{36}$$

46 We have $2(xy+yz+zx)$

$$=(x+y+z)^2-(x^2+y^2+z^2)$$
$$=(14)^2-74=196-74=122 \qquad \therefore xy+yz+zx=61$$

Hence $x^3+y^3+z^3-3xyz=(x+y+z)\{(x^2+y^2+z^2)$

$$-(xy+yz+zx)\}=14\{74-61\}=14\times 13=182$$
$$\therefore 3xyz=(x^3+y^3+z^3)-(x^3+y^3+z^3-3xyz)$$
$$=434-182=252 \qquad \therefore xyz=\mathbf{84}$$

47 $x^2+y^2+z^2=(x+y+z)^2-2(xy+yz+zx)$

$$=(13)^2-2(52)=169-104=65 \qquad \therefore x^3+y^3+z^3$$
$$=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)+3xyz$$
$$=13(65-52)+180=13\cdot 13+180=169+180=\mathbf{349}$$

Exercise 36.

The examples are to be worked out mentally by the application of the rule given in Art 4, Page 99

Exercise 37.

1 $\dfrac{a^3b^2-2a^2b^3}{a^2b^2}=\dfrac{a^3b^2}{a^2b^2}+\dfrac{-2a^2b^3}{a^2b^2}=\underline{a-2b}$

2 $\dfrac{3x^4-6x^3a}{-3x^2}=\dfrac{3x^4}{-3x^2}+\dfrac{-6x^3a}{-3x^2}=\underline{-x^2+2xa}$

3 $\dfrac{4x^3a^3-8x^4a^2}{4x^2a^2}=\dfrac{4x^3a^3}{4x^2a^2}+\dfrac{-8x^4a^2}{4x^2a^2}=\underline{xa-2x^2}$

4 $\dfrac{-9a^6+12a^3b^2}{-3a^3}=\dfrac{-9a^6}{-3a^3}+\dfrac{12a^3b^2}{-3a^3}=\underline{3a^3-4b^2}$

5 $\dfrac{14a^6b^4-21a^7b^3}{-7a^5b^2}=\dfrac{14a^6b^4}{-7a^5b^2}+\dfrac{-21a^7b^3}{-7a^5b^2}=\underline{-2ab^2+3a^2b}$

6 $$\frac{2ax^3-4a^2x^2+6a^3x}{2ax}=\frac{2ax^3}{2ax}+\frac{-4a^2x^2}{2ax}+\frac{6a^3x}{2ax}$$
$$=\underline{x^2-2ax+3a^2}$$

7 $$\frac{-3a^3x^4+6a^2x^5-9a^4x^3}{-3a^2x^3}=\frac{-3a^3x^4}{-3a^2x^3}+\frac{6a^2x^5}{-3a^2x^3}+\frac{-9a^4x^3}{-3a^2x^3}$$
$$=\underline{ax-2x^2+3a^2}$$

8 $$\frac{12x^5-8x^3a^2+20ax^4}{-4x^3}=\frac{12x^5}{-4x^3}+\frac{-8x^3a^2}{-4x^3}+\frac{20ax^4}{-4x^3}$$
$$=\underline{-3x^2+2a^2-5ax}$$

9 $$\frac{10m^5n^4-15m^7n^2-20m^3n^6}{5m^3n^2}=\frac{10m^5n^4}{5m^3n^2}+\frac{-15m^7n^2}{5m^3n^2}+\frac{-20m^3n^6}{5m^3n^2}$$
$$=\underline{2m^2n^2-3m^4-4n^4}$$

10 $$\frac{8p^4q^2-5p^3q^3-3p^2q^4}{-8p^2q^2}=\frac{8p^4q^2}{-8p^2q^2}+\frac{-5p^3q^3}{-8p^2q^2}+\frac{-3p^2q^4}{-8p^2q^2}$$
$$=\underline{-p^2+\tfrac{5}{8}pq+\tfrac{3}{8}q^2}$$

11 $$\frac{-14x^8y^5+21x^{10}y^3-28x^7y^6}{7x^7y^3}=\frac{-14x^8y^5}{7x^7y^3}+\frac{21x^{10}y^3}{7x^7y^3}+\frac{-28x^7y^6}{7x^7y^3}$$
$$=\underline{-2xy^2+3x^3-4y^3}$$

12 $$\frac{15a^4x^8-30a^7x^5-45a^6x^6}{20a^4x^5}=\frac{15a^4x^8}{20a^4x^5}+\frac{-30a^7x^5}{20a^4x^5}+\frac{-45a^6x^6}{20a^4x^5}$$
$$=\underline{\tfrac{3}{4}x^3-\tfrac{3}{2}a^3-\tfrac{9}{4}a^2x}$$

13 $$\frac{-60x^4a^5-75x^3a^6+80x^6a^4}{-20x^3a^4}=\frac{-60x^4a^5}{-20x^3a^4}+\frac{-75x^3a^6}{-20x^3a^4}+\frac{80x^6a^4}{-20x^3a^4}$$
$$=\underline{3xa+\tfrac{15}{4}a^2-4x^2}$$

14 $$\frac{25m^3n^2p-35m^2n^3p-40mnp^4}{5mnp}=\frac{25m^3n^2p}{5mnp}+\frac{-35m^2n^3p}{5mnp}$$
$$+\frac{-40mnp^4}{5mnp}=\underline{5m^2n-7mn^2-8p^3}$$

15 $$\frac{-a^2x^3y^2+a^4x^2y-ax^3y^3+a^5xy}{-axy}=\frac{-a^2x^3y^2}{-axy}+\frac{a^4x^2y}{-axy}+\frac{ax^3y^3}{-axy}$$
$$+\frac{a^5xy}{-axy}=\underline{ax^2y-a^3x+x^2y^2-a^4}$$

Exercise 38

1

$$x-3\overline{)\,x^2-5x+6\,}\left(\underline{x-2}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{x^2-3x} \\ -2x+6 \\ \underline{-2x+6} \end{array}$$

2

$$2x-1\overline{)\,2x^2-11x+5\,}\left(\underline{x-5}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{2x^2-x} \\ -10x+5 \\ \underline{-10x+5} \end{array}$$

3

$$2x-3\overline{)\,6x^2-x-12\,}\left(\underline{3x+4}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{6x^2-9x} \\ 8x-12 \\ \underline{8x-12} \end{array}$$

4

$$3x+4\overline{)\,15x^2-x-28\,}\left(\underline{5x-7}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{15x^2+20x} \\ -21x-28 \\ \underline{-21x-28} \end{array}$$

5

$$a-2b\overline{)\,2a^2-7ab+6b^2\,}\left(\underline{2a-3b}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{2a^2-4ab} \\ -3ab+6b^2 \\ \underline{-3ab+6b^2} \end{array}$$

6

$$x^2+xy+y^2\overline{)\,x^4+x^2y^2+y^4\,}\left(\underline{x^2-xy+y^2}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{x^4+x^3y+x^2y^2} \\ -x^3y+y^4 \\ \underline{-x^3y-x^2y^2-xy^3} \\ x^2y^2+xy^3+y^4 \\ \underline{x^2y^2+xy^3+y^4} \end{array}$$

7

$$2x+3a\overline{)\,4x^2-9a^2\,}\left(\underline{2x-3a}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{4x^2+6ax} \\ -6ax-9a^2 \\ \underline{-6ax-9a^2} \end{array}$$

8

$$x+a\overline{)\,x^3+a^3\,}\left(\underline{x^2-ax+a^2}\right. \quad \text{(Quotient)}$$
$$\begin{array}{r} \underline{x^3+ax^2} \\ -ax^2+a^3 \\ \underline{-ax^2-a^2x} \\ a^2x+a^3 \\ \underline{a^2x+a^3} \end{array}$$

9 $$\begin{array}{l} a-3b\,\big)\,a^3-a^2b-7ab^2+3b^3\,\big(\,\underline{a^2+2ab-b^2}\ \text{(Quotient)} \\ \qquad\quad \underline{a^3-3a^2b} \\ \qquad\qquad 2a^2b-7ab^2+3b^3 \\ \qquad\qquad \underline{2a^2b-6ab^2} \\ \qquad\qquad\qquad -ab^2+3b^3 \\ \qquad\qquad\qquad \underline{-ab^2+3b^3} \end{array}$$

10 $$\begin{array}{l} 2m-3n\,\big)\,2m^3-9m^2n+13mn^2-6n^3\,\big(\,\underline{m^2-3mn+2n^2} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Quotient)} \\ \qquad\qquad \underline{2m^3-3m^2n} \\ \qquad\qquad\quad -6m^2n+13mn^2-6n^3 \\ \qquad\qquad\quad \underline{-6m^2n+9mn^2} \\ \qquad\qquad\qquad\qquad 4mn^2-6n^3 \\ \qquad\qquad\qquad\qquad \underline{4mn^2-6n^3} \end{array}$$

11 $$\begin{array}{l} a^2-b^2\,\big)\,a^4-3a^3b+3ab^3-b^4\,\big(\,\underline{a^2-3ab+b^2}\ \text{(Quotient)} \\ \qquad\qquad \underline{a^4-a^2b^2} \\ \qquad\qquad\quad -3a^3b+a^2b^2+3ab^3-b^4 \\ \qquad\qquad\quad \underline{-3a^3b+3ab^3} \\ \qquad\qquad\qquad\qquad a^2b^2-b^4 \\ \qquad\qquad\qquad\qquad \underline{a^2b^2-b^4} \end{array}$$

12 $$\begin{array}{l} x^2+y^2\,\big)\,2x^4-3x^3y-3xy^3-2y^4\,\big(\,\underline{2x^2-3xy-2y^2} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Quotient)} \\ \qquad\qquad \underline{2x^4+2x^2y^2} \\ \qquad\qquad\quad -3x^3y-2x^2y^2-3xy^3-2y^4 \\ \qquad\qquad\quad \underline{-3x^3y-3xy^3} \\ \qquad\qquad\qquad\qquad -2x^2y^2-2y^4 \\ \qquad\qquad\qquad\qquad \underline{-2x^2y^2-2y^4} \end{array}$$

13 $$\begin{array}{l} 2a^2+8ax\,\big)\,2a^4-36a^2x^2-16ax^3\,\big(\,\underline{a^2-4ax-2x^2} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Quotient)} \\ \qquad\qquad \underline{2a^4+8a^3x} \\ \qquad\qquad\quad -8a^3x-36a^2x^2-16ax^3 \\ \qquad\qquad\quad \underline{-8a^3x-32a^2x^2} \\ \qquad\qquad\qquad\qquad -4a^2x^2-16ax^3 \\ \qquad\qquad\qquad\qquad \underline{-4a^2x^2-16ax^3} \end{array}$$

14 $$\begin{array}{l} 1+2x^2\,\big)\,3+2x+4x^2+5x^3-4x^4+2x^5\,\big(\,\underline{3+2x-2x^2+x^3} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Quotient)} \\ \qquad\qquad \underline{3+6x^2} \\ \qquad\qquad\quad 2x-2x^2+5x^3-4x^4+2x^5 \\ \qquad\qquad\quad \underline{2x+4x^3} \\ \qquad\qquad\qquad -2x^2+x^3-4x^4+2x^5 \\ \qquad\qquad\qquad \underline{-2x^2-4x^4} \\ \qquad\qquad\qquad\qquad x^3+2x^5 \\ \qquad\qquad\qquad\qquad \underline{x^3+2x^5} \end{array}$$

15 $x^2+2x-3\,)\,x^4 \quad -4x^2+12x-9\,(\,x^2-2x+3$ (Quotient)

$$\begin{array}{r}
x^4+2x^3-3x^2 \qquad\qquad\quad \\ \hline
-2x^3-x^2+12x-9 \\
-2x^3-4x^2+6x \quad \\ \hline
3x^2+6x-9 \\
3x^2+6x-9 \\ \hline
\end{array}$$

16 $2a^2-3ab+4b^2\,)\,4a^4 \quad -9a^2b^2+24ab^3-16b^4\,(\,2a^2+3ab-4b^2$ (Quotient)

$$\begin{array}{r}
4a^4-6a^3b+8a^2b^2 \qquad\qquad\qquad\quad \\ \hline
6a^3b-17a^2b^2+24ab^3-16b^4 \\
6a^3b-9a^2b^2+12ab^3 \qquad\quad \\ \hline
-8a^2b^2+12ab^3-16b^4 \\
-8a^2b^2+12ab^3-16b^4 \\ \hline
\end{array}$$

17 $a^2+2ax+4x^2\,)\,a^4 \quad +4a^2x^2 \quad +16x^4\,(\,a^2-2ax+4x^2$ (Quotient)

$$\begin{array}{r}
a^4+2a^3x+4a^2x^2 \qquad\qquad\qquad\quad \\ \hline
-2a^3x \qquad\qquad\qquad +16x^4 \\
-2a^3x-4a^2x^2-8ax^3 \qquad\quad \\ \hline
4a^2x^2+8ax^3+16x^4 \\
4a^2x^2+8ax^3+16x^4 \\ \hline
\end{array}$$

18 $a^2+2ab+2b^2\,)\,a^4 \quad +4b^4\,(\,a^2-2ab+2b^2$ (Quotient)

$$\begin{array}{r}
a^4+2a^3b+2a^2b^2 \qquad\qquad\qquad\quad \\ \hline
-2a^3b-2a^2b^2 \qquad\qquad +4b^4 \\
-2a^3b-4a^2b^2-4ab^3 \qquad\quad \\ \hline
2a^2b^2+4ab^3+4b^4 \\
2a^2b^2+4ab^3+4b^4 \\ \hline
\end{array}$$

19 $x^3-2x^2+1\,)\,2x^5-7x^4-2x^3+18x^2-3x-8\,(\,2x^2-3x-8$ (Quotient)

$$\begin{array}{r}
2x^5-4x^4 \qquad\quad +2x^2 \qquad\qquad\quad \\ \hline
-3x^4-2x^3+16x^2-3x-8 \\
-3x^4+6x^3 \qquad\qquad -3x \quad \\ \hline
-8x^3+16x^2 \qquad -8 \\
-8x^3+16x^2 \qquad -8 \\ \hline
\end{array}$$

20 $x-3\,)\,x^4 \quad -81\,(\,x^3+3x^2+9x+27$ (Quotient)

$$\begin{array}{r}
x^4-3x^3 \qquad\qquad\qquad \\ \hline
3x^3 \qquad\qquad -81 \\
3x^3-9x^2 \qquad\qquad \\ \hline
9x^2 \qquad -81 \\
9x^2-27x \qquad \\ \hline
27x-81 \\
27x-81 \\ \hline
\end{array}$$

21.

$$
\begin{array}{r|l}
a-2 & a^5 \qquad\qquad -32\ (a^4+2a^3+4a^2+8a+16 \text{ (Quotient)} \\
 & \underline{a^5-2a^4} \\
 & 2a^4 \qquad -32 \\
 & \underline{2a^4-4a^3} \\
 & 4a^3 \qquad -32 \\
 & \underline{4a^3-8a^2} \\
 & 8a^2 \qquad -32 \\
 & \underline{8a^2-16a} \\
 & 16a-32 \\
 & \underline{16a-32}
\end{array}
$$

22

$$
\begin{array}{r|l}
1-3x+x^2 & 3-9x+2x^2+5x^3-7x^4+2x^5\ (3-x^2+2x^3 \text{ (Quotient)} \\
 & \underline{3-9x+3x^2} \\
 & -x^2+5x^3-7x^4+2x^5 \\
 & \underline{-x^2+3x^3-x^4} \\
 & 2x^3-6x^4+2x^5 \\
 & \underline{2x^3-6x^4+2x^5}
\end{array}
$$

23

$$
\begin{array}{r|l}
6x^2-7x+8 & 18x^4-45x^3+82x^2-67x+40\ (3x^2-4x+5 \text{ (Quotient)} \\
 & \underline{18x^4-21x^3+24x^2} \\
 & -24x^3+58x^2-67x+40 \\
 & \underline{-24x^3+28x^2-32x} \\
 & 30x^2-35x+40 \\
 & \underline{30x^2-35x+40}
\end{array}
$$

24

$$
\begin{array}{r|l}
2-x & 64 \qquad\qquad -x^6\ (32+16x+8x^2+4x^3+2x^4+x^5 \text{ (Quotient)} \\
 & \underline{64-32x} \\
 & 32x \qquad -x^6 \\
 & \underline{32x-16x^2} \\
 & 16x^2 \qquad -x^6 \\
 & \underline{16x^2-8x^3} \\
 & 8x^3 \qquad -x^6 \\
 & \underline{8x^3-4x^4} \\
 & 4x^4 \qquad -x^6 \\
 & \underline{4x^4-2x^5} \\
 & 2x^5-x^6 \\
 & \underline{2x^5-x^6}
\end{array}
$$

25
$$
\begin{array}{l}
x^2-2x+1\,\big)\,x^6 \qquad -2x^3 \qquad +1\,\big(\,x^4+2x^3+3x^2+2x+1 \quad \text{(Quotient)}\\
\qquad \underline{x^6-2x^5+x^4}\\
\qquad\quad 2x^5-x^4-2x^3+1\\
\qquad\quad \underline{2x^5-4x^4+2x^3}\\
\qquad\qquad 3x^4-4x^3+1\\
\qquad\qquad \underline{3x^4-6x^3+3x^2}\\
\qquad\qquad\quad 2x^3-3x^2+1\\
\qquad\qquad\quad \underline{2x^3-4x^2+2x}\\
\qquad\qquad\qquad x^2-2x+1\\
\qquad\qquad\qquad \underline{x^2-2x+1}
\end{array}
$$

26
$$
\begin{array}{l}
3a^2+4ab+b^2\,\big)\,6a^4-a^3b+2a^2b^2+13ab^3+4b^4\,\big(\,2a^2-3ab+4b^2 \quad \text{(Quotient)}\\
\qquad \underline{6a^4+8a^3b+2a^2b^2}\\
\qquad\quad -9a^3b+13ab^3+4b^4\\
\qquad\quad \underline{-9a^3b-12a^2b^2-3ab^3}\\
\qquad\qquad 12a^2b^2+16ab^3+4b^4\\
\qquad\qquad \underline{12a^2b^2+16ab^3+4b^4}
\end{array}
$$

27
$$
\begin{array}{l}
a^2-2ab+3b^2\,\big)\,a^4+a^3b-8a^2b^2+19ab^3-15b^4\,\big(\,a^2+3ab-5b^2 \quad \text{(Quotient)}\\
\qquad \underline{a^4-2a^3b+3a^2b^2}\\
\qquad\quad 3a^3b-11a^2b^2+19ab^3-15b^4\\
\qquad\quad \underline{3a^3b-6a^2b^2+9ab^3}\\
\qquad\qquad -5a^2b^2+10ab^3-15b^4\\
\qquad\qquad \underline{-5a^2b^2+10ab^3-15b^4}
\end{array}
$$

28
$$
\begin{array}{l}
x^3-2x^2a+2xa^2-a^3\,\big)\,x^6 \qquad -a^6\,\big(\,x^3+2x^2a+2xa^2+a^3 \quad \text{(Quotient)}\\
\qquad \underline{x^6-2x^5a+2x^4a^2-x^3a^3}\\
\qquad\quad 2x^5a-2x^4a^2+x^3a^3-a^6\\
\qquad\quad \underline{2x^5a-4x^4a^2+4x^3a^3-2x^2a^4}\\
\qquad\qquad 2x^4a^2-3x^3a^3+2x^2a^4-a^6\\
\qquad\qquad \underline{2x^4a^2-4x^3a^3+4x^2a^4-2xa^5}\\
\qquad\qquad\quad x^3a^3-2x^2a^4+2xa^5-a^6\\
\qquad\qquad\quad \underline{x^3a^3-2x^2a^4+2xa^5-a^6}
\end{array}
$$

29
$$
\begin{array}{l}
a^2+2ab-3b^2\,\big)\,a^5-a^4b-9a^3b^2+8a^2b^3-2ab^4+3b^5\,\big(\,a^3-3a^2b-b^3 \quad \text{(Quotient)}\\
\qquad \underline{a^5+2a^4b-3a^3b^2}\\
\qquad\quad -3a^4b-6a^3b^2+8a^2b^3-2ab^4+3b^5\\
\qquad\quad \underline{-3a^4b-6a^3b^2+9a^2b^3}\\
\qquad\qquad -a^2b^3-2ab^4+3b^5\\
\qquad\qquad \underline{-a^2b^3-2ab^4+3b^5}
\end{array}
$$

$$
\begin{array}{l}
x^2-2xy+y^2\,\big)\,x^6 - 2x^3y^3 + y^6\,\big(\,x^4+2x^3y+3x^2y^2+2xy^3+y^4 \text{ (Quotient)}\\
\qquad x^6-2x^5y+x^4y^2\\
\hline
\qquad 2x^5y-x^4y^2-2x^3y^3+y^6\\
\qquad 2x^5y-4x^4y^2+2x^3y^3\\
\hline
\qquad\qquad 3x^4y^2-4x^3y^3+y^6\\
\qquad\qquad 3x^4y^2-6x^3y^3+3x^2y^4\\
\hline
\qquad\qquad\qquad 2x^3y^3-3x^2y^4+y^6\\
\qquad\qquad\qquad 2x^3y^3-4x^2y^4+2xy^5\\
\hline
\qquad\qquad\qquad\qquad x^2y^4-2xy^5+y^6\\
\qquad\qquad\qquad\qquad x^2y^4-2xy^5+y^6\\
\hline
\end{array}
$$

31

$$
\begin{array}{l}
x^2-2ax+a^2\,\big)\,x^6 - 2a^3x^3 + a^6\,\big(\,x^4+2ax^3+3a^2x^2+2a^3x+a^4 \text{ (Quotient)}\\
\qquad x^6-2ax^5+a^2x^4\\
\hline
\qquad 2ax^5-a^2x^4-2a^3x^3+a^6\\
\qquad 2ax^5-4a^2x^4+2a^3x^3\\
\hline
\qquad\qquad 3a^2x^4-4a^3x^3+a^6\\
\qquad\qquad 3a^2x^4-6a^3x^3+3a^4x^2\\
\hline
\qquad\qquad\qquad 2a^3x^3-3a^4x^2+a^6\\
\qquad\qquad\qquad 2a^3x^3-4a^4x^2+2a^5x\\
\hline
\qquad\qquad\qquad\qquad a^4x^2-2a^5x+a^6\\
\qquad\qquad\qquad\qquad a^4x^2-2a^5x+a^6\\
\hline
\end{array}
$$

32 $x^2+2xy+y^2\,)\,x^6+2x^3y^3+y^6\,(\,x^4-2x^3y+3x^2y^2-2xy^3+y^4$ (Quotient)

$$\begin{array}{l}
x^6+2x^5y+x^4y^2 \\
\hline
-2x^5y-x^4y^2+2x^3y^3 \\
-2x^5y-4x^4y^2-2x^3y^3 \\
\hline
3x^4y^2+4x^3y^3 \\
3x^4y^2+6x^3y^3+3x^2y^4 \\
\hline
-2x^3y^3-3x^2y^4 \\
-2x^3y^3-4x^2y^4-2xy^5 \\
\hline
x^2y^4+2xy^5+y^6 \\
x^2y^4+2xy^5+y^6 \\
\hline
\end{array}$$

33 $x+c\,)\,x^3+(a+b+c)x^2+(ab+ac+bc)x+abc\,(\,x^2+(a+b)x+ab$ (Quotient)

$$\begin{array}{l}
x^3+cx^2 \\
\hline
(a+b)x^2+(ab+ac+bc)x+abc \\
(a+b)x^2+(ac+bc)x \\
\hline
abx+abc \\
abx+abc \\
\hline
\end{array}$$

34 $x^2+(b-a)x-ab\,)\,x^3+(b-c-a)x^2+(ca-ab-bc)x+abc\,(\,x-c$ (Quotient)

$$\begin{array}{l}
x^3+(b-a)x^2-abx \\
\hline
-cx^2+(ca-bc)x+abc \\
-cx^2+(ca-bc)x+abc \\
\hline
\end{array}$$

35 $a^2-bc\,)\,a^3+a^2b+a^2c-abc-b^2c-bc^2\,(\,a+b+c$ (Quotient)

$$\begin{array}{l}
a^3-abc \\
\hline
a^2b+a^2c-b^2c-bc^2 \\
a^2b-b^2c \\
\hline
a^2c-bc^2 \\
a^2c-bc^2 \\
\hline
\end{array}$$

36 $$a-(b-c)\overline{\big)\,a^2(b+c)-a(b^2-bc-c^2)-bc(b-c)\,\big(}\,a(b+c)+bc=ab+ac+bc \text{ (Quotient)}$$
$$\begin{array}{l}
\underline{a^2(b+c)-a(b^2-c^2)} \\
\qquad abc \qquad -bc(b-c) \\
\qquad \underline{abc \qquad -bc(b-c)}
\end{array}$$

37 $$a+(b+c)\overline{\big)\,a^2(b+c)+a(b^2+c^2+bc)-bc(b+c)\,\big(}\,a(b+c)-bc=ab+ac-bc \text{ (Quotient)}$$
$$\begin{array}{l}
\underline{a^2(b+c)+a(b^2+c^2+2bc)} \\
\qquad -abc-bc(b+c) \\
\qquad \underline{-abc-bc(b+c)}
\end{array}$$

38 $$x-(a+b)\overline{\big)\,x^3-2ax^2+(a^2-ab-b^2)x+ab(a+b)\,\big(}\,x^2-(a-b)x-ab \text{ (Quotient)}$$
$$\begin{array}{l}
\underline{x^3-(a+b)x^2} \\
\quad -(a-b)x^2+(a^2-ab-b^2)x+ab(a+b) \\
\quad \underline{-(a-b)x^2+(a^2 \qquad -b^2)x} \\
\qquad\qquad -abx+ab(a+b) \\
\qquad\qquad \underline{-abx+ab(a+b)}
\end{array}$$

39 $$a+b+c\overline{\big)\,a^3 \qquad -3abc+b^3+c^3\,\big(}\,a^2-ab-ac+b^2-bc+c^2 \text{ (Quotient)}$$
$$\begin{array}{l}
\underline{a^3+a^2b+a^2c} \\
\quad -a^2b-a^2c \qquad -3abc+b^3+c^3 \\
\quad \underline{-a^2b-ab^2 \qquad -abc} \\
\qquad -a^2c+ab^2-2abc \qquad +b^3+c^3 \\
\qquad \underline{-a^2c \qquad -abc \quad -ac^2} \\
\qquad\quad ab^2-abc+ac^2+b^3 \qquad +c^3 \\
\qquad\quad \underline{ab^2 \qquad +b^3+b^2c} \\
\qquad\qquad -abc+ac^2 \quad -b^2c \quad +c^3 \\
\qquad\qquad \underline{-abc \qquad -b^2c-bc^2} \\
\qquad\qquad\qquad ac^2 \qquad +bc^2+c^3 \\
\qquad\qquad\qquad \underline{ac^2 \qquad +bc^2+c^3}
\end{array}$$

Or Otherwise :—

The Dividend $= a^3+b^3+c^3-3abc$

$= (a+b+c)(a^2+b^2+c^2-ab-ac-bc)$, (Art 13, Ch VI)

the Divisor $= a+b+c$, and the quotient $= a^2+b^2+c^2-ab-ac-bc$

40 $x+y-1$) $x^3 + 3xy + y^3 - 1$ ($x^2 - xy + x + y^2 + y + 1$ (Quotient)

$$
\begin{array}{l}
x^3 + x^2y - x^2 \\
\hline
-x^2y + x^2 + 3xy + y^3 - 1 \\
-x^2y - xy^2 + xy \\
\hline
x^2 + xy^2 + 2xy + y^3 - 1 \\
x^2 + xy - x \\
\hline
xy^2 + xy + x + y^3 - 1 \\
xy^2 + y^3 - y^2 \\
\hline
xy + x + y^2 - 1 \\
xy + y^2 - y \\
\hline
x + y - 1 \\
x + y - 1 \\
\hline
\end{array}
$$

Or Otherwise :—

The Dividend $= (x)^3+(y)^3+(-1)^3-3(x)(y)(-1)$

$= (x+y-1)(x^2+y^2+1-xy+x+y)$, (Art 13, Ch VI)

the Divisor $= (x+y-1)$, and the quotient $= x^2+y^2+1-xy+x+y$

41 $x-2y-3z$) $x^3 - 18xyz - 8y^3 - 27z^3$ ($x^2+2xy+3xz+4y^2-6yz+9z^2$ (Quotient)

$$
\begin{array}{l}
x^3 - 2x^2y - 3x^2z \\
\hline
2x^2y + 3x^2z - 18xyz - 8y^3 - 27z^3 \\
2x^2y - 4xy^2 - 6xyz \\
\hline
3x^2z + 4xy^2 - 12xyz - 8y^3 - 27z^3 \\
3x^2z - 6xyz - 9xz^2 \\
\hline
4xy^2 - 6xyz + 9xz^2 - 8y^3 - 27z^3 \\
4xy^2 - 8y^3 - 12y^2z \\
\hline
-6xyz + 9xz^2 + 12y^2z - 27z^3 \\
-6xyz + 12y^2z + 18yz^2 \\
\hline
9xz^2 - 18yz^2 - 27z^3 \\
9xz^2 - 18yz^2 - 27z^3 \\
\hline
\end{array}
$$

Or Otherwise —

The Dividend $=(x)^3+(-2y)^3+(-3z)^3-3(x)(-2y)(-3z)$

$=(x-2y-3z)(x^2+4y^2+9z^2+2xy+3xz-6yz)$,

the Divisor $=(x-2y-3z)$, and the Quotient

$=x^2+4y^2+9z^2+2xy+3xz-6yz$

42

$$
\begin{array}{r|l}
x-y+z & x^3+3xyz-y^3+z^3 \quad (x^2+xy-xz+y^2+yz+z^2 \text{ (Quotient)} \\
& \underline{x^3-x^2y+x^2z} \\
& x^2y-x^2z+3xyz-y^3+z^3 \\
& \underline{x^2y-xy^2+xyz} \\
& -x^2z+xy^2+2xyz-y^3+z^3 \\
& \underline{-x^2z+xyz-xz^2} \\
& xy^2+xyz+xz^2-y^3+z^3 \\
& \underline{xy^2-y^3+y^2z} \\
& xyz+xz^2-y^2z+z^3 \\
& \underline{xyz-y^2z+yz^2} \\
& xz^2-yz^2+z^3 \\
& \underline{xz^2-yz^2+z^3}
\end{array}
$$

Or Otherwise —

The Dividend $=(x)^3+(-y)^3+(z)^3-3(x)(-y)(z)$

$=(x-y+z)(x^2+y^2+z^2+xy-xz+yz)$,

the Divisor $=x-y+z$, and the Quotient —

$=x^2+y^2+z^2+xy-xz+yz$.

43 $4x^2+6xy+2xz+9y^2-3yz+z^2\ \big)\ 8x^3 - 18xyz - 27y^3 - z^3\ \big(\ 2x-3y-z$ (Quotient)

$$
\begin{array}{l}
8x^3+12x^2y+4x^2z+18xy^2-6xyz+2xz^2 \\
\hline
-12x^2y-4x^2z-18xy^2-12xyz-2xz^2-27y^3-z^3 \\
-12x^2y \qquad -18xy^2-6xyz \qquad -27y^3+9y^2z-3yz^2 \\
\hline
-4x^2z \qquad -6xyz-2xz^2-9y^2z+3yz^2-z^3 \\
-4x^2z \qquad -6xyz-2xz^2-9y^2z+3yz^2-z^3 \\
\hline
\end{array}
$$

Or Otherwise —

The Dividend $=(2x)^3+(-3y)^3+(-z)^3-3(2x)(-3y)(-z)$

$=(2x-3y-z)(4x^2+9y^2+z^2+6xy+2xz-3yz)$,

the Divisor $=4x^2+6xy+2xz+9y^2-3yz+z^2$, and the Quotient

$=2x-3y-z$

44 $a-b\ \big)\ a^2(b-c)-a(b^2-c^2)+bc(b-c)\ \big(\ a(b-c)-(bc-c^2)=ab-ac-bc+c^2$ (Quotient)

$$
\begin{array}{l}
a^2(b-c)-a(b^2-bc) \\
\hline
-a(bc-c^2)+bc(b-c) \\
-a(bc-c^2)+bc(b-c) \\
\hline
\end{array}
$$

45 $x^2+x(a-b)-ab\ \big)\ x^3+x^2(a-b+c)-x(ab-ac+bc)-abc\ \big(\ x+c$ (Quotient)

$$
\begin{array}{l}
x^3+x^2(a-b) \qquad -xab \\
\hline
x^2c \qquad +xc(a-b) \qquad -abc \\
x^2c \qquad +xc(a-b) \qquad -bbc \\
\hline
\end{array}
$$

46 $x^2-x(b+c)+bc\ \big)\ x^3+x^2(a-b-c)-x(ab+ac-bc)+abc\ \big(\ x+a$ (Quotient)

$$
\begin{array}{l}
x^3+x^2(\ \ -b-c)-x(-bc) \\
\hline
x^2a \qquad -xa(b+c) \qquad +abc \\
x^2a \qquad -xa(b+c) \qquad +abc \\
\hline
\end{array}
$$

47. $a(b-c)-b(b-c)\ \big)\ a^3(b-c)-a(b^3-c^3)+bc(b^2-c^2)\ \big(\ a^2+ab-c(b+c)=a^2+ab-bc-c^2$ (Quotient)

$$a^3(b-c)-a^2b(b-c)$$
$$a^2b(b-c)-a(b^3-c^3)+bc(b^2-c^2)$$
$$a^2b(b-c)-ab^2(b-c)$$
$$-ac(b^2-c^2)\quad +bc(b^2-c^2)$$
$$-ac(b^2-c^2)\quad +bc(b^2-c^2)$$

48. $a^2(b+c)-ac^2-bc^2\ \big)\ a^3(b^2-c^2)-a^2(b^3-c^3)+b^2c^2(b-c)\ \big(\ a(b-c)-b(b-c)$ (Quotient)

$$a^3(b^2-c^2)-a^2(bc^2-c^3)-abc^2(b-c)$$
$$-a^2b(b^2-c^2)+abc^2(b-c)+b^2c^2(b-c)$$
$$-a^2b(b^2-c^2)+abc^2(b-c)+b^2c^2(b-c)$$

49 $y-(x-z)\ \big)\ y^3(x+2z)-y^2xz-y(x^3-xz^2+2z^3)+x^3z-xz^3\ \big(\ y^2(x+2z)+y(x^2-2z^2)-xz(x+z)=y^2x+2y^2z+yx^2-2yz^2-x^2z-xz^2$ (Quotient)

$$y^3(x+2z)-y^2(x^2+xz-2z^2)$$
$$y^2(x^2-2z^2)-y(x^3-xz^2+2z^3)$$
$$y^2(x^2-2z^2)-y(x^3-2xz^2-x^2z+2z^3)$$
$$-yxz(x+z)+xz(x^2-z^2)$$
$$-yxz(x+z)+xz(x^2-z^2)$$

50. $x(a+b)+a(a+b)\ \big)\ x^3(a+b)+a^3(a+b)\ \big(\ x^2-ax+a^2$ (Quotient)

$$x^3(a+b)+ax^2(a+b)$$
$$-ax^2(a+b)\quad +a^3(a+b)$$
$$-ax^2(a+b)-a^2x(a+b)$$
$$xa^2(a+b)+a^3(a+b)$$
$$xa^2(a+b)+a^3(a+b)$$

51 $c^2(a-b)-cb^2+ab^2\ \big)\ c^3(a-b)+c^2(a^2-2ab)+cb^3+ab^2(a-b)\ \big(\ c+(a-b)=c+a-b$ (Quotient)

$$c^3(a-b)-c^2b^2+cab^2$$
$$c^2(a^2-2ab+b^2)-cb^2(a-b)+ab^2(a-b)$$
$$c^2(a^2-2ab+b^2)-cb^2(a-b)+ab^2(a-b)$$

52 $x^2(a^2-ab+b^2)+3aby^2\,\big)\,2x^3(a^3+b^3)+6abxy^2(a+b)\,\big(\,2(a+b)x$ (Quotient)

$$\underline{2x^3(a^3+b^3)+6abxy^2(a+b)}$$

53 $x(y+z)+(yz+1)\,\big)\,x^2(y+z)(yz+1)+x\{(y+z)^2+(yz+1)^2\}+(y+z)(yz+1)\,\big(\,x(yz+1)+(y+z)=x+y+z+xyz$ (Quotient)

$$\underline{x^2(y+z)(yz+1)+x(yz+1)^2}$$
$$x(y+z)^2+(y+z)(yz+1)$$
$$\underline{x(y+z)^2+(y+z)(yz+1)}$$

54 $x^2+y^2-a^2\,\big)\,16x^6-24x^4a^2+9a^4x^2+16y^6-24y^4a^2+9a^4y^2-a^6\,\big(\,16x^4-8x^2(2y^2+a^2)+(4y^2-a^2)^2$ (Quotient)

$$\underline{16x^6-16x^4a^2+16x^4y^2}$$
$$-8x^4(2y^2+a^2)+9a^4x^2+16y^6-24y^4a^2+9a^4y^2-a^6$$
$$\underline{-8x^4(2y^2+a^2)-8x^2(2y^4-y^2a^2-a^4)}$$
$$x^2(16y^4-8y^2a^2+a^4)+16y^6-24y^4a^2+9a^4y^2-a^6$$
$$\underline{x^2(16y^4-8y^2a^2+a^4)+16y^6-24y^4a^2+9a^4y^2-a^6}$$

55 and 56 Worked out in the book

57 $x^{\frac{5}{3}} \div x^{\frac{2}{3}} = x^{\frac{5}{3}-\frac{2}{3}} = x^{\frac{3}{3}} = \underline{x}$

58 $x^{-\frac{3}{4}} \div x^{-\frac{7}{4}} = x^{-\frac{3}{4}-(-\frac{7}{4})} = x^{-\frac{3}{4}+\frac{7}{4}} = \underline{x}$

59 $a^2b^{\frac{2}{3}} \div a^{-1}b^{-\frac{1}{3}} = a^{2+1}b^{\frac{2}{3}+\frac{1}{3}} = \underline{a^3b}$

60. $a^{-2}b^{\frac{1}{2}}c^{\frac{5}{2}} \div a^{-3}b^{\frac{3}{2}}c^2 = a^{-2+3}b^{\frac{1}{2}-\frac{3}{2}}c^{\frac{5}{2}-2} = \underline{ab^{-1}c^{\frac{1}{2}}}$

61 $15xyz \div \left(-5x^{\frac{2}{3}}y^{\frac{4}{5}}z^{\frac{4}{3}}\right) = -3x^{1-\frac{2}{3}}y^{1-\frac{4}{5}}z^{1-\frac{4}{3}} = -3x^{\frac{1}{3}}y^{\frac{1}{5}}z^{-\frac{1}{3}}$

62 $3x^{\frac{2}{3}}+4y^{\frac{1}{3}}\Big)\,9x^{\frac{4}{3}} \quad -16y^{\frac{2}{3}}\Big(\,3x^{\frac{2}{3}}-4y^{\frac{1}{3}}$ (Quotient)

$$\begin{array}{r}
\underline{9x^{\frac{4}{3}}+12x^{\frac{2}{3}}y^{\frac{1}{3}}\qquad\qquad} \\
-12x^{\frac{2}{3}}y^{\frac{1}{3}}-16y^{\frac{2}{3}} \\
\underline{-12x^{\frac{2}{3}}y^{\frac{1}{3}}-16y^{\frac{2}{3}}}
\end{array}$$

63 $a^{\frac{1}{3}}+b^{\frac{1}{3}}\Big)\,a \quad +b\Big(\,a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}$ (Quotient)

$$\begin{array}{r}
\underline{a+a^{\frac{2}{3}}b^{\frac{1}{3}}\qquad\qquad} \\
-a^{\frac{2}{3}}b^{\frac{1}{3}}\qquad +b \\
\underline{-a^{\frac{2}{3}}b^{\frac{1}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}\quad} \\
a^{\frac{1}{3}}b^{\frac{2}{3}}+b \\
\underline{a^{\frac{1}{3}}b^{\frac{2}{3}}+b}
\end{array}$$

64 $a^{\frac{3}{2}}+a^{\frac{3}{4}}b^{\frac{3}{4}}+b^{\frac{3}{2}}\Big)\,a^3 \quad +a^{\frac{3}{2}}b^{\frac{3}{2}} \quad +b^3\Big(\,a^{\frac{3}{2}}-a^{\frac{3}{4}}b^{\frac{3}{4}}+b^{\frac{3}{2}}$ (Quotient)

$$\begin{array}{r}
\underline{a^3+a^{\frac{9}{4}}b^{\frac{3}{4}}+a^{\frac{3}{2}}b^{\frac{3}{2}}\qquad\quad} \\
-a^{\frac{9}{4}}b^{\frac{3}{4}}\qquad\qquad\qquad +b^3 \\
\underline{-a^{\frac{9}{4}}b^{\frac{3}{4}}-a^{\frac{3}{2}}b^{\frac{3}{2}}-a^{\frac{3}{4}}b^{\frac{9}{4}}\quad} \\
a^{\frac{3}{2}}b^{\frac{3}{2}}+a^{\frac{3}{4}}b^{\frac{9}{4}}+b^3 \\
\underline{a^{\frac{3}{2}}b^{\frac{3}{2}}+a^{\frac{3}{4}}b^{\frac{9}{4}}+b^3}
\end{array}$$

65 $2x^{\frac{4}{3}}+5x^{\frac{2}{3}}y^{\frac{2}{3}}-3y^{\frac{4}{3}}\Big)\,4x^{\frac{8}{3}}-37x^{\frac{4}{3}}y^{\frac{4}{3}}+9y^{\frac{8}{3}}\Big(\,2x^{\frac{4}{3}}-5x^{\frac{2}{3}}y^{\frac{2}{3}}-3y^{\frac{4}{3}}$ (Quotient)

$$\begin{array}{r}
\underline{4x^{\frac{8}{3}}+10x^2y^{\frac{2}{3}}-6x^{\frac{4}{3}}y^{\frac{4}{3}}\qquad\qquad\qquad} \\
-10x^2y^{\frac{2}{3}}-31x^{\frac{4}{3}}y^{\frac{4}{3}}+9y^{\frac{8}{3}}\qquad\quad \\
\underline{-10x^2y^{\frac{2}{3}}-25x^{\frac{4}{3}}y^{\frac{4}{3}}+15x^{\frac{2}{3}}y^2} \\
-6x^{\frac{4}{3}}y^{\frac{4}{3}}-15x^{\frac{2}{3}}y^2+9y^{\frac{8}{3}} \\
\underline{-6x^{\frac{4}{3}}y^{\frac{4}{3}}-15x^{\frac{2}{3}}y^2+9y^{\frac{8}{3}}}
\end{array}$$

66 $a^{\frac{1}{4}}-b^{\frac{1}{2}}\,\big)\,a \qquad\qquad -b^2\,\big(\,a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{4}}b+b^{\frac{3}{2}}$ (Quotient)

$$\begin{array}{l}
\underline{a-a^{\frac{3}{4}}b^{\frac{1}{2}}} \\
a^{\frac{3}{4}}b^{\frac{1}{2}} \qquad\qquad -b^2 \\
\underline{a^{\frac{3}{4}}b^{\frac{1}{2}}-a^{\frac{1}{2}}b} \\
\qquad a^{\frac{1}{2}}b \qquad\quad -b^2 \\
\qquad \underline{a^{\frac{1}{2}}b-a^{\frac{1}{4}}b^{\frac{3}{2}}} \\
\qquad\qquad a^{\frac{1}{4}}b^{\frac{3}{2}}-b^2 \\
\qquad\qquad \underline{a^{\frac{1}{4}}b^{\frac{3}{2}}-b^2}
\end{array}$$

67 $2a^{-5}+3a^{-\frac{5}{2}}b^{-\frac{1}{2}}-5b^{-3}\,\big)\,4a^{-10}+12a^{-\frac{15}{2}}b^{-\frac{1}{2}}+9a^{-5}b^{-3}-25b^{-6}\,\big(\,2a^{-5}+3a^{-\frac{5}{2}}b^{-\frac{1}{2}}+5b^{-3}$ (Quotient)

$$\begin{array}{l}
\underline{4a^{-10}+6a^{-\frac{15}{2}}b^{-\frac{1}{2}}-10a^{-5}b^{-3}} \\
\qquad 6a^{-\frac{15}{2}}b^{-\frac{1}{2}}+19a^{-5}b^{-3}-25b^{-6} \\
\qquad \underline{6a^{-\frac{15}{2}}b^{-\frac{1}{2}}+9a^{-5}b^{-3}-15a^{-\frac{5}{2}}b^{-\frac{7}{2}}} \\
\qquad\qquad 10a^{-5}b^{-3}+15a^{-\frac{5}{2}}b^{-\frac{7}{2}}-25b^{-6} \\
\qquad\qquad \underline{10a^{-5}b^{-3}+15a^{-\frac{5}{2}}b^{-\frac{7}{2}}-25b^{-6}}
\end{array}$$

68

$$
\begin{array}{l}
3x^{-\frac{5}{4}}+5x^{-\frac{5}{8}}y^{-\frac{3}{8}}-7y^{-\frac{3}{4}}\,\big)\,9x^{-\frac{5}{2}}-25x^{-\frac{5}{4}}y^{-\frac{3}{4}}+70x^{-\frac{5}{8}}y^{-\frac{9}{8}}-49y^{-\frac{3}{2}}\,\big(\,3x^{-\frac{5}{4}}-5x^{-\frac{5}{8}}y^{-\frac{3}{8}}+7y^{-\frac{3}{4}} \text{ (Quotient)}\\
\underline{9x^{-\frac{5}{2}}+15x^{-\frac{15}{8}}y^{-\frac{3}{8}}-21x^{-\frac{5}{4}}y^{-\frac{3}{4}}}\\
-15x^{-\frac{15}{8}}y^{-\frac{3}{8}}-4x^{-\frac{5}{4}}y^{-\frac{3}{4}}+70x^{-\frac{5}{8}}y^{-\frac{9}{8}}-49y^{-\frac{3}{2}}\\
\underline{-15x^{-\frac{15}{8}}y^{-\frac{3}{8}}-25x^{-\frac{5}{4}}y^{-\frac{3}{4}}+35x^{-\frac{5}{8}}y^{-\frac{9}{8}}}\\
21x^{-\frac{5}{4}}y^{-\frac{3}{4}}+35x^{-\frac{5}{8}}y^{-\frac{9}{8}}-49y^{-\frac{3}{2}}\\
\underline{21x^{-\frac{5}{4}}y^{-\frac{3}{4}}+35x^{-\frac{5}{8}}y^{-\frac{9}{8}}-49y^{-\frac{3}{2}}}
\end{array}
$$

69

$$
\begin{array}{l}
a^{\frac{1}{2}}-b^{\frac{1}{3}}\,\big)\,a^{3}-b^{2}\,\big(\,a^{\frac{5}{2}}+a^{2}b^{\frac{1}{3}}+a^{\frac{3}{2}}b^{\frac{2}{3}}+ab+a^{\frac{1}{2}}b^{\frac{4}{3}}+b^{\frac{5}{3}} \text{ (Quotient)}\\
\underline{a^{3}-a^{\frac{5}{2}}b^{\frac{1}{3}}}\\
a^{\frac{5}{2}}b^{\frac{1}{3}}-b^{2}\\
\underline{a^{\frac{5}{2}}b^{\frac{1}{3}}-a^{2}b^{\frac{2}{3}}}\\
a^{2}b^{\frac{2}{3}}-b^{2}\\
\underline{a^{2}b^{\frac{2}{3}}-a^{\frac{3}{2}}b}\\
a^{\frac{3}{2}}b-b^{2}\\
\underline{a^{\frac{3}{2}}b-ab^{\frac{4}{3}}}\\
ab^{\frac{4}{3}}-b^{2}\\
\underline{ab^{\frac{4}{3}}-a^{\frac{1}{2}}b^{\frac{5}{3}}}\\
a^{\frac{1}{2}}b^{\frac{5}{3}}-b^{2}\\
\underline{a^{\frac{1}{2}}b^{\frac{5}{3}}-b^{2}}
\end{array}
$$

70

$$
\begin{array}{l}
x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}\big)\, x - 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} + y + z \big(x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}z^{\frac{1}{3}} + y^{\frac{2}{3}} - y^{\frac{1}{3}}z^{\frac{1}{3}} + z^{\frac{2}{3}} \quad \text{(Quotient)} \\
\underline{x + x^{\frac{2}{3}}y^{\frac{1}{3}} + x^{\frac{2}{3}}z^{\frac{1}{3}}} \\
-x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{2}{3}}z^{\frac{1}{3}} - 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} + y + z \\
\underline{-x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}} \\
-x^{\frac{2}{3}}z^{\frac{1}{3}} + x^{\frac{1}{3}}y^{\frac{2}{3}} - 2x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} + y + z \\
\underline{-x^{\frac{2}{3}}z^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} - x^{\frac{1}{3}}z^{\frac{2}{3}}} \\
x^{\frac{1}{3}}y^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} + x^{\frac{1}{3}}z^{\frac{2}{3}} + y + z \\
\underline{x^{\frac{1}{3}}y^{\frac{2}{3}} + y + y^{\frac{2}{3}}z^{\frac{1}{3}}} \\
-x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} + x^{\frac{1}{3}}z^{\frac{2}{3}} - y^{\frac{2}{3}}z^{\frac{1}{3}} + z \\
\underline{-x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} - y^{\frac{2}{3}}z^{\frac{1}{3}} - y^{\frac{1}{3}}z^{\frac{2}{3}}} \\
x^{\frac{1}{3}}z^{\frac{2}{3}} + y^{\frac{1}{3}}z^{\frac{2}{3}} + z \\
\underline{x^{\frac{1}{3}}z^{\frac{2}{3}} + y^{\frac{1}{3}}z^{\frac{2}{3}} + z}
\end{array}
$$

Or Otherwise —

The Dividend $= \left(x^{\frac{1}{3}}\right)^3 + \left(y^{\frac{1}{3}}\right)^3 + \left(z^{\frac{1}{3}}\right)^3 - 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$

$= \left(x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}-x^{\frac{1}{3}}z^{\frac{1}{3}}-y^{\frac{1}{3}}z^{\frac{1}{3}}\right)$,

the Divisor $= x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$, and the quotient

$= x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}-x^{\frac{1}{3}}z^{\frac{1}{3}}-y^{\frac{1}{3}}z^{\frac{1}{3}}$

Exercise 39.

1

$$x+a\,\big)\,x^3 + a^3\,\big(\,x^2 - ax + a^2 \quad \text{(Quotient)}$$
$$\begin{array}{r}
\underline{x^3 + ax^2 \qquad\qquad} \\
-ax^2 \qquad + a^3 \\
\underline{-ax^2 - a^2x \qquad} \\
a^2x + a^3 \\
\underline{a^2x + a^3}
\end{array}$$

2

$$x+a\,\big)\,x^4 - a^4\,\big(\,x^3 - ax^2 + a^2x - a^2x \quad \text{(Quotient)}$$
$$\begin{array}{r}
\underline{x^4 + ax^3 \qquad\qquad\qquad\qquad} \\
-ax^3 \qquad - a^4 \qquad\qquad\qquad \\
\underline{-ax^3 - a^2x^2 \qquad\qquad\qquad} \\
a^2x^2 - a^4 \qquad\qquad \\
\underline{a^2x^2 + a^3x \qquad\qquad} \\
-a^3x - a^4 \\
\underline{-a^3x - a^4}
\end{array}$$

3

$$x+a\,\big)\,x^5 + a^5\,\big(\,x^4 - ax^3 + a^2x^2 - a^3x + a^4 \quad \text{(Quotient)}$$
$$\begin{array}{r}
\underline{x^5 + ax^4 \qquad\qquad\qquad\qquad\qquad\qquad} \\
-ax^4 + a^5 \qquad\qquad\qquad\qquad\qquad \\
\underline{-ax^4 - a^2x^3 \qquad\qquad\qquad\qquad} \\
a^2x^3 + a^5 \qquad\qquad\qquad \\
\underline{a^2x^3 + a^3x^2 \qquad\qquad\qquad} \\
-a^3x^2 + a^5 \qquad\qquad \\
\underline{-a^3x^2 - a^4x \qquad\qquad} \\
a^4x + a^5 \\
\underline{a^4x + a^5}
\end{array}$$

4

$$x+a\,\big)\,x^6 - a^6\,\big(\,x^5 - ax^4 + a^2x^3 - a^3x^2 + a^4x - a^5 \quad \text{(Quotient)}$$
$$\begin{array}{r}
\underline{x^6 + ax^5 \qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\
-ax^5 - a^6 \qquad\qquad\qquad\qquad\qquad\qquad \\
\underline{-ax^5 - a^2x^4 \qquad\qquad\qquad\qquad\qquad} \\
a^2x^4 - a^6 \qquad\qquad\qquad\qquad \\
\underline{a^2x^4 + a^3x^3 \qquad\qquad\qquad\qquad} \\
-a^3x^3 \qquad - a^6 \qquad\qquad \\
\underline{-a^3x^3 - a^4x^2 \qquad\qquad\qquad} \\
a^4x^2 - a^6 \qquad \\
\underline{a^4x^2 + a^5x \qquad} \\
-a^5x - a^6 \\
\underline{-a^5x - a^6}
\end{array}$$

5 $x+a \overline{) x^7 + a^7} \left(x^6 - ax^5 + a^2x^4 - a^3x^3 + a^4x^2 - a^5x + a^6 \right.$ (Quotient)

$$
\begin{array}{l}
\underline{x^7 + ax^6} \\
-ax^6 + a^7 \\
\underline{-ax^6 - a^2x^5} \\
a^2x^5 + a^7 \\
\underline{a^2x^5 + a^3x^4} \\
-a^3x^4 + a^7 \\
\underline{-a^3x^4 - a^4x^3} \\
a^4x^3 + a^7 \\
\underline{a^4x^3 + a^5x^2} \\
-a^5x^2 + a^7 \\
\underline{-a^5x^2 - a^6x} \\
a^6x + a^7 \\
\underline{a^6x + a^7}
\end{array}
$$

6 $x+a \overline{) x^8 - a^8} \left(x^7 - ax^6 + a^2x^5 - a^3x^4 + a^4x^3 - a^5x^2 + a^6x - a^7 \right.$ (Quotient)

$$
\begin{array}{l}
\underline{x^8 + ax^7} \\
-ax^7 - a^8 \\
\underline{-ax^7 - a^2x^6} \\
a^2x^6 - a^8 \\
\underline{a^2x^6 + a^3x^5} \\
-a^3x^5 - a^8 \\
\underline{-a^3x^5 - a^4x^4} \\
a^4x^4 - a^8 \\
\underline{a^4x^4 + a^5x^3} \\
-a^5x^3 - a^8 \\
\underline{-a^5x^3 - a^6x^2} \\
a^6x^2 - a^8 \\
\underline{a^6x^2 + a^7x} \\
-a^7x - a^8 \\
\underline{-a^7x - a^8}
\end{array}
$$

7 $x+a \overline{) x^3 - a^3} \left(x^2 - ax + a^2 \right.$ (Quotient)

$$
\begin{array}{l}
\underline{x^3 + ax^2} \\
-ax^2 - a^3 \\
\underline{-ax^2 - a^2x} \\
a^2x - a^3 \\
\underline{a^2x + a^3} \\
-2a^3
\end{array}
$$

8

8 $$x+a\,)\,x^4 + a^4\,(\,x^3 - ax^2 + a^2x - a^3 \quad \text{(Quotient)}$$
$$\begin{array}{l}
\underline{x^4 + ax^3} \\
-ax^3 + a^4 \\
\underline{-ax^3 - a^2x^2} \\
a^2x^2 + a^4 \\
\underline{a^2x^2 + a^3x} \\
-a^3x + a^4 \\
\underline{-a^3x - a^4} \\
2a^4
\end{array}$$

9 $$x+a\,)\,x^5 - a^5\,(\,x^4 - ax^3 + a^2x^2 - a^3x + a^4 \quad \text{(Quotient)}$$
$$\begin{array}{l}
\underline{x^5 + ax^4} \\
-ax^4 - a^5 \\
\underline{-ax^4 - a^2x^3} \\
a^2x^3 - a^5 \\
\underline{a^2x^3 + a^3x^2} \\
-a^3x^2 - a^5 \\
\underline{-a^3x^2 - a^4x} \\
a^4x - a^5 \\
\underline{a^4x + a^5} \\
-2a^5
\end{array}$$

10 $$x+a\,)\,x^6 + a^6\,(\,x^5 - ax^4 + a^2x^3 - a^3x^2 + a^4x - a^5 \quad \text{(Quotient)}$$
$$\begin{array}{l}
\underline{x^6 + ax^5} \\
-ax^5 + a^6 \\
\underline{-ax^5 - a^2x^4} \\
a^2x^4 + a^6 \\
\underline{a^2x^4 + a^3x^3} \\
-a^3x^3 + a^6 \\
\underline{-a^3x^3 - a^4x^2} \\
a^4x^2 + a^6 \\
\underline{a^4x^2 + a^5x} \\
-a^5x + a^6 \\
\underline{-a^5x - a^6} \\
2a^6
\end{array}$$

11

$$x+a\overline{)\,x^7-a^7\,}\left(x^6-ax^5+a^2x^4-a^3x^3+a^4x^2-a^5x+a^6 \quad \text{(Quotient)}\right.$$

$$\begin{array}{r}
\underline{x^7+ax^6\qquad\qquad} \\
-ax^6-a^7 \\
\underline{-ax^6-a^2x^5} \\
a^2x^5-a^7 \\
\underline{a^2x^5+a^3x^4} \\
-a^3x^4-a^7 \\
\underline{-a^3x^4-a^4x^3} \\
a^4x^3-a^7 \\
\underline{a^4x^3+a^5x^2} \\
-a^5x^2-a^7 \\
\underline{-a^5x^2-a^6x} \\
a^6x-a^7 \\
\underline{a^6x+a^7} \\
-2a^7
\end{array}$$

12

$$x+a\overline{)\,x^8+a^8\,}\left(x^7-ax^6+a^2x^5-a^3x^4+a^4x^3-a^5x^2+a^6x-a^7 \quad \text{(Quotient)}\right.$$

$$\begin{array}{r}
\underline{x^8+ax^7\qquad\qquad} \\
-ax^7+a^8 \\
\underline{-ax^7-a^2x^6} \\
a^2x^6+a^8 \\
\underline{a^2x^6+a^3x^5} \\
-a^3x^5+a^8 \\
\underline{-a^3x^5-a^4x^4} \\
a^4x^4+a^8 \\
\underline{a^4x^4+a^5x^3} \\
-a^5x^3+a^8 \\
\underline{-a^5x^3-a^6x^2} \\
a^6x^2+a^8 \\
\underline{a^6x^2+a^7x} \\
-a^7x+a^8 \\
\underline{-a^7x-a^8} \\
2a^8
\end{array}$$

13 $x^4-1=x^3(x-1)+x^2(x-1)+x(x-1)+(x-1)$,

$$\frac{x^4-1}{x-1}=\underline{x^3+x^2+x+1}$$

14 $x^4-y^4=x^3(x+y)-x^2y(x+y)+xy^2(x+y)-y^3(x+y)$

$$\frac{x^4-y^4}{x+y}=\underline{x^3-x^2y+xy^2-y^3}$$

15 $x^5 - 1 = x^4(x-1) + x^3(x-1) + x^2(x-1) + x(x-1) + (x-1)$

$$\frac{x^5-1}{x-1} = \underline{x^4 + x^3 + x^2 + x + 1}$$

16 $x^5 + y^5 = x^4(x+y) - x^3y(x+y) + x^2y^2(x+y) - xy^3(x+y) + y^4(x+y)$,

$$\frac{x^5+y^5}{x+y} = \underline{x^4 - x^3y + x^2y^2 - xy^3 + y^4}$$

17 $x^5 - 1 = x^4(x-1) + x^3(x-1) + x^2(x-1) + x(x-1) + (x-1)$,

$$\frac{x^5-1}{x-1} = \underline{x^4 + x^3 + x^2 + x + 1}$$

18 $x^6 - y^6 = x^5(x+y) - x^4y(x+y) + x^3y^2(x+y) - x^2y^3(x+y) + xy^4(x+y) - y^5(x+y)$,

$$\frac{x^6-y^6}{x+y} = \underline{x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5}$$

19 $x^7 - 1 = x^6(x-1) + x^5(x-1) + x^4(x-1) + x^3(x-1) + x^2(x-1) + x(x-1) + (x+1)$,

$$\frac{x^7-1}{x-1} = \underline{x^6 + x^5 + x^4 + x^3 + x^2 + x + 1}$$

20 $x^7 + y^7 = x^6(x+y) - x^5y(x+y) + x^4y^2(x+y) - x^3y^3(x+y) + x^2y^4(x+y) - xy^5(x+y) + y^6(x+y)$.

$$\frac{x^7+y^7}{x+y} = \underline{x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 - xy^5 + y^6}$$

Exercise 40

1 $ab + ac = a(b+c)$

2 $a^2b^3 + a^3b^2 = a^2b^2(b+a)$

3 $x^3y^4 - 2x^4y^3 = x^3y^3(y - 2x)$

4 $2x^2yz + 4xy^2z - 6xyz^2 = 2xyz(x + 2y - 3z)$

5 $4a^5b - 6a^4b^2 - 8a^3b^3 = 2a^3b(2a^2 - 3ab - 4b^2)$

6 $ax^2y - 5a^2x^3y^2 + 3ax^3 = ax^2(y - 5axy^2 + 3x)$

7 $3x^4y^3z^2 - 12x^3y^4z^3 + 21x^5y^2z^4 = 3x^3y^2z^2(xy - 4y^2z + 7x^2z^2)$

8 $28a^8b^5 - 42a^6b^8 = 14a^6b^5(2a^3 - 3b^3)$

9 $72x^{10}y^8 + 108x^8y^{10} = 36x^8y^8(2x + 3y^2)$

10 $39a^5b^7c^7 - 65b^6c^7a^7 - 91c^6a^7b^7 = 13a^5b^5c^6(3b^2c - 5c^2a^2 - 7a^2b^2)$.

Exercise 41

1 $9a^2-16b^2=(3a)^2-(4b)^2=(3a+4b)(3a-4b)$

2 $4a^3-25ax^2=a(4a^2-25x^2)=a\{(2a)^2-(5x)^2\}$
$=a(2a+5x)(2a-5x)$

3 $36x^4-1=(6x^2)^2-1=(6x^2+1)(6x^2-1)$

4 $16x^4-1=(4x^2)^2-1=(4x^2+1)(4x^2-1)=(4x^2+1)\{(2x)^2-1\}$
$=(4x^2+1)(2x+1)(2x-1)$

5 $16x^5-9x=x(16x^4-9)=x\{(4x^2)^2-(3)^2\}=x(4x^2+3)(4x^2-3)$

6 $16x^5-81x=x(16x^4-81)=x\{(4x^2)^2-(9)^2\}$
$=x(4x^2+9)(4x^2-9)$
$=x(4x^2+9)\{(2x)^2-(3)^2\}$
$=x(4x^2+9)(2x+3)(2x-3)$

7 $1-16a^4=1-(4a^2)^2=(1+4a^2)(1-4a^2)=(1+4a^2)\{1-(2a)^2\}$
$=(1+4a^2)(1+2a)(1-2a)$

8 $x^2-81x^6=x^2(1-81x^4)=x^2\{1-(9x^2)^2\}$
$=x^2(1+9x^2)(1-9x^2)$
$=x^2(1+9x^2)\{1-(3x)^2\}$
$=x^2(1+9x^2)(1+3x)(1-3x)$

9 $36-x^4a^2=(6)^2-(x^2a)^2=(6+x^2a)(6-x^2a)$

10 $64a^4-49x^6=(8a^2)^2-(7x^3)^2=(8a^2+7x^3)(8a^2-7x^3)$

11 $121-m^6=(11)^2-(m^3)^2=(11+m^3)(11-m^3)$

12 $49x^6a^{10}-81=(7x^3a^5)^2-(9)^2=(7x^3a^5+9)(7x^3a^5-9)$

13 $a^2b^2-25c^2d^2=(ab)^2-(5cd)^2=(ab+5cd)(ab-5cd)$

14 $81x^{12}-64a^{10}=(9x^6)^2-(8a^5)^2=(9x^6+8a^5)(9x^6-8a^5)$

15 $p^2q^4-100p^2=p^2\{(q^2)^2-(10)^2\}=p^2(q^2+10)(q^2-10)$

16 $144x^7-25x^3a^4=x^3\{(12x^2)^2-(5a^2)^2\}=x^3(12x^2+5a^2)(12x^2-5a^2)$

17 $192a^9-243a^5x^4=3a^5(64a^4-81x^4)=3a^5\{(8a^2)^2-(9x^2)^2\}$
$=3a^5(8a^2+9x^2)(8a^2-9x^2)$

18 $98a^3x^5-128ax=2ax(49a^2x^4-64)=2ax\{(7ax^2)^2-(8)^2\}$
$=2ax(7ax^2+8)(7ax^2-8)$

19 $324x^{18}a^6 - 484x^6a^2 = 4x^6a^2(81x^{12}a^4 - 121) = 4x^6a^2\{(9x^6a^2)^2 - (11)^2\}$
$= 4x^6a^2(9x^6a^2 + 11)(9x^6a^2 - 11)$

20 $245m^{19}n^{10} - 605m^{11}n^2 = 5m^{11}n^2(49m^8n^8 - 121)$
$= 5m^{11}n^2\{(7m^4n^4)^2 - (11)^2\}$
$= 5m^{11}n^2(7m^4n^4 + 11)(7m^4n^4 - 11)$

21 $(a+3b)^2 - 25c^2 = (a+3b)^2 - (5c)^2$
$= (a+3b+5c)(a+3b-5c)$

22 $a^2 - (3b-5c)^2 = \{a+(3b-5c)\}\{a-(3b-5c)\}$
$= (a+3b-5c)(a-3b+5c)$

23 $(x+y)^2 - (x-y)^2 = \{(x+y)+(x-y)\}\{(x+y)-(x-y)\}$
$= 2x \cdot 2y = 4xy$

24 $(3a+2x)^2 - (2a+x)^2 = \{(3a+2x)+(2a+x)\}\{(3a+2x)-(2a+x)\}$
$= (5a+3x)(a+x)$

25 $4(a-b)^2 - 9(c-d)^2 = \{2(a-b)\}^2 - \{3(c-d)\}^2$
$= \{2(a-b)+3(c-d)\}\{2(a-b)-3(c-d)\}$
$= (2a-2b+3c-3d)(2a-2b-3c+3d)$

26 $49x^2 - (5y-3z)^2 = (7x)^2 - (5y-3z)^2$
$= (7x+5y-3z)(7x-5y+3z)$

27 $(8x+5)^2 - (2x-7)^2 = \{(8x+5)+(2x-7)\}\{(8x+5)-(2x-7)\}$
$= (10x-2)(6x+12)$
$= 2(5x-1)6(x+2) = 12(5x-1)(x+2)$

28 $(a+b-c)^2 - (a-b+c)^2$
$= \{(a+b-c)+(a-b+c)\}\{(a+b-c)-(a-b+c)\}$
$= 2a(2b-2c) = 4a(b-c)$

29 $(2a-3b+4c)^2 - (a+4b-5c)^2$
$= \{(2a-3b+4c)+(a+4b-5c)\}\{(2a-3b+4c)-(a+4b-5c)\}$
$= (3a+b-c)(a-7b+9c)$

30 $64(a+3x-4y)^2 - 9(2a-x+3y)^2$
$= \{8(a+3x-4y)\}^2 - \{3(2a-x+3y)\}^2$
$= \{8(a+3x-4y)+3(2a-x+3y)\}\{8(a+3x-4y)-3(2a-x+3y)\}$
$= (14a+21x-23y)(2a+27x-41y)$

31 $(4x^2-5a^2)^2-(5x^2-4a^2)^2$

$=\{(4x^2-5a^2)+(5x^2-4a^2)\}\{(4x^2-5a^2)-(5x^2-4a^2)\}$

$=(9x^2-9a^2)(-x^2-a^2)=-9(x^2-a^2)(x^2+a^2)$

$=-9(x+a)(x-a)(x^2+a^2)$

32 $(5a^2-3a+7)^2-(5a^2-3a-7)^2$

$=\{(5a^2-3a+7)+(5a^2-3a-7)\}\{(5a^2-3a+7)-(5a^2-3a-7)\}$

$=(10a^2-6a)14=2a(5a-3)14=28a(5a-3)$

Exercise 42

1 $x^4+x^2+1=x^4+2x^2+1-x^2$

$=(x^2+1)^2-x^2=(x^2+1+x)(x^2+1-x)$

$=(x^2+x+1)(x^2-x+1)$

2 $x^8+x^4+1=x^8+2x^4+1-x^4$

$=(x^4+1)^2-(x^2)^2$

$=(x^4+1+x^2)(x^4+1-x^2)$

$=(x^4-x^2+1)(x^2+x+1)(x^2-x+1)$

3 $a^4+a^2x^2+x^4=a^4+2a^2x^2+x^4-a^2x^2$

$=(a^2+x^2)^2-(ax)^2=(a^2+x^2+ax)(a^2+x^2-ax)$

$=(a^2+ax+x^2)(a^2-ax+x^2)$

4 $a^8+a^4x^4+x^8=a^8+2a^4x^4+x^8-a^4x^4$

$=(a^4+x^4)^2-(a^2x^2)^2$

$=(a^4+x^4+a^2x^2)(a^4+x^4-a^2x^2)$

$=(a^2+ax+x^2)(a^2-ax+x^2)(a^4-a^2x^2+x^4)$

5 $x^4+64=x^4+16x^2+64-16x^2$

$=(x^2+8)^2-(4x)^2=(x^2+8+4x)(x^2+8-4x)$

$=(x^2+4x+8)(x^2-4x+8)$

6 $4x^4+81=4x^4+36x^2+81-36x^2$

$=(2x^2+9)^2-(6x)^2$

$=(2x^2+9+6x)(2x^2+9-6x)$

$=(2x^2+6x+9)(2x^2-6x+9)$

7 $9x^4+36=9(x^4+4)=9(x^4+4x^2+4-4x^2)$

$=9\{(x^2+2)^2-(2x)^2\}$

$=9(x^2+2x+2)(x^2-2x+2)$

8 $a^4+2a^2+9=a^4+6a^2+9-4a^2$

$=(a^2+3)^2-(2a)^2=(a^2+2a+3)(a^2-2a+3)$

9 $x^4-7x^2+9=x^4-6x^2+9-x^2$
$=(x^2-3)^2-(x)^2=(x^2+x-3)(x^2-x-3)$

10 $4x^4+8x^2+9=4x^4+12x^2+9-4x^2$
$=(2x^2+3)^2-(2x)^2=(2x^2+2x+3)(2x^2-2x+3)$

11 $4x^4-16x^2+9=4x^4-12x^2+9-4x^2$
$=(2x^2-3)^2-(2x)^2=(2x^2+2x-3)(2x^2-2x-3)$

12 $4x^4+3x^2+9=4x^4+12x^2+9-9x^2=(2x^2+3)^2-(3x)^2$
$=(2x^2+3x+3)(2x^2-3x+3)$

13 $4a^4-37a^2+9=4a^4-12a^2+9-25a^2$
$=(2a^2-3)^2-(5a)^2=(2a^2+5a-3)(2a^2-5a-3)$

14 $4a^4+625=4a^4+100a^2+625-100a^2$
$=(2a^2+25)^2-(10a)^2$
$=(2a^2+10a+25)(2a^2-10a+25)$

15 $9x^4+23x^2+16=9x^4+24x^2+16-x^2=(3x^2+4)^2-x^2$
$=(3x^2+x+4)(3x^2-x+4)$

16 $9a^4-25a^2+16=9a^4-24a^2+16-a^2$
$=(3a^2-4)^2-a^2=(3a^2+a-4)(3a^2-a-4)$

17 $9x^4-33x^2+16=9x^4-24x^2+16-9x^2$
$=(3x^2-4)^2-(3x)^2$
$=(3x^2+3x-4)(3x^2-3x-4)$

18 $9a^4-a^2+16=9a^4+24a^2+16-25a^2$
$=(3a^2+4)^2-(5a)^2=(3a^2+5a+4)(3a^2-5a+4)$

19 $16x^4+4x^2a^2+25a^4=16x^4+40x^2a^2+25a^4-36a^2x^2$
$=(4x^2+5x^2)^2-(6ax)^2$
$=(4x^2+6ax+5a^2)(4x^2-6ax+5a^2)$

20 $9a^4-19a^2x^2+25x^4=9a^4+30a^2x^2+25x^4-49a^2x^2$
$=(3a^2+5x^2)^2-(7ax)^2$
$=(3a^2+7ax+5x^2)(3a^2-7ax+5x^2)$

21 $x^4+8x^2+144=x^4+24x^2+144-16x^2$
$=(x^2+12)^2-(4x)^2$
$=(x^2+4x+12)(x^2-4x+12)$

22 $a^4-35a^2b^2+25b^4=a^4-10a^2b^2+25b^4-25a^2b^2$
$$=(a^2-5b^2)^2-(5ab)^2$$
$$=(a^2+5ab-5b^2)(a^2-5ab-5b^2)$$

23 $36a^4-16a^2b^2+b^4=36a^4-12a^2b^2+b^4-4a^2b^2$
$$=(6a^2-b^2)^2-(2ab)^2$$
$$=(6a^2+2ab-b^2)(6a^2-2ab-b^2)$$

24 $49m^4+16n^4-60m^2n^2$
$$=49m^4-56m^2n^2+16n^4-4m^2n^2$$
$$=(7m^2-4n^2)^2-(2mn)^2$$
$$=(7m^2+2mn-4n^2)(7m^2-2mn-4n^2)$$

25 $64a^4+81x^4=64a^4+144a^2x^2+81x^4-144a^2x^2$
$$=(8a^2+9x^2)^2-144a^2x^2$$
$$=(8a^2+12ax+9x^2)(8a^2-12ax+9x^2)$$

26 $4x^4+(7a)^4=4x^4+4x^2(7a)^2+(7a)^4-4x^2(7a)^2$
$$=\{2x^2+(7a)^2\}^2-(14ax)^2$$
$$=(2x^2+14ax+49a^2)(2x^2-14ax+49a^2)$$

27 $x^2-y^2+2yz-z^2=x^2-(y-z)^2=(x+y-z)(x-y+z)$

28 $4a^2-b^2-9c^2+6bc=(2a)^2-(b-3c)^2=(2a+b-3c)(2a-b+3c)$

29 $9x^2-4y^2+12yz-9z^2=(3x)^2-(2y-3z)^2$
$$=(3x+2y-3z)(3x-2y+3z)$$

30 $a^2-4b^2-25c^2+20bc=a^2-(2b-5c)^2$
$$=(a+2b-5c)(a-2b+5c)$$

31 $30xz+16y^2-9x^2-25z^2$
$$=16y^2-(9x^2-30xz+25z^2)$$
$$=(4y)^2-(3x-5z)^2$$
$$=(4y+3x-5z)(4y-3x+5z)$$

32 $a^2+4b^2-9c^2-4d^2-4ab+12cd$
$$=(a^2-4ab+4b^2)-(9c^2-12cd+4d^2)$$
$$=(a-2b)^2-(3c-2d)^2$$
$$=(a-2b+3c-2d)(a-2b-3c+2d)$$

33 $(x^2-2xy)-(z^2-2yz)$
$$=(x^2-2xy+y^2)-(z^2-2yz+y^2)$$
$$=(x-y)^2-(z-y)^2=\{(x-y)+(z-y)\}\{(x-y)-(z-y)\}$$
$$=(x-2y+z)(x-z)$$

34 $4x^2-1+9a^2-25b^2+12xa-10b$

$$=(4x^2+12ax+9a^2)-(25b^2+10b+1)$$
$$=(2x+3a)^2-(5b+1)^2$$
$$=(2x+3a+5b+1)(2x+3a-5b-1)$$

35 $9x^2-4y^2-49z^2-30x+28yz+25$

$$=(9x^2-30x+25)-(4y^2-28yz+49z^2)$$
$$=(3x-5)^2-(2y-7z)^2$$
$$=(3x+2y-7z-5)(3x-2y+7z-5)$$

36 $16a^2-16c^2-9b^2-24a+24bc+9$

$$=(16a^2-24a+9)-(16c^2-24bc+9b^2)$$
$$=(4a-3)^2-(4c-3b)^2$$
$$=(4a-3b+4c-3)(4a+3b-4c-3)$$

37. $49y^2+20z+x^2-14xy-25z^2-4$

$$=(x^2-14xy+49y^2)-(25z^2-20z+4)$$
$$=(x-7y)^2-(5z-2)^2$$
$$=(x-7y+5z-2)(x-7y-5z+2)$$

38 $16x^2+42by-9y^2+40xa-49b^2+25a^2$

$$=(16x^2+40ax+25a^2)-(9y^2-42by+49b^2)$$
$$=(4x+5a)^2-(3y-7b)^2$$
$$=(4x+5a+3y-7b)(4x+5a-3y+7b)$$

39 $49x^2-1+16y^2-64z^2+16z-56xy$

$$=(49x^2-56xy+16y^2)-(64z^2-16z+1)$$
$$=(7x-4y)^2-(8z-1)^2$$
$$=(7x-4y+8z-1)(7x-4y-8z+1)$$

40 $a^2-b^2-c^2+d^2-2(ad-bc)$

$$=(a^2-2ad+d^2)-(b^2-2bc+c^2)$$
$$=(a-d)^2-(b-c)^2$$
$$=(a+b-c-d)(a-b+c-d)$$

Exercise 43

1 $a^3-8b^3=a^3-(2b)^3=(a-2b)(a^2+2ab+4b^2)$

2 $a^4-27ax^3=a\{(a)^3-(3x)^3\}=a(a-3x)(a^2+3ax+9x^2)$

3 $512x^9+1=(8x^3)^3+1$
$=(8x^3+1)\{(8x^3)^2-8x^3+1\}$
$=\{(2x)^3+1\}\{64x^6-8x^3+1\}$
$=(2x+1)(4x^2-2x+1)(64x^6-8x^3+1)$

4 $a^9-512b^9=(a^3)^3-(8b^3)^3$
$=(a^3-8b^3)(a^6+8a^3b^3+64b^6)$
$=\{a^3-(2b)^3\}(a^6+8a^3b^3+64b^6)$
$=(a-2b)(a^2+2ab+4b^2)(a^6+8a^3b^3+64b^6)$

5 $27a^6+125x^6=(3a^2)^3+(5x^2)^3$
$=(3a^2+5x^2)\{(3a^2)^2-(3a^2)(5x^2)+(5x^2)^2\}$
$=(3a^2+5x^2)(9a^4-15a^2x^2+25x^4)$

6 $m^6-n^6=(m^2)^3-(n^2)^3$
$=(m^2-n^2)(m^4+m^2n^2+n^4)$
$=(m+n)(m-n)(m^2+mn+n^2)(m^2-mn+n^2)$

7 $343x^3+512y^3=(7x)^3+(8y)^3$
$=(7x+8y)(49x^2-56xy+64y^2)$

8 $64x^{12}-1=(4x^4)^3-1$
$=(4x^4-1)(16x^8+4x^4+1)$
$=(2x^2+1)(2x^2-1)\{16x^8+8x^4+1-4x^4\}$
$=(2x^2+1)(2x^2-1)\{(4x^4+1)^2-(2x^2)^2\}$
$=(2x^2+1)(2x^2-1)(4x^4+2x^2+1)(4x^4-2x^2+1)$

9 $a^6-64x^{12}=(a^3)^2-(8x^6)^2$
$=(a^3+8x^6)(a^3-8x^6)$
$=\{a^3+(2x^2)^3\}\{a^3-(2x^2)^3\}$
$=(a+2x^2)(a^2-2ax^2+4x^4)(a-2x^2)(a^2+2ax^2+4x^4)$

10 $125x^9-216a^9=(5x^3)^3-(6a^3)^3$
$=(5x^3-6a^3)(25x^6+30a^3x^3+36a^6)$

11 $64a^{13}b+343ab^{13}=ab(64a^{12}+343b^{12})$
$=ab\{(4a^4)^3+(7b^4)^3\}$
$=ab(4a^4+7b^4)(16a^8-28a^4b^4+49b^8)$

12 $729x^{20}y^2-64x^2y^{20}=x^2y^2(729x^{18}-64y^{18})$
$=x^2y^2\{(9x^6)^3-(4y^6)^3\}$
$=x^2y^2(9x^6-4y^6)(81x^{12}+36x^6y^6+16y^{12})$

$$=x^2y^2(3x^3+2y^3)(3x^3-2y^3)\{(81x^{12}+72x^6y^6+16y^{12})-36x^6y^6\}$$
$$=x^2y^2(3x^3+2y^3)(3x^3-2y^3)\{(9x^6+4y^6)^2-(6x^3y^3)^2\}$$
$$=x^2y^2(3x^3+2y^3)(3x^3-2y^3)(9x^6+6x^3y^3+4y^6)(9x^6-6x^3y^3+4y^6)$$

13 $(a^2+b^2)^3+8a^3b^3=(a^2+b^2)^3+(2ab)^3$
$$=(a^2+b^2+2ab)\{(a^2+b^2)^2-2ab(a^2+b^2)+(2ab)^2\}$$
$$=(a^2+b^2+2ab)(a^4+2a^2b^2+b^4-2a^3b-2ab^3+4a^2b^2)$$
$$=(a+b)^2(a^4-2a^3b+6a^2b^2-2ab^3+b^4)$$

14 $(2x^2-3y^2)^3+y^6=\{(2x^2-3y^2)+y^2\}\{(2x^2-3y^2)^2-y^2(2x^2-3y^2)+y^4\}$
$$=2(x^2-y^2)\{4x^4-12x^2y^2+9y^4-2x^2y^2+3y^4+y^4\}$$
$$=2(x+y)(x-y)(4x^4-14x^2y^2+13y^4)$$

15 $(2a^3-b^3)^3-b^9=\{(2a^3-b^3)-b^3\}\{(2a^3-b^3)^2+b^3(2a^3-b^3)+b^6\}$
$$=2(a^3-b^3)(4a^6-4a^3b^3+b^6+2a^3b^3-b^6+b^6)$$
$$=2(a-b)(a^2+ab+b^2)(4a^6-2a^3b^3+b^6)$$

Exercise 44

1 $3+1=4$ and $3\times 1=3$, $\quad x^2+4x+3=(x+1)(x+3)$

2 $3+2=5$ and $3\times 2=6$, $\quad x^2+5x+6=(x+2)(x+3)$

3 $3+4=7$ and $3\times 4=12$, $\quad x^2+7x+12=(x+3)(x+4)$

4 $4+5=9$ and $4\times 5=20$, $\quad x^2+9x+20=(x+4)(x+5)$

5 $6+3=9$ and $6\times 3=18$, $\quad x^2+9x+18=(x+6)(x+3)$

6 $7+4=11$ and $7\times 4=28$, $\quad x^2+11x+28=(x+7)(x+4)$

7 $-6-4=-10$ and $(-6)\times(-4)=24$,
$$x^2-10x+24=(x-6)(x-4)$$

8 $-5-3=-8$ and $(-5)\times(-3)=15$,
$$x^2-8x+15=(x-5)(x-3)$$

9 $-5-6=-11$ and $(-5)\times(-6)=30$,
$$x^2-11x+30=(x-5)(x-6)$$

10 $-4-8=-12$ and $(-4)\times(-8)=32$,
$$x^2-12x+32=(x-4)(x-8)$$

11 $-12-2=-14$ and $(-12)\times(-2)=24$,
$$x^2-14x+24=(x-12)(x-2)$$

12 $-20-2=-22$ and $(-20)\times(-2)=40$,
$$x^2-22x+40=(x-20)(x-2)$$

13 $10-3=7$ and $10\times(-3)=-30$,
$$x^2+7x-30=(x+10)(x-3)$$

14 $8-6=2$ and $8\times(-6)=-48$,
$$x^2+2x-48=(x+8)(x-6)$$

15 $18-2=16$ and $18\times(-2)=-36$,
$$x^2+16x-36=(x+18)(x-2)$$

16 $12-3=9$ and $12\times(-3)=-36$,
$$x^2+9x-36=(x+12)(x-3)$$

17 $14-3=11$ and $14\times(-3)=-42$,
$$x^2+11x-42=(x+14)(x-3)$$

18 $18-4=14$ and $18\times(-4)=-72$,
$$x^2+14x-72=(x+18)(x-4)$$

19 $5-8=-3$ and $5\times(-8)=-40$,
$$x^2-3x-40=(x+5)(x-8)$$

20 $-16+5=-11$ and $(-16)\times 5=-80$
$$x^2-11x-80=(x-16)(x+5)$$

21 $-32+3=-29$ and $(-32)\times 3=-96$,
$$x^2-29x-96=(x-32)(x+3)$$

22 $-14+4=-10$ and $(-14)\times 4=-56$,
$$x^2-10x-56=(x-14)(x+4)$$

23 $-7+6=-1$ and $(-7)\times 6=-42$,
$$x^2-x-42=(x-7)(x+6)$$

24 $-9+8=-1$ and $(-9)\times 8=-72$,
$$x^2-x-72=(x-9)(x+8)$$

25 $\left.\begin{array}{l}12+10=22\\ 12\times 10=120\end{array}\right\}$ $\begin{array}{l}x^2+22x+120\\ \qquad=(x+12)(x+10)\end{array}$

26 $\left.\begin{array}{l}20-4=16\\ 20\times(-4)=-80\end{array}\right\}$ $\begin{array}{l}x^2+16x-80\\ \qquad=(x+20)(x-4)\end{array}$

27 $\left.\begin{array}{l}-24+3=-21\\ (-24)\times 3=-72\end{array}\right\}$ $\begin{array}{l}x^2-21x-72\\ \qquad=(x-24)(x+3)\end{array}$

28 $\left.\begin{array}{l}12-7=5\\ 12\times(-7)=-84\end{array}\right\}$ $\begin{array}{l}x^2+5x-84\\ \qquad=(x+12)(x-7)\end{array}$

29 $\left.\begin{array}{l}-12-8=-20\\ (-12)\times(-8)=96\end{array}\right\}$ $\begin{array}{l}x^2-20x+96\\ \qquad=(x-12)(x-8)\end{array}$

30 $\left.\begin{array}{l}26-3=23\\ 26\times(-3)=-78\end{array}\right\}$ $\begin{array}{l}x^2+23x-78\\ \qquad=(x+26)(x-3)\end{array}$

31 $\left.\begin{array}{l}-12+6=-6\\ (-12)\times 6=-72\end{array}\right\}$ $\begin{array}{l}x^2-6x-72\\ \qquad=(x-12)(x+6)\end{array}$

32 $\left.\begin{array}{l}-21-4=-25\\ (-21)\times(-4)=84\end{array}\right\}$ $\begin{array}{l}x^2-25x+84\\ \qquad=(x-21)(x-4)\end{array}$

33 $\left.\begin{array}{l}-22-4=-26\\ (-22)\times(-4)=88\end{array}\right\}$ $\begin{array}{l}x^2-26x+88\\ \qquad=(x-22)(x-4)\end{array}$

34 $\left.\begin{array}{l}15-8=7\\ 15\times(-8)=-120\end{array}\right\}$ $\begin{array}{l}x^2+7x-120\\ \qquad=(x+15)(x-8)\end{array}$

35 $\left.\begin{array}{l}-10+8=-2\\ (-10)\times 8=-80\end{array}\right\}$ $\begin{array}{l}x^2-2x-80\\ \qquad=(x-10)(x+8)\end{array}$

36 $\left.\begin{array}{l}14-6=8\\ (14)\times(-6)=-84\end{array}\right\}$ $\begin{array}{l}x^2+8x-84\\ \qquad=(x+14)(x-6)\end{array}$

37 $\left.\begin{array}{l}-8+7=-1\\ (-8)\times 7=-56\end{array}\right\}$ $\begin{array}{l}a^2-a-56\\ \qquad=(a-8)(a+7)\end{array}$

38 $\left.\begin{array}{l}-15+6=-9\\ (-15)\times 6=-90\end{array}\right\}$ $\begin{array}{l}m^2-9m-90\\ \qquad=(m-15)(m+6)\end{array}$

39 $\left.\begin{array}{l}20-3=17\\ 20\times(-3)=-60\end{array}\right\}$ $\begin{array}{l}a^2+17a-60\\ \qquad=(a+20)(a-3)\end{array}$

40 $\left.\begin{array}{l}-6-9=-15\\ (-6)\times(-9)=54\end{array}\right\}$ $\begin{array}{l}a^2-15a+54\\ \qquad=(a-6)(a-9)\end{array}$

41 $\left.\begin{array}{l}-24+2=-22\\ (-24)\times 2=-48\end{array}\right\}$ $\begin{array}{l}p^2-22p-48\\ \qquad=(p-24)(p+2)\end{array}$

42 $\left.\begin{array}{l}9-8=1\\ 9\times(-8)=-72\end{array}\right\}$ $\begin{array}{l}m^2+m-72\\ \qquad=(m+9)(m-8)\end{array}$

43 $\left.\begin{array}{l}30-3=27\\ \text{and } 30\times(-3)=-90\end{array}\right\}$ $\begin{aligned}&m^2+27m-90\\&\quad=(m+30)(m-3)\end{aligned}$

44 $\left.\begin{array}{l}-24-5=-29\\ (-24)\times(-5)=120\end{array}\right\}$ $\begin{aligned}&a^2-29a+120\\&\quad=(a-24)(a-5)\end{aligned}$

45 $\left.\begin{array}{l}13-6=7\\ 13\times(-6)=-78\end{array}\right\}$ $\begin{aligned}&x^2+7x-78\\&\quad=(x+13)(x-6)\end{aligned}$

46 $\left.\begin{array}{l}-51+2=-49\\ (-51)\times2=-102\end{array}\right\}$ $\begin{aligned}&a^2-49a-102\\&\quad=(a-51)(a+2)\end{aligned}$

47 $\left.\begin{array}{l}-15-4=-19\\ (-15)\times(-4)=60\end{array}\right\}$ $\begin{aligned}&a^2-19a+60\\&\quad=(a-15)(a-4)\end{aligned}$

48 $\left.\begin{array}{l}16-4=12\\ 16\times(-4)=-64\end{array}\right\}$ $\begin{aligned}&x^2+12x-64\\&\quad=(x+16)(x-4)\end{aligned}$

49 $\left.\begin{array}{l}-30+4=-26\\ (-30)\times4=-120\end{array}\right\}$ $\begin{aligned}&a^2-26a-120\\&\quad=(a-30)(a+4)\end{aligned}$

50 $\left.\begin{array}{l}15-7=8\\ 15\times(-7)=-105\end{array}\right\}$ $\begin{aligned}&x^2+8x-105\\&\quad=(x+15)(x-7)\end{aligned}$

51 $\left.\begin{array}{l}6-7=-1\\ 6\times(-7)=-42\end{array}\right\}$ $\begin{aligned}&x^2-xy-42y^2\\&\quad=(x-7y)(x+6y)\end{aligned}$

52 $\left.\begin{array}{l}-8-4=-12\\ (-8)\times(-4)=32\end{array}\right\}$ $\begin{aligned}&a^2-12ab+32b^2\\&\quad=(a-8b)(a-4b)\end{aligned}$

53 $\left.\begin{array}{l}6-5=1\\ 6\times(-5)=-30\end{array}\right\}$ $\begin{aligned}&m^2+mn-30n^2\\&\quad=(m+6n)(m-5n)\end{aligned}$

54 $\left.\begin{array}{l}4-3=1\\ 4\times(-3)=-12\end{array}\right\}$ $\begin{aligned}&a^2+ab-12b^2\\&\quad=(a+4b)(a-3b)\end{aligned}$

55 $\left.\begin{array}{l}3-5=-2\\ 3\times(-5)=-15\end{array}\right\}$ $\begin{aligned}&a^2-2ab-15b^2\\&\quad=(a+3b)(a-5b)\end{aligned}$

56 $\left.\begin{array}{l}-8+1=-7\\ (-8)\times1=-8\end{array}\right\}$ $\begin{aligned}&x^2-7xy-8y^2\\&\quad=(x-8y)(x+y)\end{aligned}$

57 $\left.\begin{array}{l}8-5=3\\ 8\times(-5)=-40\end{array}\right\}$ $\begin{aligned}&x^2+3xy-40y^2\\&\quad=(x+8y)(x-5y)\end{aligned}$

58 $\left.\begin{array}{l}-8-6=-14\\ (-8)\times(-6)=48\end{array}\right\}$ $\begin{aligned}&p^2-14pq+48q^2\\&\quad=(p-8q)(p-6q)\end{aligned}$

59 $\left.\begin{array}{l}10-8=2\\ 10\times(-8)=-80\end{array}\right\}$ $\begin{aligned}&p^2+2pq-80q^2\\&\quad=(p+10q)(p-8q)\end{aligned}$

60 $\left.\begin{array}{l}24-4=20\\ 24\times(-4)=-96\end{array}\right\}$ $\begin{aligned}&x^2+20xy-96y^2\\&\quad=(x+24y)(x-4y)\end{aligned}$

61 $\left.\begin{array}{l} 5-1=4 \\ 5\times(-1)=-5 \end{array}\right\}$
$$\begin{aligned} &a^4+4a^2-5 \\ &\quad=(a^2+5)(a^2-1) \\ &\quad=(a^2+5)(a+1)(a-1) \end{aligned}$$

62 $\left.\begin{array}{l} 5-3=2 \\ 5\times(-3)=-15 \end{array}\right\}$
$$\begin{aligned} &x^4+2x^2-15 \\ &\quad=(x^2+5)(x^2-3) \end{aligned}$$

63 $\left.\begin{array}{l} 7-4=3 \\ 7\times(-4)=-28 \end{array}\right\}$
$$\begin{aligned} &x^4+3x^2-28 \\ &\quad=(x^2+7)(x^2-4) \\ &\quad=(x^2+7)(x+2)(x-2) \end{aligned}$$

64 $\left.\begin{array}{l} 3-1=2 \\ 3\times(-1)=-3 \end{array}\right\}$
$$\begin{aligned} &x^6+2x^3-3 \\ &\quad=(x^3+3)(x^3-1) \\ &\quad=(x^3+3)(x-1)(x^2+x+1) \end{aligned}$$

65 $\left.\begin{array}{l} -8-2=-10 \\ (-8)\times(-2)=16 \end{array}\right\}$
$$\begin{aligned} &a^6-10a^3+16 \\ &\quad=(a^3-8)(a^3-2) \\ &\quad=(a-2)(a^2+2a+4)(a^3-2) \end{aligned}$$

66 $\left.\begin{array}{l} 27-1=26 \\ 27\times(-1)=-27 \end{array}\right\}$
$$\begin{aligned} &x^6+26x^3-27 \\ &\quad=(x^3+27)(x^3-1) \\ &\quad=(x+3)(x^2-3x+9)(x-1) \\ &\qquad\qquad(x^2+x+1) \end{aligned}$$

67 $\left.\begin{array}{l} 8-1=7 \\ 8\times(-1)=-8 \end{array}\right\}$
$$\begin{aligned} &a^6+7a^3-8 \\ &\quad=(a^3+8)(a^3-1) \\ &\quad=(a+2)(a^2-2a+4)(a-1) \\ &\qquad\qquad(a^2+a+1) \end{aligned}$$

68 $\left.\begin{array}{l} -16-4=-20 \\ (-16)\times(-4)=64 \end{array}\right\}$
$$\begin{aligned} &x^8-20x^4+64 \\ &\quad=(x^4-16)(x^4-4) \\ &\quad=(x^2+4)(x^2-4)(x^2+2)(x^2-2) \\ &\quad=(x^2+4)(x+2)(x-2)(x^2+2) \\ &\qquad\qquad(x^2-2) \end{aligned}$$

69 $\left.\begin{array}{l} -16+5=-11 \\ (-16)\times 5=-80 \end{array}\right\}$
$$\begin{aligned} &a^8-11a^4-80 \\ &\quad=(a^4-16)(a^4+5) \\ &\quad=(a^2+4)(a^2-4)(a^4+5) \\ &\quad=(a^2+4)(a+2)(a-2)(a^4+5) \end{aligned}$$

70 $\left.\begin{array}{l} -8+1=-7 \\ (-8)\times 1=-8 \end{array}\right\}$
$$\begin{aligned} &x^{12}-7x^6-8 \\ &\quad=(x^6-8)(x^6+1) \\ &\quad=(x^2-2)(x^4+2x^2+4)(x^2+1) \\ &\qquad\qquad(x^4-x^2+1) \\ &\quad=(x^2+1)(x^2-2)(x^4-x^2+1) \\ &\qquad\qquad(x^4+2x^2+4) \end{aligned}$$

71 Putting x for (a^2+2a), the given expression

$$=x^2-x-2=(x+1)(x-2)=(a^2+2a+1)(a^2+2a-2)$$
$$=(a^2+2a-2)(a+1)^2$$

72 Putting a for (x^2+3x), the given expression

$$=a^2+3a+2=(a+1)(a+2)=(x^2+3x+1)(x^2+3x+2)$$
$$=(x^2+3x+1)(x+1)(x+2)$$

73 Putting a for (x^2-2x), the given expression

$$=a^2-2a-3=(a-3)(a+1)=(x^2-2x-3)(x^2-2x+1)$$
$$=(x-3)(x+1)(x-1)^2$$

74 Putting x for (a^2-3a), the given expression

$$=x^2-3x-4=(x-4)(x+1)=(a^2-3a-4)(a^2-3a+1)$$
$$=(a+1)(a-4)(a^2-3a+1)$$

75 Putting a for (x^2-4x), the given expression

$$=a^2-4a-5=(a-5)(a+1)=(x^2-4x-5)(x^2-4x+1)$$
$$=(x+1)(x-5)(x^2-4x+1)$$

76 Putting a for (x^2-x), the given expression

$$=a^2-8a+12=(a-6)(a-2)$$
$$=(x^2-x-6)(x^2-x-2)$$
$$=(x-3)(x+2)(x+1)(x-2).$$

77 Putting a for (x^2-5x), the given expression

$$=a^2+10a+24=(a+6)(a+4)$$
$$=(x^2-5x+6)(x^2-5x+4)$$
$$=(x-2)(x-3)(x-1)(x-4)$$

78 Putting x for (a^2+7a), the given expression

$$=x^2-8x-180=(x-18)(x+10)$$
$$=(a^2+7a-18)(a^2+7a+10)$$
$$=(a-2)(a+9)(a+2)(a+5)$$

79 Putting x for (a^2+6a), the given expression

$$=x^2-32x-320=(x-40)(x+8)$$
$$=(a^2+6a-40)(a^2+6a+8)$$
$$=(a-4)(a+10)(a+2)(a+4)$$

80 Putting a for (x^2-8x), the given expression

$$=a^2-29a+180=(a-9)(a-20)$$
$$=(x^2-8x-9)(x^2-8x-20)$$
$$=(x-9)(x+1)(x-10)(x+2)$$

81 Putting a for $(3x+4y)$ and b for $(x+2y)$, the given expression $=a^2+ab-2b^2$

$$=(a-b)(a+2b)$$
$$=\{(3x+4y)-(x+2y)\}\{(3x+4y)+2(x+2y)\}$$
$$=(2x+2y)-(5x+8y)=2(x+y)(5x+8y)$$

82 Putting x for $(2a-5b)$ and y for $(a+2b)$, the given expression $=x^2+2xy-3y^2=(x+3y)(x-y)$

$$=\{(2a-5b)+3(a+2b)\}\{(2a-5b)-(a+2b)\}$$
$$=(5a+b)(a-7b)$$

83 Putting a for $(4m-3n)$ and b for $(m+5n)$, the given expression $=a^2+3ab-4b^2$

$$=(a+4b)(a-b)$$
$$=\{(4m-3n)+4m(+5n)\}\{(4m-3n)-(m+5n)\}$$
$$=(8m+17n)(3m-8n)$$

84 Putting x for $(5a+3b)$ and y for $(a+b)$, the given expression

$$=x^2+4xy-5y^2=(x+5y)(x-y)$$
$$=\{(5a+3b)+5(a+b)\}\{(5a+3b)-(a+b)\}$$
$$=(10a+8b)(4a+2b)$$
$$=4(5a+4b)(2a+b)$$

85 Putting a for $(5x+7y)$ and b for $(x-y)$, the given expression

$$=a^2-2ab-8b^2=(a+2b)(a-4b)$$
$$=\{(5x+7y)+2(x-y)\}\{(5x+7y)-4(x-y)\}$$
$$=(7x+5y)(x+11y)$$

86 Putting x for $(7a+3b)$ and y for $(a-4b)$, the given expression

$$=x^2-3xy-10y^2$$
$$=(x-5y)(x+2y)$$
$$=\{(7a+3b)-5(a-4b)\}\{(7a+3b)+2(a-4b)\}$$
$$=(2a+23b)(9a-5b)$$

87 Putting x for $(9a-4b)$ and y for $(a-3b)$, the given

$$
\begin{aligned}
\text{expression} &= x^2-5xy-6y^2 \\
&=(x-6y)(x+y) \\
&=\{(9a-4b)-6(a-3b)\}\{(9a-4b)+(a-3b)\} \\
&=(3a+14b)(10a-7b)
\end{aligned}
$$

88 Putting x for $(2m^2+3n^2)$ and y for (m^2-2n^2), the given expres

$$
\begin{aligned}
\text{sion} = x^2+5xy-14y^2 &= (x-2y)(x+7y) \\
&=\{(2m^2+3n^2)-2(m^2-2n^2)\}\{(2m^2+3n^2) \\
&\qquad +7(m^2-2n^2)\} \\
&=7n^2(9m^2-11n^2)
\end{aligned}
$$

89 Putting a for (x^2+6xy) and b for $(xy-6y^2)$, the given

$$
\begin{aligned}
\text{expression} &= a^2-3ab+2b^2 \\
&=(a-b)(a-2b) \\
&=\{(x^2+6xy)-(xy-6y^2)\}\{(x^2+6xy)-2(xy-6y^2)\} \\
&=(x^2+4xy+12y^2)(x^2+5xy+6y^2) \\
&=(x^2+4xy+12y^2)(x+3y)(x+2y)
\end{aligned}
$$

90 Putting x for (a^2-5ab) and y for $(ab-4b^2)$, the given

$$
\begin{aligned}
\text{expression} &= x^2-9xy+18y^2 \\
&=(x-3y)(x-6y) \\
&=\{(a^2-5ab)-3(ab-4b^2)\}\{(a^2-5ab)-6(ab-4b^2)\} \\
&=(a^2-8ab+12b^2)(a^2-11ab+24b^2) \\
&=(a-6b)(a-2b)(a-3b)(a-8b) \\
&=(a-2b)(a-3b)(a-6b)(a-8b)
\end{aligned}
$$

91

$$
\begin{aligned}
2x^2+x-15 &= \tfrac{1}{2}(2\cdot 2x^2+2\cdot x-15\times 2) \\
&=\tfrac{1}{2}(a^2+a-30) \quad [\text{putting } a \text{ for } 2x] \\
&=\tfrac{1}{2}(a+6)(a-5)=\tfrac{1}{2}(2x+6)(2x-5) \\
&\qquad\qquad =(x+3)(2x-5)
\end{aligned}
$$

92

$$
\begin{aligned}
6a^2-a-15 &= \tfrac{1}{6}(6\cdot 6a^2-6\cdot a-15\times 6) \\
&=\tfrac{1}{6}(x^2-x-90) \quad [\text{putting } x \text{ for } 6a] \\
&=\tfrac{1}{6}(x+9)(x-10) \\
&=\tfrac{1}{6}(6a+9)(6x-10) \\
&=\tfrac{1}{3}(6a+9)\cdot\tfrac{1}{2}(6x-10)=(2a+3)(3x-5)
\end{aligned}
$$

93 $8m^2-6m-9=\frac{1}{8}(8\cdot 8m^2-6\cdot 8m-72)$

$=\frac{1}{8}(a^2-6a-72)$ [putting a for $8m$]

$=\frac{1}{8}(a-12)(a+6)$

$=\frac{1}{8}(8m-12)(8m+6)=\frac{1}{4}(8m-12)\cdot\frac{1}{2}(8m+6)$

$=(2m-3)(4m+3)$

94 $6x^2+7xy-24y^2=\frac{1}{6}(6\cdot 6x^2+7\cdot 6xy-144y^2)$

$=\frac{1}{6}(a^2+7ay-144y^2)$ [putting a for $6x$]

$=\frac{1}{6}(a+16y)(a-9y)$

$=\frac{1}{6}(6x+16y)(6x-9y)=\frac{1}{2}(6x+16y)\cdot\frac{1}{3}(6x-9y)$

$=(3x+8y)(2x-3y)$

95 $10a^2-41ab+21b^2=\frac{1}{10}(10\cdot 10a^2-41\cdot 10a\cdot b+210b^2)$

$=\frac{1}{10}(x^2-41xb+210b^2)$ [putting x for $10a$]

$=\frac{1}{10}(x-6b)(x-35b)$

$=\frac{1}{10}(10a-6b)(10a-35b)$

$=\frac{1}{2}(10a-6b)\cdot\frac{1}{5}(10a-35b)=(5a-3b)(2a-7b)$

96 $12m^2-mn-20n^2=\frac{1}{12}(12\cdot 12m^2-12mn-240n^2)$

$=\frac{1}{12}(x^2-xn-240n^2)$ [putting x for $12m$]

$=\frac{1}{12}(x-16n)(x+15n)$

$=\frac{1}{12}(12m-16n)(12m+15n)$

$=\frac{1}{4}(12m-16n)\cdot\frac{1}{3}(12m+15n)$

$=(3m-4n)(4m+5n)$

97 $12x^2+28xy-5y^2=\frac{1}{12}(12\cdot 12x^2+28\cdot 12x\cdot y-60y^2)$

$=\frac{1}{12}(a^2+28ay-60y^2)$ [putting a for $12x$]

$=\frac{1}{12}(a+30y)(a-2y)$

$=\frac{1}{12}(12x+30y)(12x-2y)$

$=\frac{1}{6}(12x+30y)\cdot\frac{1}{2}(12x-2y)=(2x+5y)(6x-y)$

98 $20a^2+ab-30b^2=\frac{1}{20}(20\cdot 20a^2+20a\cdot b-600b^2)$

$=\frac{1}{20}(x^2+xb-600b^2)$ [putting x for $20a$]

$=\frac{1}{20}(x+25b)(x-24b)$

$=\frac{1}{20}(20a+25b)(20a-24b)$

$=\frac{1}{5}(20a+25b)\cdot\frac{1}{4}(20a-24b)$

$=(4a+5b)(5a-6b)$

99 $18x^2-51xy+35y^2=\frac{1}{18}(18\cdot18x^2-51\cdot18xy+630y^2)$

$=\frac{1}{18}(a^2-51ay+630y^2)$ [putting a for $18x$]

$=\frac{1}{18}(a-21y)(a-30y)$

$=\frac{1}{18}(18x-21y)(18x-30y)$

$=\frac{1}{3}(18x-21y)\frac{1}{6}(18x-30y)$

$=(6x-7y)(3x-5y)$

100 $12x^2+23xy-24y^2=\frac{1}{12}(12\cdot12x^2+23\cdot12xy-288y^2)$

$=\frac{1}{12}(a^2+23ay-288y^2)$ [putting a for $12x$]

$=\frac{1}{12}(a+32y)(a-9y)$

$=\frac{1}{12}(12x+32y)(12x-9y)$

$=\frac{1}{4}(12x+32y)\frac{1}{3}(12x-9y)$

$=(3x+8y)(4x-3y)$

Exercise 45.

1 $x^2+4x+3=x^2+4x+4-1$

$=(x+2)^2-1=(x+2+1)(x+2-1)=(x+3)(x+1)$

2 $x^2+6x+5=x^2+6x+9-4=(x+3)^2-(2)^2$

$=\{(x+3)+2\}\{(x+3)-2\}$

$=(x+5)(x+1)$

3 $x^2+8x+15=x^2+8x+16-1=(x+4)^2-1$

$=\{(x+4)+1\}\{(x+4)-1\}$

$=(x+5)(x+3)$

4 $x^2-10x+21=x^2-10x+25-4=(x-5)^2-(2)^2$

$=\{(x-5)+2\}\{(x-5)-2\}$

$=(x-3)(x-7)$

5 $x^2-2x-48=x^2-2x+1-49=(x-1)^2-(7)^2$

$=\{(x-1)+7\}\{(x-1)-7\}$

$=(x+6)(x-8)$

6 $x^2-4x-45=x^2-4x+4-49=(x-2)^2-(7)^2$

$=\{(x-2)+7\}\{(x-2)-7\}$

$=(x+5)(x-9)$

7 $x^2 - 12x + 32 = x^2 - 12x + 36 - 4 = (x-6)^2 - (2)^2$
$= \{(x-6)+2\}\{(x-6)-2\}$
$= (x-4)(x-8)$

8 $x^2 - 6x - 55 = x^2 - 6x + 9 - 64 = (x-3)^2 - (8)^2$
$= \{(x-3)+8\}\{(x-3)-8\}$
$= (x+5)(x-11)$

9 $a^2 + 2ab - c^2 + 2bc = (a^2 + 2ab + b^2) - (b^2 + c^2 - 2bc)$
$= (a+b)^2 - (b-c)^2$
$= \{(a+b)+(b-c)\}\{(a+b)-(b-c)\}$
$= (a+2b-c)(a+c)$

10 $x^2 + 2x - y^2 + 2y = (x^2 + 2x + 1) - (1 + y^2 - 2y)$
$= (x+1)^2 - (y-1)^2$
$= \{(x+1)+(y-1)\}\{(x+1)-(y-1)\}$
$= (x+y)(x-y+2)$

11 $x^2 + 6x - y^2 + 4y + 5 = (x^2 + 6x + 9) - (4 + y^2 - 4y)$
$= (x+3)^2 - (y-2)^2$
$= \{(x+3)+(y-2)\}\{(x+3)-(y-2)\}$
$= (x+y+1)(x-y+5)$

12 $a^2 + 4ab - 5b^2 - c^2 + 6bc = (a^2 + 4ab + 4b^2) - (9b^2 + c^2 - 6bc)$
$= (a+2b)^2 - (3b-c)^2$
$= \{(a+2b)+(3b-c)\}\{(a+2b)-(3b-c)\}$
$= (a+5b-c)(a-b+c)$

13 $x^2 - 6xy + 5y^2 - z^2 + 4yz = (x^2 - 6xy + 9y^2) - (4y^2 + z^2 - 4yz)$
$= (x-3y)^2 - (2y-z)^2$
$= \{(x-3y)+(2y-z)\}\{(x-3y)-(2y-z)\}$
$= (x-y-z)(x-5y+z)$

14 $x^2 - 10xy + 16y^2 - 4z^2 + 12yz$
$= (x^2 - 10xy + 25y^2) - (9y^2 + 4z^2 - 12yz)$
$= (x-5y)^2 - (3y-2z)^2$
$= \{(x-5y)+(3y-2z)\}\{(x-5z)-(3y-2z)\}$
$= (x-2y-2z)(x-8y+2z)$

15 $a^2-12ab-13b^2-9c^2+42bc$

$$=(a^2-12ab+36b^2)-(49b^2+9c^2-42bc)$$
$$=(a-6b)^2-(7b-3c)^2$$
$$=\{(a-6b)+(7b-3c)\}\{(a-6b)-(7b-3c)\}$$
$$=(a+b-3c)(a-13b+3c)$$

16 $x^2+12xy-9z^2+36yz$

$$=(x^2+12xy+36y^2)-(36y^2+9z^2-36yz)$$
$$=(x+6y)^2-(6y-3z)^2$$
$$=\{(x+6y)+(6y-3z)\}\{(x+6y)-(6y-3z)\}$$
$$=(x+12y-3z)(x+3z)$$

17 $x^2-14xy-15y^2-25z^2+80yz$

$$=(x^2-14xy+49y^2)-(64y^2+25z^2-80yz)$$
$$=(x-7y)^2-(8y-5z)^2$$
$$=\{(x-7y)+(8y-5z)\}\{(x-7y)-(8y-5z)\}$$
$$=(x+y-5z)(x-15y+5z)$$

18 $2x^2-5x-3=2\left(x^2-\frac{5}{2}x-\frac{3}{2}\right)$

$$=2\left\{x^2-\frac{5}{2}x+\frac{25}{16}-\left(\frac{25}{16}+\frac{3}{2}\right)\right\}$$
$$=2\left\{\left(x-\frac{5}{4}\right)^2-\left(\frac{49}{16}\right)\right\}$$
$$=2\left\{\left(x-\frac{5}{4}\right)^2-\left(\frac{7}{4}\right)^2\right\}$$
$$=2\left\{\left(x-\frac{5}{4}\right)+\frac{7}{4}\right\}\left\{\left(x-\frac{5}{4}\right)-\frac{7}{4}\right\}$$
$$=2\left(x+\frac{1}{2}\right)(x-3)=(2x+1)(x-3)$$

19 $3x^2-5x-2=3\left(x^2-\frac{5}{3}x-\frac{2}{3}\right)$

$$=3\left\{\left(x^2-\frac{5}{3}x+\frac{25}{36}\right)-\left(\frac{25}{36}+\frac{2}{3}\right)\right\}$$
$$=3\left\{\left(x-\frac{5}{6}\right)^2-\left(\frac{49}{36}\right)\right\}$$
$$=3\left\{\left(x-\frac{5}{6}\right)^2-\left(\frac{7}{6}\right)^2\right\}$$
$$=3\left\{\left(x-\frac{5}{6}\right)+\frac{7}{6}\right\}\left\{\left(x-\frac{5}{6}\right)-\frac{7}{6}\right\}$$
$$=3\left(x+\frac{1}{3}\right)(x-2)=(3x+1)(x-2)$$

20 $3x^2+14x+8=3\left(x^2+\frac{14}{3}x+\frac{8}{3}\right)$

$$=3\left\{x^2+\frac{14}{3}x+\frac{49}{9}-\left(\frac{49}{9}-\frac{8}{3}\right)\right\}$$

$$=3\{(x+\tfrac{7}{3})^2-(\tfrac{25}{9})\}$$
$$=3\{(x+\tfrac{7}{3})^2-(\tfrac{5}{3})^2\}$$
$$=3\{(x+\tfrac{7}{3})+\tfrac{5}{3}\}\{(x+\tfrac{7}{3})-\tfrac{5}{3}\}$$
$$=3(x+4)(x+\tfrac{2}{3})=(x+4)(3x+2)$$

21 $4x^2+7x-2=4(x^2+\tfrac{7}{4}x-\tfrac{1}{2})$
$$=4\{x^2+\tfrac{7}{4}x+\tfrac{49}{64}-(\tfrac{49}{64}+\tfrac{1}{2})\}$$
$$=4\{(x+\tfrac{7}{8})^2-(\tfrac{81}{64})\}$$
$$=4\{(x+\tfrac{7}{8})^2-(\tfrac{9}{8})^2\}$$
$$=4\{(x+\tfrac{7}{8})+\tfrac{9}{8}\}\{(x+\tfrac{7}{8})-\tfrac{9}{8}\}$$
$$=4(x+2)(x-\tfrac{1}{4})=(x+2)(4x-1)$$

22 $6x^2+x-2=6\left(x^2+\dfrac{x}{6}-\dfrac{1}{3}\right)$
$$=6\left\{x^2+\frac{x}{6}+\frac{1}{144}-\left(\frac{1}{144}+\frac{1}{3}\right)\right\}$$
$$=6\left\{\left(x+\frac{1}{12}\right)^2-\left(\frac{49}{144}\right)\right\}$$
$$=6\left\{\left(x+\frac{1}{12}\right)^2-\left(\frac{7}{12}\right)^2\right\}$$
$$=6\left\{\left(x+\frac{1}{12}\right)+\frac{7}{12}\right\}\left\{\left(x+\frac{1}{12}\right)-\frac{7}{12}\right\}$$
$$=6\left(x+\frac{2}{3}\right)\left(x-\frac{1}{2}\right)=3\left(x+\frac{2}{3}\right)2\left(x-\frac{1}{2}\right)$$
$$=(3x+2)(2x-1)$$

23 $6x^2-5x-4=6\left(x^2-\dfrac{5}{6}x-\dfrac{2}{3}\right)$
$$=6\left\{x^2-\frac{5}{6}x+\frac{25}{144}-\left(\frac{25}{144}+\frac{2}{3}\right)\right\}$$
$$=6\left\{\left(x-\frac{5}{12}\right)^2-\left(\frac{121}{144}\right)\right\}$$
$$=6\left\{\left(x-\frac{5}{12}\right)^2-\left(\frac{11}{12}\right)^2\right\}$$
$$=6\left\{\left(x-\frac{5}{12}\right)+\frac{11}{12}\right\}\left\{\left(x-\frac{5}{12}\right)-\frac{11}{12}\right\}$$
$$=6\left(x+\frac{1}{2}\right)\left(x-\frac{4}{3}\right)=2\left(x+\frac{1}{2}\right)3\left(x-\frac{4}{3}\right)$$
$$=(2x+1)(3x-4)$$

24 $$6x^2+7x-3=6\left(x^2+\frac{7}{6}x-\frac{1}{2}\right)$$
$$=6\left\{x^2+\frac{7}{6}x+\frac{49}{144}-\left(\frac{49}{144}+\frac{1}{2}\right)\right\}$$
$$=6\left\{\left(x+\frac{7}{12}\right)^2-\frac{121}{144}\right\}$$
$$=6\left\{\left(x+\frac{7}{12}\right)^2-\left(\frac{11}{12}\right)^2\right\}$$
$$=6\left\{\left(x+\frac{7}{12}\right)+\frac{11}{12}\right\}\left\{\left(x+\frac{7}{12}\right)-\frac{11}{12}\right\}$$
$$=6\left(x+\frac{3}{2}\right)\left(x-\frac{1}{3}\right)=2\left(x+\frac{3}{2}\right)3\left(x-\frac{1}{3}\right)$$
$$=(2x+3)(3x-1)$$

25 $$8x^2+2x-15=8\left(x^2+\frac{1}{4}x-\frac{15}{8}\right)$$
$$=8\left\{x^2+\frac{x}{4}+\frac{1}{64}-\left(\frac{1}{64}+\frac{15}{8}\right)\right\}$$
$$=8\left\{\left(x+\frac{1}{8}\right)^2-\frac{121}{64}\right\}$$
$$=8\left\{\left(x+\frac{1}{8}\right)^2-\left(\frac{11}{8}\right)^2\right\}$$
$$=8\left\{\left(x+\frac{1}{8}\right)+\frac{11}{8}\right\}\left\{\left(x+\frac{1}{8}\right)-\frac{11}{8}\right\}$$
$$=8\left(x+\frac{3}{2}\right)\left(x-\frac{5}{4}\right)=(2x+3)(4x-5)$$

26 $$4x^2+4x-35=4\left(x^2+x-\frac{35}{4}\right)$$
$$=4\left\{x^2+x+\frac{1}{4}-\left(\frac{1}{4}+\frac{35}{4}\right)\right\}$$
$$=4\left\{\left(x+\frac{1}{2}\right)^2-(9)\right\}$$
$$=4\left\{\left(x+\frac{1}{2}\right)+3\right\}\left\{\left(x+\frac{1}{2}\right)-3\right\}$$
$$=4\left(x+\frac{7}{2}\right)\left(x-\frac{5}{2}\right)=(2x+7)(2x-5)$$

27 $$\begin{aligned} 6x^2 - x - 12 &= 6\left(x^2 - \frac{1}{6}x - 2\right) \\ &= 6\left\{x^2 - \frac{x}{6} + \frac{1}{144} - \left(\frac{1}{144} + 2\right)\right\} \\ &= 6\left\{\left(x - \frac{1}{12}\right)^2 - \frac{289}{144}\right\} \\ &= 6\left\{\left(x - \frac{1}{12}\right)^2 - \left(\frac{17}{12}\right)^2\right\} \\ &= 6\left\{\left(x - \frac{1}{12}\right) + \frac{17}{12}\right\}\left\{\left(x - \frac{1}{12}\right) - \frac{17}{12}\right\} \\ &= 6\left(x + \frac{4}{3}\right)\left(x - \frac{3}{2}\right) = (3x + 4)(2x - 3) \end{aligned}$$

28 $$\begin{aligned} 3x^2 - 16x - 12 &= 3\left(x^2 - \frac{16x}{3} - 4\right) \\ &= 3\left\{\left(x^2 - \frac{16x}{3} + \frac{64}{9}\right) - \left(\frac{64}{9} + 4\right)\right\} \\ &= 3\left\{\left(x - \frac{8}{3}\right)^2 - \frac{100}{9}\right\} \\ &= 3\left\{\left(x - \frac{8}{3}\right)^2 - \left(\frac{10}{3}\right)^2\right\} \\ &= 3\left\{\left(x - \frac{8}{3}\right) + \frac{10}{3}\right\}\left\{\left(x - \frac{8}{3}\right) - \frac{10}{3}\right\} \\ &= 3\left(x + \frac{2}{3}\right)(x - 6) = (3x + 2)(x - 6) \end{aligned}$$

29 $$\begin{aligned} 2x^2 - 9x - 35 &= 2\left(x^2 - \frac{9}{2}x - \frac{35}{2}\right) \\ &= 2\left\{x^2 - \frac{9}{2}x + \frac{81}{16} - \left(\frac{81}{16} + \frac{35}{2}\right)\right\} \\ &= 2\left\{\left(x - \frac{9}{4}\right)^2 - \left(\frac{19}{4}\right)^2\right\} \\ &= 2\left\{\left(x - \frac{9}{4}\right) + \frac{19}{4}\right\}\left\{\left(x - \frac{9}{4}\right) - \frac{19}{4}\right\} \\ &= 2\left(x + \frac{5}{2}\right)(x - 7) = (2x + 5)(x - 7) \end{aligned}$$

30
$$\begin{aligned}
2x^2+5x-42 &= 2\left(x^2+\frac{5}{2}x-21\right)\\
&= 2\left\{x^2+\frac{5}{2}x+\frac{25}{16}-\left(\frac{25}{16}+21\right)\right\}\\
&= 2\left\{\left(x+\frac{5}{4}\right)^2-\frac{361}{16}\right\}\\
&= 2\left\{\left(x+\frac{5}{4}\right)^2-\left(\frac{19}{4}\right)^2\right\}\\
&= 2\left\{\left(x+\frac{5}{4}\right)+\frac{19}{4}\right\}\left\{\left(x+\frac{5}{4}\right)-\frac{19}{4}\right\}\\
&= 2(x+6)\left(x-\frac{7}{2}\right)=(x+6)(2x-7)
\end{aligned}$$

31
$$\begin{aligned}
3x^2+13x-30 &= 3\left(x^2+\frac{13}{3}x-10\right)\\
&= 3\left\{x^2+\frac{13}{3}x+\frac{169}{36}-\left(\frac{169}{36}+10\right)\right\}\\
&= 3\left\{\left(x+\frac{13}{6}\right)^2-\frac{529}{36}\right\}\\
&= 3\left\{\left(x+\frac{13}{6}\right)^2-\left(\frac{23}{6}\right)^2\right\}\\
&= 3\left\{\left(x+\frac{13}{6}\right)+\frac{23}{6}\right\}\left\{\left(x+\frac{13}{6}\right)-\frac{23}{6}\right\}\\
&= 3(x+6)\left(x-\frac{5}{3}\right)=(x+6)(3x-5)
\end{aligned}$$

32
$$\begin{aligned}
12x^2+x-6 &= 12\left(x^2+\frac{x}{12}-\frac{1}{2}\right)\\
&= 12\left\{x^2+\frac{x}{12}+\frac{1}{576}-\left(\frac{1}{576}+\frac{1}{2}\right)\right\}\\
&= 12\left\{\left(x+\frac{1}{24}\right)^2-\frac{289}{576}\right\}\\
&= 12\left\{\left(x+\frac{1}{24}\right)^2-\left(\frac{17}{24}\right)^2\right\}\\
&= 12\left\{\left(x+\frac{1}{24}\right)+\frac{17}{24}\right\}\left\{\left(x+\frac{1}{24}\right)-\frac{17}{24}\right\}\\
&= 12\left(x+\frac{3}{4}\right)\left(x-\frac{2}{3}\right)=(4x+3)(3x-2)
\end{aligned}$$

33
$$\begin{aligned}
2a^2+7ab-15b^2 &= 2\left(a^2+\frac{7}{2}b-\frac{15}{2}b^2\right)\\
&= 2\left\{a^2+\frac{7}{2}ab+\frac{49}{16}b^2-\left(\frac{49}{16}+\frac{15}{2}\right)b^2\right\}\\
&= 2\left\{\left(a+\frac{7}{4}b\right)^2-\frac{169b^2}{16}\right\}\\
&= 2\left\{\left(a+\frac{7}{4}b\right)^2-\left(\frac{13}{4}b\right)^2\right\}\\
&= 2\left\{\left(a+\frac{7}{4}b\right)+\frac{13}{4}b\right\}\left\{\left(a+\frac{7}{4}b\right)-\frac{13}{4}b\right\}\\
&= 2(a+5b)(a-\tfrac{1}{2}b)=(a+5b)(2a-3b)
\end{aligned}$$

34
$$\begin{aligned}
6x^2-13xy+6y^2 &= 6\left(x^2-\frac{13}{6}xy+y^2\right)\\
&= 6\left\{x^2-\frac{13}{6}xy+\frac{169}{144}y^2-\left(\frac{169}{144}-1\right)y^2\right\}\\
&= 6\left\{\left(x-\frac{13}{12}y\right)^2-\frac{25}{144}y^2\right\}\\
&= 6\left\{\left(x-\frac{13}{12}y\right)^2-\left(\frac{5}{12}y\right)^2\right\}\\
&= 6\left\{\left(x-\frac{13}{12}y\right)+\frac{5}{12}y\right\}\left\{\left(x-\frac{13}{12}y\right)-\frac{5}{12}y\right\}\\
&= 6\left(x-\frac{2}{3}y\right)\left(x-\frac{3}{2}y\right)=(3x-2y)(2x-3y)
\end{aligned}$$

35
$$\begin{aligned}
6m^2-11mn-10n^2 &= 6(m^2-\tfrac{11}{6}mn-\tfrac{5}{3}n^2)\\
&= 6\{m^2-\tfrac{11}{6}mn+\tfrac{121}{144}n^2-(\tfrac{121}{144}+\tfrac{5}{3})n^2\}\\
&= 6\{(m-\tfrac{11}{12}n)^2-\tfrac{361}{144}n^2\}\\
&= 6\{(m-\tfrac{11}{12}n)^2-(\tfrac{19}{12}n)^2\}\\
&= 6\{(m-\tfrac{11}{12}n)+\tfrac{19}{12}n\}\{(m-\tfrac{11}{12}n)-\tfrac{19}{12}n\}\\
&= 6(m+\tfrac{2}{3}n)(m-\tfrac{5}{2}n)=(3m+2n)(2m-5n)
\end{aligned}$$

36
$$\begin{aligned}
3p^2+5pq-12q^2 &= 3(p^2+\tfrac{5}{3}pq-4q^2)\\
&= 3\{p^2+\tfrac{5}{3}pq+\tfrac{25}{36}q^2-(\tfrac{25}{36}+4)q^2\}\\
&= 3\{(p+\tfrac{5}{6}q)^2-\tfrac{169}{36}q^2\}\\
&= 3\{(p+\tfrac{5}{6}q)^2-(\tfrac{13}{6})^2\}\\
&= 3\{(p+\tfrac{5}{6}q)+\tfrac{13}{6}q\}\{(p+\tfrac{5}{6}q)-\tfrac{13}{6}q\}\\
&= 3(p+3q)(p-\tfrac{4}{3}q)=(p+3q)(3p-4q)
\end{aligned}$$

37 $$8a^2-14ab-15b^2=8(a^2-\tfrac{7}{4}ab-\tfrac{15}{8}b^2)$$
$$=8(a^2-\tfrac{7}{4}ab-\tfrac{15}{8}b^2)$$
$$=8\{a^2-\tfrac{7}{4}ab+\tfrac{49}{64}b^2-(\tfrac{49}{64}+\tfrac{15}{8})b^2\}$$
$$=8\{(a-\tfrac{7}{8}b)^2-\tfrac{169}{64}b^2\}$$
$$=8\{(a-\tfrac{7}{8}b)^2-(\tfrac{13}{8}b)^2\}$$
$$=8\{(a-\tfrac{7}{8}b)+\tfrac{13}{8}b\}\{(a-\tfrac{7}{8}b)-\tfrac{13}{8}b\}$$
$$=8(a+\tfrac{3}{4}b)(a-\tfrac{5}{2}b)=(4a+3b)(2a-5b)$$

38 $$10m^2+11mn-6n^2=10(m^2+\tfrac{11}{10}mn-\tfrac{3}{5}n^2)$$
$$=10\{m^2+\tfrac{11}{10}mn+\tfrac{121}{400}n^2-(\tfrac{121}{400}+\tfrac{3}{5})n^2\}$$
$$=10\{(m+\tfrac{11}{20}n)^2-\tfrac{361}{400}n^2\}$$
$$=10\{(m+\tfrac{11}{20}n)^2-(\tfrac{19}{20}n)^2\}$$
$$=10\{(m+\tfrac{11}{20}n)+\tfrac{19}{20}n\}\{(m+\tfrac{11}{20}n)-\tfrac{19}{20}n\}$$
$$=10(m+\tfrac{3}{2}n)(m-\tfrac{2}{5}n)=(2m+3n)(5m-2n)$$

39 $$12x^2+13xy-4y^2=12(x^2+\tfrac{13}{12}xy-\tfrac{1}{3}y^2)$$
$$=12\{x^2+\tfrac{13}{12}xy+\tfrac{169}{576}y^2-(\tfrac{169}{576}+\tfrac{1}{3})y^2\}$$
$$=12\{(x+\tfrac{13}{24}y)^2-\tfrac{361}{576}y^2\}=12\{(x+\tfrac{13}{24}y)^2-(\tfrac{19}{24}y)^2\}$$
$$=12\{(x+\tfrac{13}{24}y)+\tfrac{19}{24}y\}\{(x+\tfrac{13}{24}y)-\tfrac{19}{24}y\}$$
$$=12(x+\tfrac{4}{3}y)(x-\tfrac{1}{4}y)=(3x+4y)(4x-y)$$

40 $$15a^2-11ab-12b^2=15(a^2-\tfrac{11}{15}ab-\tfrac{4}{5}b^2)$$
$$=15\{a^2-\tfrac{11}{15}ab+\tfrac{121}{900}b^2-(\tfrac{121}{900}+\tfrac{4}{5})b^2\}$$
$$=15\{(a-\tfrac{11}{30}b)^2-\tfrac{841}{900}b^2\}$$
$$=15\{(a-\tfrac{11}{30}b)^2-(\tfrac{29}{30}b)^2\}$$
$$=15\{(a-\tfrac{11}{30}b)+\tfrac{29}{30}b\}\{(a-\tfrac{11}{30}b)-\tfrac{29}{30}b\}$$
$$=15(a+\tfrac{3}{5}b)(a-\tfrac{4}{3}b)=(5a+3b)(3a-4b)$$

41 $$2a^2-5ab+2b^2=2(a^2-\tfrac{5}{2}ab+b^2)$$
$$=2\{a^2-\tfrac{5}{2}ab+\tfrac{25}{16}b^2-(\tfrac{25}{16}-1)b^2\}$$
$$=2\{(a-\tfrac{5}{4}b)^2-\tfrac{9}{16}b^2\}$$
$$=2\{(a-\tfrac{5}{4}b)^2-(\tfrac{3}{4}b)^2\}$$
$$=2\{(a-\tfrac{5}{4}b)+\tfrac{3}{4}b\}\{(a-\tfrac{5}{4}b)-\tfrac{3}{4}b\}$$
$$=2(a-\tfrac{1}{2}b)(a-2b)=(2a-b)(a-2b)$$

42 $$3a^2-8ab-3b^2=3(a^2-\tfrac{8}{3}ab-b^2)$$
$$=3\{a^2-\tfrac{8}{3}ab+\tfrac{16}{9}b^2-(\tfrac{16}{9}+1)b^2\}$$
$$=3\{(a-\tfrac{4}{3}b)^2-\tfrac{25}{9}b^2\}$$

$$=3\{(a-\tfrac{4}{3}b)^2-(\tfrac{5}{3}b)^2\}$$
$$=3\{(a-\tfrac{4}{3}b)+\tfrac{5}{3}b\}\{(a-\tfrac{4}{3}b)-\tfrac{5}{3}b\}$$
$$=3(a+\tfrac{1}{3}b)(a-3b)=(3a+b)(a-3b).$$

43 $3x^2+8xy-3y^2=3(x^2+\tfrac{8}{3}xy-y^2)$
$$=3\{x^2+\tfrac{8}{3}xy+\tfrac{16}{9}y^2-(\tfrac{16}{9}+1)y^2\}$$
$$=3\{(x+\tfrac{4}{3}y)^2-\tfrac{25}{9}y^2\}$$
$$=3\{(x+\tfrac{4}{3}y)^2-(\tfrac{5}{3}y)^2\}$$
$$=3\{(x+\tfrac{4}{3}y)-\tfrac{5}{3}y\}\{(x+\tfrac{4}{3}y)+\tfrac{5}{3}y\}$$
$$=3(x+3y)(x-\tfrac{1}{3}y)=(x+3y)(3x-y).$$

44 $4a^2-15a-4=4(a^2-\tfrac{15}{4}a-1)$
$$=4\{a^2-\tfrac{15}{4}a+\tfrac{225}{64}-(\tfrac{225}{64}+1)\}$$
$$=4\{(a-\tfrac{15}{8})^2-\tfrac{289}{64}\}$$
$$=4\{(a-\tfrac{15}{8})^2-(\tfrac{17}{8})^2\}$$
$$=4\{(a-\tfrac{15}{8})-\tfrac{17}{8}\}\{(a-\tfrac{15}{8})+\tfrac{17}{8}\}$$
$$=4(a-4)(a+\tfrac{1}{4})=(a-4)(4a+1).$$

45 $4a^2-17ab+4b^2=4(a^2-\tfrac{17}{4}ab+b^2)$
$$=4\{a^2-\tfrac{17}{4}ab+\tfrac{289}{64}b^2-(\tfrac{289}{64}-1)b^2\}$$
$$=4\{(a-\tfrac{17}{8}b)^2-\tfrac{225}{64}b^2\}$$
$$=4\{(a-\tfrac{17}{8}b)^2-(\tfrac{15}{8}b)^2\}$$
$$=4\{(a-\tfrac{17}{8}b)-\tfrac{15}{8}b\}\{(a-\tfrac{17}{8}b)+\tfrac{15}{8}b\}$$
$$=4(a-\tfrac{1}{4}b)(a-4b)=(4a-b)(a-4b).$$

46 $5x^2-24x-5=5(x^2-\tfrac{24}{5}x-1)$
$$=5\{x^2-\tfrac{24}{5}x+\tfrac{144}{25}-(\tfrac{144}{25}+1)\}$$
$$=5\{(x-\tfrac{12}{5})^2-\tfrac{169}{25}\}$$
$$=5\{(x-\tfrac{12}{5})^2-(\tfrac{13}{5})^2\}$$
$$=5\{(x-\tfrac{12}{5})+\tfrac{13}{5}\}\{(x-\tfrac{12}{5})-\tfrac{13}{5}\}$$
$$=5(x+\tfrac{1}{5})(x-5)=(5x+1)(x-5).$$

47 $5x^2-26xy-5y^2=5(x^2-\tfrac{26}{5}xy-y^2)$
$$=5\{x^2-\tfrac{26}{5}xy+\tfrac{169}{25}y^2-(\tfrac{169}{25}-1)y^2\}$$
$$=5\{(x-\tfrac{13}{5}y)^2-\tfrac{144}{25}y^2\}$$
$$=5\{(x-\tfrac{13}{5}y)^2-(\tfrac{12}{5}y)^2\}$$
$$=5\{(x-\tfrac{13}{5}y)+\tfrac{12}{5}y\}\{(x-\tfrac{13}{5}y)-\tfrac{12}{5}y\}$$
$$=5(x-\tfrac{1}{5}y)(x-5y)=(5x-y)(x-5y).$$

48 $6x^2+37x+6=6\left(x^2+\frac{37x}{6}+1\right)$

$$=6\{x^2+\tfrac{37}{6}x+\tfrac{1369}{144}-(\tfrac{1369}{144}-1)\}$$
$$=6\{(x+\tfrac{37}{12})^2-\tfrac{1225}{144}\}$$
$$=6\{(x+\tfrac{37}{12})^2-(\tfrac{35}{12})^2\}$$
$$=6\{(x+\tfrac{37}{12})+\tfrac{35}{12}\}\{(x+\tfrac{37}{12})-\tfrac{35}{12}\}$$
$$=6(x+6)(x+\tfrac{1}{6})=(x+6)(6x+1)$$

49 $6a^2+35ab-6b^2=6\{a^2+\tfrac{35}{6}ab-b^2\}$

$$=6\{a^2+\tfrac{35}{6}ab+\tfrac{1225}{144}b^2-(\tfrac{1225}{144}+1)b^2\}$$
$$=6\{(a+\tfrac{35}{12}b)^2-\tfrac{1369}{144}b^2\}$$
$$=6\{(a+\tfrac{35}{12}b)^2-(\tfrac{37}{12}b)^2\}$$
$$=6\{(a+\tfrac{35}{12}b)+\tfrac{37}{12}b\}\{(a+\tfrac{35}{12}b)-\tfrac{37}{12}b\}$$
$$=6(a+6b)(a-\tfrac{1}{6}b)=(a+6b)(6a-b)$$

50 $6a^2-35ab-6b^2=6(a^2-\tfrac{35}{6}ab-b^2)$

$$=6\{a^2-\tfrac{35}{6}ab+\tfrac{1225}{144}b^2-(\tfrac{1225}{144}+1)b^2\}$$
$$=6\{(a-\tfrac{35}{12}b)^2-\tfrac{1369}{144}b^2\}$$
$$=6\{(a-\tfrac{35}{12}b)^2-(\tfrac{37}{12}b)^2\}$$
$$=6\{(a-\tfrac{35}{12}b)+\tfrac{37}{12}b\}\{(a-\tfrac{35}{12}b)-\tfrac{37}{12}b\}$$
$$=6(a+\tfrac{1}{6}b)(a-6b)=(6a+b)(a-6b)$$

51 $7a^2-50ab+7b^2=7(a^2-\tfrac{50}{7}ab+b^2)$

$$=7\{a^2-\tfrac{50}{7}ab+\tfrac{625}{49}b^2-(\tfrac{625}{49}-1)b^2\}$$
$$=7\{(a-\tfrac{25}{7}b)^2-\tfrac{576}{49}b^2\}$$
$$=7\{(a-\tfrac{25}{7}b)^2-(\tfrac{24}{7}b)^2\}$$
$$=7\{(a-\tfrac{25}{7}b)+\tfrac{24}{7}b\}\{(a-\tfrac{25}{7}b)-\tfrac{24}{7}b\}$$
$$=7(a-\tfrac{1}{7}b)(a-7b)=(7a-b)(a-7b)$$

52 $7a^2+48ab-7b^2=7(a^2+\tfrac{48}{7}ab-b^2)$

$$=7\{a^2+\tfrac{48}{7}ab+\tfrac{576}{49}b^2-(\tfrac{576}{49}+1)b^2\}$$
$$=7\{(a+\tfrac{24}{7}b)^2-\tfrac{625}{49}b^2\}$$
$$=7\{(a+\tfrac{24}{7}b)^2-(\tfrac{25}{7}b)^2\}$$
$$=7\{(a+\tfrac{24}{7}b)+\tfrac{25}{7}b\}\{(a+\tfrac{24}{7}b)-\tfrac{25}{7}b\}$$
$$=7(a+7b)(a-\tfrac{1}{7}b)=(a+7b)(7a-b)$$

53 $7a^2-48ab-7b^2=7(a^2-\tfrac{48}{7}ab-b^2)$

$$=7\{a^2-\tfrac{48}{7}ab+\tfrac{576}{49}b^2-(\tfrac{576}{49}+1)b^2\}$$
$$=7\{(a-\tfrac{24}{7}b)^2-\tfrac{625}{49}b^2\}$$

$$=7\{(a-\tfrac{24}{7}b)^2-(\tfrac{25}{7}b)^2\}$$
$$=7\{(a-\tfrac{24}{7}b)+\tfrac{25}{7}b\}\{(a-\tfrac{24}{7}b)-\tfrac{25}{7}b\}$$
$$=7(a+\tfrac{1}{7}b)(a-7b)=(7a+b)(a-7b)$$

54 $8x^2+63xy-8y^2=8(x^2+\tfrac{63}{8}xy-y^2)$
$$=8\{x^2+\tfrac{63}{8}xy+\tfrac{3969}{256}y^2-(\tfrac{3969}{256}+1)y^2\}$$
$$=8\{(x+\tfrac{63}{16}y)^2-\tfrac{4225}{256}y^2\}$$
$$=8\{(x+\tfrac{63}{16}y)^2-(\tfrac{65}{16}y)^2\}$$
$$=8\{(x+\tfrac{63}{16}y)+\tfrac{65}{16}y\}\{(x+\tfrac{63}{16}y)-\tfrac{65}{16}y\}$$
$$=8(x+8y)(x-\tfrac{1}{8}y)=(x+8y)(8x-y)$$

55 $9x^2-82xy+9y^2=9(x^2-\tfrac{82}{9}xy+y^2)$
$$=9\{x^2-\tfrac{82}{9}xy+\tfrac{1681}{81}y^2-(\tfrac{1681}{81}-1)y^2\}$$
$$=9\{(x-\tfrac{41}{9}y)^2-\tfrac{1600}{81}y^2\}$$
$$=9\{(x-\tfrac{41}{9}y)^2-(\tfrac{40}{9}y)^2\}$$
$$=9\{(x-\tfrac{41}{9}y)+\tfrac{40}{9}y\}\{(x-\tfrac{41}{9}y)-\tfrac{40}{9}y\}$$
$$=9(x-\tfrac{1}{9}y)(x-9y)=(9x-y)(x-9y)$$

56 $10x^2+99xy-10y^2=10(x^2+\tfrac{99}{10}xy-y^2)$
$$=10\{x^2+\tfrac{99}{10}xy+\tfrac{9801}{400}y^2-(\tfrac{9801}{400}+1)y^2\}$$
$$=10\{(x+\tfrac{99}{20}y)^2-\tfrac{10201}{400}y^2\}$$
$$=10\{(x+\tfrac{99}{20}y)^2-(\tfrac{101}{20}y)^2\}$$
$$=10\{(x+\tfrac{99}{20}y)+(\tfrac{101}{20}y)\}\{(x+\tfrac{99}{20}y)-\tfrac{101}{20}y\}$$
$$=10(x+10y)(x-\tfrac{1}{10}y)=(x+10y)(10x-y)$$

57 Putting x for $(a+b)$, the given expression
$$=2x^2+3x-2=2(x^2+\tfrac{3}{2}x-1)$$
$$=2\{x^2+\tfrac{3}{2}x+\tfrac{9}{16}-(\tfrac{9}{16}+1)\}$$
$$=2\{(x+\tfrac{3}{4})^2-\tfrac{25}{16}\}$$
$$=2\{(x+\tfrac{3}{4})^2-(\tfrac{5}{4})^2\}$$
$$=2\{(x+\tfrac{3}{4})+\tfrac{5}{4}\}\{(x+\tfrac{3}{4})-\tfrac{5}{4}\}$$
$$=2(x+2)(x-\tfrac{1}{2})$$
$$=(x+2)(2x-1)=(a+b+2)(2a+2b-1)$$

58 Putting a for (x^2+y^2) and b for xy, the given expression
$$=2a^2-3ab-2b^2=2(a^2-\tfrac{3}{2}ab-b^2)$$
$$=2\{a^2-\tfrac{3}{2}ab+\tfrac{9}{16}b^2-(\tfrac{9}{16}+1)b^2\}$$
$$=2\{(a-\tfrac{3}{4}b)^2-\tfrac{25}{16}b^2\}$$
$$=2\{(a-\tfrac{3}{4}b)^2-(\tfrac{5}{4}b)^2\}$$

$$=2\{(a-\tfrac{3}{4}b)+\tfrac{5}{4}b\}\{(a-\tfrac{3}{4}b)-\tfrac{5}{4}b\}$$
$$=2(a+\tfrac{1}{2}b)(a-2b)$$
$$=(2a+b)(a-2b)$$
$$=(2x^2+2y^2+xy)(x^2+y^2-2xy)$$
$$=(2x^2+2y^2+xy)(x-y)^2$$

59 Putting x for (a^2+b^2) and y for ab, the given expression
$$=2x^2+5xy+2y^2=2(x^2+\tfrac{5}{2}xy+y^2)$$
$$=2\{x^2+\tfrac{5}{2}xy+\tfrac{25}{16}y^2-(\tfrac{25}{16}-1)y^2\}$$
$$=2\{(x+\tfrac{5}{4}y)^2-\tfrac{9}{16}y^2\}$$
$$=2\{(x+\tfrac{5}{4}y)^2-(\tfrac{3}{4}y)^2\}$$
$$=2\{(x+\tfrac{5}{4}y)+\tfrac{3}{4}y\}\{(x+\tfrac{5}{4}y)-\tfrac{3}{4}y\}$$
$$=2(x+2y)(x+\tfrac{1}{2}y)$$
$$=(x+2y)(2x+y)$$
$$=(a^2+b^2+2ab)(2a^2+2b^2+ab)$$
$$=(a+b)^2(2a^2+2b^2+ab)$$

60 Putting a for $(x^2-4xy+y^2)$ and b for xy, the given expression $=4a^2+15ab-4b^2$
$$=4(a^2+\tfrac{15}{4}ab-b^2)$$
$$=4\{a^2+\tfrac{15}{4}ab+\tfrac{225}{64}b^2-(\tfrac{225}{64}+1)b^2\}$$
$$=4\{(a+\tfrac{15}{8}b)^2-\tfrac{289}{64}b^2\}$$
$$=4\{(a+\tfrac{15}{8}b)^2-(\tfrac{17}{8}b)^2\}$$
$$=4\{(a+\tfrac{15}{8}b)+\tfrac{17}{8}b\}\{(a+\tfrac{15}{8}b)-\tfrac{17}{8}b\}$$
$$=4(a+4b)(a-\tfrac{1}{4}b)$$
$$=(a+4b)(4a-b)$$
$$=(x^2-4xy+y^2+4xy)(4x^2-16xy+4y^2-4xy)$$
$$=(x^2+y^2)(4x^2-12xy+4y^2)$$
$$=(x^2+y^2)4(x-\tfrac{1}{4}y)(x-4y)$$
$$=(x^2+y^2)(4x-y)(x-4y)$$

61 $2x^4-5x^2-12=2(x^4-\tfrac{5}{2}x^2-6)$
$$=2\{x^4-\tfrac{5}{2}x^2+\tfrac{25}{16}-(\tfrac{25}{16}+6)\}$$
$$=2\{(x^2-\tfrac{5}{4})^2-\tfrac{121}{16}\}=2\{(x^2-\tfrac{5}{4})^2-(\tfrac{11}{4})^2\}$$
$$=2\{(x^2-\tfrac{5}{4})+\tfrac{11}{4}\}\{(x^2-\tfrac{5}{4})-\tfrac{11}{4}\}$$
$$=2(x^2+\tfrac{3}{2})(x^2-4)=(2x^2+3)(x+2)(x-2)$$

62 $8a^4-14a^2b^2-9b^4=8(a^4-\frac{7}{4}a^2b^2-\frac{9}{8}b^4)$

$$=8\{a^4-\tfrac{7}{4}a^2b^2+\tfrac{49}{64}b^4-b^4(\tfrac{49}{64}+\tfrac{9}{8})\}$$
$$=8\{(a^2-\tfrac{7}{8}b^2)^2-\tfrac{121}{64}b^4\}$$
$$=8\{(a^2-\tfrac{7}{8}b^2)^2-(\tfrac{11}{8}b^2)^2\}$$
$$=8\{(a^2-\tfrac{7}{8}b^2)+\tfrac{11}{8}b^2\}\{(a^2-\tfrac{7}{8}b^2)-\tfrac{11}{8}b^2\}$$
$$=8(a^2+\tfrac{1}{2}b^2)(a^2-\tfrac{9}{4}b^2)$$
$$=(2a^2+b^2)(4a^2-9b^2)$$
$$=(2a^2+b^2)(2a+3b)(2a-3b)$$

63 $9a^4+2a^2b^2-32b^4=9(a^4+\frac{2}{9}a^2b^2-\frac{32}{9}b^4)$

$$=9\{a^4+\tfrac{2}{9}a^2b^2+\tfrac{1}{81}b^4-b^4(\tfrac{1}{81}+\tfrac{32}{9})\}$$
$$=9\{(a^2+\tfrac{1}{9}b^2)^2-\tfrac{289}{81}b^4\}$$
$$=9\{(a^2+\tfrac{1}{9}b^2)^2-(\tfrac{17}{9}b^2)^2\}$$
$$=9\{(a^2+\tfrac{1}{9}b^2)+\tfrac{17}{9}b^2\}\{(a^2+\tfrac{1}{9}b^2)-\tfrac{17}{9}b^2\}$$
$$=9(a^2+2b^2)(a^2-\tfrac{16}{9}b^2)$$
$$=(a^2+2b^2)(9a^2-16b^2)$$
$$=(a^2+2b^2)(3a+4b)(3a-4b)$$

64 $8x^6-65x^3+8=8(x^6-\frac{65}{8}x^3+1)$

$$=8\{x^6-\tfrac{65}{8}x^3+\tfrac{4225}{256}-(\tfrac{4225}{256}-1)\}$$
$$=8\{(x^3-\tfrac{65}{16})^2-\tfrac{3969}{256}\}$$
$$=8\{(x^3-\tfrac{65}{16})^2-(\tfrac{63}{16})^2\}$$
$$=8\{x^3-\tfrac{65}{16}+\tfrac{63}{16}\}\{x^3-\tfrac{65}{16}-\tfrac{63}{16}\}$$
$$=8(x^3-\tfrac{1}{8})(x^3-8)$$
$$=(8x^3-1)(x^3-8)$$
$$=(2x-1)(4x^2+2x+1)(x-2)(x^2+2x+4)$$

65 $4a^8-17a^4b^4+4b^8=4(a^8-\frac{17}{4}a^4b^4+b^8)$

$$=4\{a^8-\tfrac{17}{4}a^4b^4+\tfrac{289}{64}b^8-(\tfrac{289}{64}-1)b^8\}$$
$$=4\{(a^4-\tfrac{17}{8}b^4)^2-\tfrac{225}{64}b^8\}$$
$$=4\{(a^4-\tfrac{17}{8}b^4)^2-(\tfrac{15}{8}b^4)^2\}$$
$$=4\{(a^4-\tfrac{17}{8}b^4+\tfrac{15}{8}b^4)(a^4-\tfrac{17}{8}b^4-\tfrac{15}{8}b^4)\}$$
$$=4(a^4-\tfrac{1}{4}b^4)(a^4-4b^4)$$
$$=(4a^4-b^4)(a^4-4b^4)$$
$$=(2a^2+b^2)(2a^2-b^2)(a^2+2b^2)(a^2-2b^2)$$

Exercise 46.

1 $x^3+x^2+x+1=x^2(x+1)+(x+1)=(x^2+1)(x+1)$

2 $x^3+x^2-x-1=x^2(x+1)-(x+1)$
$$=(x+1)(x^2-1)=(x+1)(x+1)(x-1)$$
$$=(x+1)^2(x-1)$$

3 $x^3-x^2-x+1=x^2(x-1)-(x-1)$
$$=(x-1)(x^2-1)=(x-1)(x+1)(x-1)$$
$$=(x-1)^2(x+1)$$

4 $bc(a^2+1)+a(b^2+c^2)=bca^2+ab^2+ac^2+bc$
$$=ab(ac+b)+c(ac+b)=(ab+c)(ac+b)$$

5 $x^4-ab^3+xb^3-x^3a=x^4+xb^3-ab^3-x^3a$
$$=x(x^3+b^3)-a(x^3+b^3)$$
$$=(x-a)(x^3+b^3)=(x-a)(x+b)(x^2-xb+b^2)$$

6 $ab(x^2+y^2)+xy(a^2+b^2)=abx^2+xya^2+xyb^2+aby^2$
$$=ax(bx+ay)+by(bx+ay)$$
$$=(ax+by)(bx+ay)$$

7 $x^2+xy-yz-z^2=x^2-z^2+xy-yz$
$$=(x+z)(x-z)+y(x-z)=(x+y+z)(x-z)$$

8 $xb-ac-xc+ab=xb-xc+ab-ac$
$$=x(b-c)+a(b-c)=(x+a)(b-c)$$

9 $(2x^2+3b^2)a-(2a^2+3x^2)b$
$$=2ax^2-3x^2b-2a^2b+3ab^2$$
$$=x^2(2a-3b)-ab(2a-3b)$$
$$=(x^2-ab)(2a-3b)$$

10 $a(a+c)-b(b+c)=a^2-b^2+ac-bc$
$$=(a+b)(a-b)+c(a-b)$$
$$=(a-b)(a+b+c)$$

11 $4a^2+8ac-12bc-9b^2$
$$=4a^2-9b^2+4c(2a-3b)$$
$$=(2a+3b)(2a-3b)+4c(2a-3b)$$
$$=(2a-3b)(2a+3b+4c)$$

12 $a^2x^2+acxz-b^2y^2-bcyz$

$$=(ax+by)(ax-by)+cz(ax-by)$$
$$=(ax-by)(ax+by+cz)$$

13 $x^4-y^3z+y^2x^2-y^2z^2$

$$=x^4-y^2z^2+y^2x^2-y^3z$$
$$=(x^2-yz)(x^2+yz)+y^2(x^2-yz)$$
$$=(x^2-yz)(x^2+yz+y^2)$$

14 $16x^2-15ab+12bx-25a^2$

$$=16x^2-25a^2-12bx-15ab$$
$$=(4x+5a)(4x-5a)+3b(4x-5a)$$
$$=(4x-5a)(4x+5a+3b)$$

15 $a^2(a+2b)+b^2(2a+b)$

$$=a^3+2a^2b+2ab^2+b^3$$
$$=a^3+b^3+2a^2b+2ab^2$$
$$=(a+b)(a^2-ab+b^2)+2ab(a+b)$$
$$=(a+b)(a^2+ab+b^2)$$

16 $m^3-2m^2n+2mn^2-n^3$

$$=m^3-n^3-2m^2n+2mn^2$$
$$=(m-n)(m^2+mn+n^2)-2mn(m-n)$$
$$=(m-n)(m^2-mn+n^2)$$

17 $a^4+2a^3b-2ab^3-b^4=a^4-b^4+2a^3b-2ab^3$

$$=(a^2-b^2)(a^2+b^2)+2ab(a^2-b^2)$$
$$=(a^2-b^2)(a^2+b^2+2ab)$$
$$=(a-b)(a+b)(a+b)^2=(a-b)(a+b)^3$$

18 $x^3(x-2y)+y^3(2x-y)=x^4-y^4-2x^3y+2xy^3$

$$=(x^2-y^2)(x^2+y^2)-2xy(x^2-y^2)$$
$$=(x^2-y^2)(x^2+y^2-2xy)$$
$$=(x-y)(x+y)(x-y)^2=(x-y)^3(x+y)$$

19 $a^3+5a^2+10a+8=a^3+8+5a^2+10a$

$$=(a+2)(a^2-2a+4)+5a(a+2)$$
$$=(a+2)(a^2+3a+4)$$

20 $x^3-17x^2+85x-125=x^3-125-17x^2+85x$

$$=(x-5)(x^2+5x+25)-17x(x-5)$$
$$=(x-5)(x^2-12x+25)$$

21 $8a^3+18a^2b-27ab^2-27b^3$

$$
\begin{aligned}
&=8a^3-27b^3+18a^2b-27ab^2\\
&=(2a-3b)(4a^2+6ab+9b^2)+9ab(2a-3b)\\
&=(2a-3b)(4a^2+15ab+9b^2)\\
&=(2a-3b)(4a^2+12ab+3ab+9b^2)\\
&=(2a-3b)\{4a(a+3b)+3b(a+3b)\}\\
&=(2a-3b)(a+3b)(4a+3b)
\end{aligned}
$$

22 $x^2-2xy+y^2-x+y=(x-y)^2-(x-y)$

$$=(x-y)(x-y-1)$$

23 $4a^2-4ab+b^2-6a+3b=(2a-b)^2-3(2a-b)$

$$=(2a-b)(2a-b-3)$$

24 $x^4-2ax^3+2a^2x^2-2a^3x+a^4$

$$
\begin{aligned}
&=x^4+2a^2x^2+a^4-2ax^3-2a^3x\\
&=(x^2+a^2)^2-2ax(x^2+a^2)\\
&=(x^2+a^2)(x^2+a^2-2ax)=(x^2+a^2)(x-a)^2
\end{aligned}
$$

25 $a^4-3a^3b+4a^2b^2-6ab^3+4b^4$

$$
\begin{aligned}
&=a^4+4a^2b^2+4b^4-3a^3b-6ab^3\\
&=(a^2+2b^2)^2-3ab(a^2+2b^2)\\
&=(a^2+2b^2)(a^2+2b^2-3ab)\\
&=(a^2+2b^2)(a-2b)(a-b)
\end{aligned}
$$

26 $a^2+3ab+2b^2+ac+2bc$

$$
\begin{aligned}
&=(a+b)(a+2b)+c(a+2b)\\
&=(a+2b)(a+b+c)
\end{aligned}
$$

27 $x^2-4xy+3y^2+xz-3yz$

$$
\begin{aligned}
&=(x-y)(x-3y)+z(x-3y)\\
&=(x-3y)(x-y+z)
\end{aligned}
$$

28 $m^2+2pm-5mn-4pn+6n^2$

$$
\begin{aligned}
&=m^2+2pm-3mn-2mn-4pn+6n^2\\
&=m(m+2p-3n)-2n(m+2p-3n)\\
&=(m+2p-3n)(m-2n)
\end{aligned}
$$

29 $a^2-10ab-15bc+21b^2+5ac$

$$
\begin{aligned}
&=a^2-7ab+5ac-3ab+21b^2-15bc\\
&=a(a-7b+5c)-3b(a-7b+5c)\\
&=(a-7b+5c)(a-3b)
\end{aligned}
$$

30 $2x^2+4a(4b-3a)+x(4b+5a)$

$$=2x^2-3ax+4bx+8ax+4a(4b-3a)$$
$$=x(2x-3a+4b)+4a(2x+4b-3a)$$
$$=(2x-3a+4b)(x+4a)$$

31 $a^2-3a(2b-1)+4b(2b-3)$

$$=a^2-2ab+3a-4ab+4b(2b-3)$$
$$=a(a-2b+3)-4b(a-2b+3)$$
$$=(a-2b+3)(a-4b)$$

32 $3x(x+2)-2y(4x-1)-3y^2$

$$=3x^2-9xy+6x+xy-3y^2+2y$$
$$=3x(x-3y+2)+y(x-3y+2)$$
$$=(x-3y+2)(3x+y)$$

33 $a^2-b^2-c^2-2bc+a-b-c$

$$=a^2-(b+c)^2+(a-b-c)$$
$$=(a+b+c)(a-b-c)+(a-b-c)$$
$$=(a-b-c)(a+b+c+1)$$

34 $x^2-4y^2-9z^2+12yz+4x-8y+12z$

$$=x^2-(2y-3z)^2+4(x-2y+3z)$$
$$=(x+2y-3z)(x-2y+3z)+4(x-2y+3z)$$
$$=(x-2y+3z)(x+2y-3z+4)$$

35 $9x^2-4z^2-24xy+16y^2+20y-15x+10z$

$$=9x^2-24xy+16y^2-4z^2-15x+20y+10z$$
$$=(3x-4y)^2-(2z)^2-5(3x-4y-2z)$$
$$=(3x-4y+2z)(3x-4y-2z)-5(3x-4y-2z)$$
$$=(3x-4y-2z)(3x-4y+2z-5)$$

36 $2a^2x^4-5a^4x^2+3a^6-2b^2x^4+5a^2b^2x^2-3a^4b^2$

$$=2a^2x^4-2b^2x^4-5a^4x^2+5a^2b^2x^2+3a^6-3a^4b^2$$
$$=2x^4(a^2-b^2)-5a^2x^2(a^2-b^2)+3a^4(a^2-b^2)$$
$$=(a^2-b^2)(2x^4-5a^2x^2+3a^4)$$
$$=(a^2-b^2)(x^2-a^2)(2x^2-3a^2)$$
$$=(a+b)(a-b)(x+a)(x-a)(2x^2-3a^2)$$

37 $2x^3+(2a-3b)x^2-(2b+3ab)x+3b^2$

$$=2x^3+2ax^2-2bx-3bx^2-3abx+3b^2$$

$$=2x(x^2+ax-b)-3b(x^2+ax-b)$$
$$=(x^2+ax-b)(2x-3b)$$

38 $(a^2+b^2)x^2-a^2b(2a+b)+a(2bx^2-a^3)$
$$=(a^2+b^2)x^2+2abx^2-a^4-a^2b(2a+b)$$
$$=x^2(a^2+b^2+2ab)-a^2(a^2+2ab+b^2)$$
$$=(x^2-a^2)(a^2+b^2+2ab)=(x+a)(x-a)(a+b)^2$$

39 $a^2(b-c)+b^2(c-a)+c^2(a-b)$
$$=a^2(b-c)-a(b^2-c^2)+b^2c-bc^2$$
$$=a^2(b-c)-a(b-c)(b+c)+bc(b-c)$$
$$=(b-c)\{a^2-a(b+c)+bc\}$$
$$=(b-c)\{a(a-b)-c(a-b)\}$$
$$=(b-c)(a-b)(a-c)$$

40 $a^2(b-c)+b^2(c+a)+c^2(a+b)+2abc$
$$=a^2(b+c)+a(b^2+c^2+2bc)+b^2c+bc^2$$
$$=a^2(b+c)+a(b+c)^2+bc(b+c)$$
$$=(b+c)\{a^2+a(b+c)+bc\}$$
$$=(b+c)\{a(a+b)+c(a+b)\}$$
$$=(b+c)(a+b)(a+c)$$

41 $a^3(b^2-c^2)+b^3(c^2-a^2)+c^3(a^2-b^2)$
$$=a^3(b^2-c^2)-a^2(b^3-c^3)+b^3c^2-b^2c^3$$
$$=a^3(b^2-c^2)-a^2(b-c)(b^2+bc+c^2)+b^2c^2(b-c)$$
$$=(b-c)\{a^3(b+c)-a^2(b^2+bc+c^2)+b^2c^2\}$$
$$=(b-c)(a^3b-a^2b^2+a^3c-a^2bc-a^2c^2+b^2c^2)$$
$$=(b-c)\{a^2b(a-b)+a^2c(a-b)-c^2(a^2-b^2)\}$$
$$=(b-c)(a-b)(a^2b+a^2c-ac^2-bc^2)$$
$$=(b-c)(a-b)\{b(a^2-c^2)+ac(a-c)\}$$
$$=(b-c)(a-b)(a-c)(ab+bc+ac)$$

42 $a^4(b-c)+b^4(c-a)+c^4(a-b)$
$$=a^4(b-c)-a(b^4-c^4)+bc(b^3-c^3)$$
$$=(b-c)\{a^4-a(b^3+b^2c+bc^2+c^3)+bc(b^2+bc+c^2)\}$$
$$=(b-c)\{-b^3(a-c)-b^2c(a-c)-bc^2(a-c)+a(a^3-c^3)\}$$
$$=(b-c)(a-c)\{-b^3-b^2c-bc^2+a(a^2+ac+c^2)\}$$
$$=(b-c)(a-c)\{c^2(a-b)+c(a^2-b^2)+(a^3-b^3)\}$$

$$=(b-c)(a-c)(a-b)(c^2+ac+bc+a^2+ab+b^2)$$
$$=(b-c)(a-c)(a-b)(a^2+b^2+c^2+ab+ac+bc)$$

43 $a^4(b^2-c^2)+b^4(c^2-a^2)+c^4(a^2-b^2)$

$$=a^4(b^2-c^2)-a^2(b^4-c^4)+b^2c^2(b^2-c^2)$$
$$=(b^2-c^2)\{a^4-a^2(b^2+c^2)+b^2c^2\}$$
$$=(b^2-c^2)(a^2-b^2)(a^2-c^2)$$
$$=(a-b)(a-c)(b-c)(a+b)(b+c)(c+a)$$

44 $a^5(b-c)+b^5(c-a)+c^5(a-b)$

$$=a^5(b-c)-a(b^5-c^5)+bc(b^4-c^4)$$
$$=(b-c)\{a^5-a(b^4+b^3c+b^2c^2+bc^3+c^4)$$
$$+bc(b^3+b^2c+bc^2+c^3)\}$$
$$=(b-c)\{-b^4(a-c)-b^3c(a-c)-b^2c^2(a-c)$$
$$-bc^3(a-c)+a(a^4-c^4)\}$$
$$=(b-c)(a-c)\{-b^4-b^3c-b^2c^2-bc^3+$$
$$a(a^3+a^2c+ac^2+c^3)\}$$
$$=(b-c)(a-c)\{c^3(a-b)+c^2(a^2+b^2)+c(a^3-b^3)$$
$$+(a^4-b^4)\}$$
$$=(b-c)(a-c)(a-b)\{c^3+c^2(a+b)+c(a^2+ab+b^2)$$
$$+(a^3+a^2b+ab^2+b^3)\}$$
$$=(a-b)(a-c)(b-c)(a^3+b^3+c^3+a^2b+a^2c$$
$$+b^2a+b^2c+c^2a+c^2b+abc)$$

Exercise 47

1 $a^3+b^3-c^3+3abc=\{(a+b)^3-c^3\}-3ab(a+b)+3abc$

$$=(a+b-c)\{(a+b)^2+(a+b)c+c^2\}-3ab$$
$$(a+b-c)$$
$$=(a+b-c)(a^2+b^2+c^2-ab+ac+bc),$$

the required quotient $=a^2+b^2+c^2-ab+ac+bc$

2 $x^3-y^3-1-3xy=(x-y)^3-1+3xy(x-y)-3xy$

$$=\{(x-y)-1\}\{(x-y)^2+(x-y)+1\}+3xy$$
$$(x-y-1)$$
$$=(x-y-1)(x^2+y^2-2xy+x-y+1+3xy)$$
$$=(x-y-1)(x^2+y^2+1+xy+x-y),$$

the required quotient $=x^2+y^2+1+xy+x-y$

3 $x^3-8y^3+27z^3+18xyz$

$$=x^3-\{(2y-3z)^3+3\cdot 2\cdot 3yz(2y-3z)\}+18xyz$$
$$=\{x-(2y-3z)\}\{x^2+x(2y-3z)+(2y-3z)^2\}$$
$$+18yz(x-2y+3z)$$
$$=(x-2y+3z)\{x^2+2xy-3xz+4y^2+9z^2-12yz+18yz\}$$
$$=(x-2y+3z)(x^2+4y^2+9z^2+2xy-3xz+6yz)\ ,$$

the required quotient $=x^2+4y^2+9z^2+2xy-3xz+6yz$

4 $8a^3-27b^3-c^3-18abc$

$$=(2a)^3+(-3b)^3+(-c)^3-3\cdot 2a\cdot -3b\cdot -c$$
$$=(2a-3b-c)(4a^2+9b^2+c^2+6ab-3bc+2ac)\ ,$$

the required quotient $=2a-3b-c$

5 $x^6-26x^3-27=x^6-27x^3+x^3-27$

$$=x^3(x^3-27)+(x^3-27)$$
$$=(x^3-27)(x^3+1)$$
$$=(x-3)(x^2+3x+9)(x+1)(x^2-x+1)\ ,$$

the required quotient $=(x-3)(x+1)=x^2-2x-3$

6 $(a^2-bc)^3+27b^3c^3$

$$=\{(a^2-bc)+3bc\}\{(a^2-bc)^2-3bc(a^2-bc)+9b^2c^2\}$$
$$=(a^2+2bc)(a^4+b^2c^2-2a^2bc-3a^2bc+3b^2c^2+9b^2c^2)$$
$$=(a^2+2bc)(a^4-5a^2bc+13b^2c^2)\ ,$$

the required quotient $=a^2+2bc$

7 $125-8a^3+b^3+30ab$

$$=(5-2a)^3+b^3+30a(5-2a)+30ab$$
$$=(5-2a+b)\{(5-2a)^2-b(5-2a)+b^2\}+30a(5-2a+b)$$
$$=(5-2a+b)(25+4a^2-20a-5b+2ab+b^2)+30a$$
$$=(5-2a+b)\{(a^2+2ab+b^2)+(a^2+10a+25)+2a^2-5b\}$$
$$=(5-2a+b)\{(a+b)^2+(a+5)^2+2a^2-5b\}\ ,$$

the required quotient $=5-2a+b$

8 $2x^3-x^2+x+4=2x^3+2x^2-3x^2-3x+4x+4$

$$=2x^2(x+1)-3x(x+1)+4(x+1)$$
$$=(x+1)(2x^2-3x+4)\ ,$$

the required quotient $=2x^2-3x+4$

9 $x^4-4x^3+7x^2-11x+10=x^4-4x^3+4x^2+3x^2-6x-5x+10$

$$=x^2(x-2)^2+3x(x-2)-5(x-2)$$
$$=(x-2)\{x^2(x-2)+3x-5\}\ ,$$

the required quotient $=x^2(x-2)+3x-5$

$=x^3-2x^2+3x-5$

10 $x^4-3x^3-13x^2+13x-6$

$=x^4+3x^3-6x^3-18x^2+5x^2+15x-2x-6$

$=x^3(x+3)-6x^2(x+3)+5x(x+3)-2(x+3)$

$=(x+3)(x^3-6x^2+5x-2)$,

the required quotient $=x^3-6x^2+5x-2$

11 $x^3+4x-39=x^3-3x^2+3x^2-9x+13x-39$

$=x^2(x-3)+3x(x-3)+13(x-3)$

$=(x-3)(x^2+3x+13)$,

the required quotient $=x^2+3x+13$

12 $a^3-4a^2b+24b^3=a^3+2a^2b-6a^2b+24b^3$

$=a^2(a+2b)-6b(a^2-4b^2)$

$=a^2(a+2b)-6b(a+2b)(a-2b)$

$=(a+2b)(a^2-6ab+12b^2)$,

the required quotient $=a^2-6ab+12b^2$

13 $a^4-5a^3b+8a^2b^2-7ab^3+3b^4$

$=a^4-3a^3b-2a^3b+6a^2b^2+2a^2b^2-6ab^3-ab^3+3b^4$

$=a^3(a-3b)-2a^2b(a-3b)+2ab^2(a-3b)-b^3(a-3b)$

$=(a-3b)(a^3-2a^2b+2ab^2-b^3)$,

the required quotient is $a^3-2a^2b+2ab^2-b^3$

14 $x^4+x^3-10x^2+7x-4$

$=x^4+4x^3-3x^3-12x^2+2x^2+8x-x-4$

$=x^3(x+4)-3x^2(x+4)+2x(x+4)-(x+4)$

$=(x+4)(x^3-3x^2+2x-1)$,

the required quotient is x^3-3x^2+2x-1

15 $x^3-3xy^2-76y^3$

$=x^3-16xy^2+19xy^2-76y^3$

$=x(x^2-16y^2)+19y^2(x-4y)$

$=(x-4y)(x^2+4xy+19y^2)$,

the required quotient is $x^2+4xy+19y^2$

16 $x^3+8x^2+19x+12$

$=x^3+8x^2+7x+12x+12$

$$=x(x+1)(x+7)+12(x+1)$$
$$=(x+1)(x^2+7x+12)=(x+1)(x+3)(x+4)$$

17. $x^3+9x^2+26x+24$
$$=x^3+9x^2+14x+12x+24$$
$$=x(x^2+9x+14)+12(x+2)$$
$$=x(x+2)(x+7)+12(x+2)$$
$$=(x+2)(x^2+7x+12)=(x+2)(x+3)(x+4)$$

18 $x^3-6x^2+11x-6=x^3-6x^2+9x+2x-6$
$$=x(x^2-6x+9)+2(x-3)$$
$$=x(x-3)^2+2(x-3)$$
$$=(x-3)(x^2-3x+2)$$
$$=(x-3)(x-1)(x-2)$$
$$=(x-1)(x-2)(x-3)$$

19 $x^3+5x^2-2x-24=x^3+5x^2-14x+12x-24$
$$=x(x^2+5x-14)+12(x-2)$$
$$=x(x+7)(x-2)+12(x-2)$$
$$=(x-2)(x^2+7x+12)=(x-2)(x+3)(x+4)$$

20 $x^3-4x^2+x+2=x^3-4x^2+3x-2x+2$
$$=x(x^2-4x+3)-2(x-1)$$
$$=x(x-3)(x-1)-2(x-1)$$
$$=(x-1)\{x(x-3)-2\}=(x-1)(x^2-3x-2)$$

21 $x^3+5x^2+4x-6x-6$
$$=x(x^2+5x+4)-6(x+1)$$
$$=x(x+4)(x+1)-6(x+1)=(x+1)(x^2+4x-6)$$

22 $x^3-6x^2+13x-10$
$$=x^3-6x^2+8x+5x-10$$
$$=x(x^2-6x+8)+5(x-2)=x(x-4)(x-2)+5(x-2)$$
$$=(x-2)(x^2-4x+5)$$

23 $x^4-3x^3-9x^2+12x+20$
$$=x^4-3x^3-5x^2-4x^2+12x+20$$
$$=x^2(x^2-3x-5)-4(x^2-3x-5)$$
$$=(x^2-3x-5)(x+2)(x-2)$$

24 $x^4-3x^3-x^2+13x-10$
$$=x^4-x^3-2x^3+2x^2-3x^2+3x+10x-10$$
$$=x^3(x-1)-2x^2(x-1)-3x(x-1)+10(x-1)$$

$$=(x-1)(x^3-2x^2-3x+10)$$
$$=(x-1)(x^3+2x^2-4x^2-8x+5x+10)$$
$$=(x-1)\{x^2(x+2)-4x(x+2)+5(x+2)\}$$
$$=(x-1)(x+2)(x^2-4x+5)$$

25 $x^4-5x^3+x^2+13x+6$
$$=x^4-5x^3-6x^2+7x^2+13x+6$$
$$=x^2(x^2-5x-6)+(7x^2+7x+6x+6)$$
$$=x^2(x+1)(x-6)+\{7x(x+1)+6(x+1)\}$$
$$=x^2(x+1)(x-6)+(x+1)(7x+6)$$
$$=(x+1)(x^3-6x^2+7x+6)$$
$$=(x+1)(x^3-6x^2+9x-2x+6)$$
$$=(x+1)\{x(x-3)^2-2(x-3)\}=(x+1)(x-3)(x^2-3x-2)$$

26 $x^4+5x^3-8x^2-30x+36$
$$=x^4+3x^3+2x^3+6x^2-14x^2-42x+12x+36$$
$$=x^3(x+3)+2x^2(x+3)-14x(x+3)+12(x+3)$$
$$=(x+3)(x^3+2x^2-14x+12)$$
$$=(x+3)(x^3-2x^2+4x^2-8x-6x+12)$$
$$=(x+3)\{x^2(x-2)+4x(x-2)-6(x-2)\}$$
$$=(x+3)(x-2)(x^2+4x-6)$$

27 $x^4-7x^3+9x^2+26x-56$
$$=x^4-7x^3+12x^2-3x^2+26x-56$$
$$=x^2(x^2-7x+12)-(3x^2-12x-14x+56)$$
$$=x^2(x-3)(x-4)-\{3x(x-4)-14(x-4)\}$$
$$=x^2(x-3)(x-4)-(x-4)(3x-14)$$
$$=(x-4)(x^3-3x^2-3x+14)$$
$$=(x-4)(x^3+2x^2-5x^2-10x+7x+14)$$
$$=(x-4)\{x^2(x+2)-5x(x+2)+7(x+2)\}$$
$$=(x-4)(x+2)(x^2-5x+7)$$

28 $x^3-7x^2+13x-15=x^3-7x^2+10x+3x-15$
$$=x(x^2-7x+10)+3(x-5)$$
$$=x(x-5)(x-2)+3(x-5)$$
$$=(x-5)(x^2-2x+3)$$

29 $x^3-5x+12=x^3+27-5x-15$
$$=(x+3)(x^2-3x+9)-5(x+3)$$
$$=(x+3)(x^2-3x+9-5)=(x+3)(x^2-3x+4)$$

30 $$x^3-6x^2+32=x^3+8-6(x^2-4)$$
$$=(x+2)(x^2-2x+4)-6(x+2)(x-2)$$
$$=(x+2)(x^2-2x+4-6x+12)$$
$$=(x+2)(x^2-8x+16)=(x+2)(x-4)^2$$

31 $$2x^3-3x^2-4=2x^3-16-3x^2+12$$
$$=2(x^3-8)-3(x^2-4)$$
$$=2(x-2)(x^2+2x+4)-3(x-2)(x+2)$$
$$=(x-2)(2x^2+4x+8-3x-6)$$
$$=(x-2)(2x^2+x+2)$$

32 $$x^3-9xy^2-10y^3=x^3-4xy^2-5xy^2-10y^3$$
$$=x(x^2-4y^2)-5y^2(x+2y)$$
$$=x(x-2y)(x+2y)-5y^2(x+2y)$$
$$=(x+2y)(x^2-2xy-5y^2)$$

33 $$a^3+4a^2b-9b^3=a^3+3a^2b+a^2b-9b^3$$
$$=a^2(a+3b)+b(a^2-9b^2)$$
$$=a^2(a+3b)+b(a+3b)(a-3b)$$
$$=(a+3b)(a^2+ab-3b^2)$$

34 $$5a^3-3a^2b-28b^3=5a^3-10a^2b+7a^2b-28b^3$$
$$=5a^2(a-2b)+7b(a^2-4b^2)$$
$$=5a^2(a-2b)+7b(a-2b)(a+2b)$$
$$=(a-2b)(5a^2+7ab+14b^2)$$

35 $$8x^3+4x-3=8x^3-1+4x-2$$
$$=(2x-1)(4x^2+2x+1)+2(2x-1)$$
$$=(2x-1)(4x^2+2x+3)$$

36 $$2x^3+5x^2-4x-3$$
$$=2x^3-2x^2+7x^2-7x+3x-3$$
$$=2x^2(x-1)+7x(x-1)+3(x-1)$$
$$=(x-1)(2x^2+7x+3)$$
$$=(x-1)(2x^2+6x+x+3)$$
$$=(x-1)\{2x(x+3)+(x+3)\}=(x-1)(x+3)(2x+1)$$

37 $$x^3-3x-2=x^3-8-3x+6$$
$$=(x-2)(x^2+2x+4)-3(x-2)$$
$$=(x-2)(x^2+2x+4-3)$$
$$=(x-2)(x^2+2x+1)=(x-2)(x+1)^2$$

38 $2a^3 - a^2b - b^3 = a^3 - b^3 + a^3 - a^2b$

$= (a-b)(a^2+ab+b^2) + a^2(a-b)$

$= (a-b)(a^2+ab+b^2+a^2) = (a-b)(2a^2+ab+b^2)$

39 $3x^3+8x^2-8x-3 = 3x^3-3+8x^2-8x$

$=3(x^3-1)+8x(x-1)$

$=3(x-1)(x^2+x+1)+8x(x-1)$

$=(x-1)(3x^2+3x+3+8x)$

$=(x-1)(3x^2+11x+3)$

40 $x^3-6xy^2+9y^3 = x^3-9xy^2+3xy^2+9y^3$

$=x(x^2-9y^2)+3y^2(x+3y)$

$=x(x-3y)(x+3y)+3y^2(x+3y)$

$=(x+3y)(x^2-3xy+3y^2)$

41 $x^2+bx-(a^2-3ab+2b^2) = x^2+bx-(a-b)(a-2b)$

$=x^2+\{(a-b)-(a-2b)\}x-(a-b)(a-2b)$

$=\{x+(a-b)\}\{x-(a-2b)\}$

$=(x+a-b)(x-a+2b)$

42 $x^4+4abx^2y^2-(a^2-b^2)^2y^4$

$=x^4+4abx^2y^2+4a^2b^2y^4-y^4\{(a^2-b^2)^2+4a^2b^2\}$

$=(x^2+2aby^2)^2-y^4(a^2+b^2)^2$

$=\{x^2+2aby^2+y^2(a^2+b^2)\}\{x^2+2aby^2-y^2(a^2+b^2)\}$

$=\{x^2+y^2(a+b)^2\}\{x^2-y^2(a-b)^2\}$

$=\{x^2+y^2(a+b)^2\}\{x+y(a-b)\}\{x-y(a-b)\}$

43 $a^4+2(x^2+y^2)a^2b^2+(x^2-y^2)^2b^4$

$=a^4+2(x^2+y^2)a^2b^2+(x^2+y^2)^2b^4-(x^2+y^2)^2b^4$
$+(x^2-y^2)^2b^4$

$=\{a^2+(x^2+y^2)b^2\}^2-b^4\{(x^2+y^2)^2-(x^2-y^2)^2\}$

$=\{a^2+(x^2+y^2)b^2\}^2-b^4(4x^2y^2)$

$=\{a^2+(x^2+y^2)b^2+2b^2xy\}\{a^2+(x^2+y^2)b^2-2b^2xy\}$

$=\{a^2+b^2(x^2+y^2+2xy)\}\{a^2+b^2(x^2+y^2-2xy)\}$

$=\{a^2+b^2(x+y)^2\}\{a^2+b^2(x-y)^2\}$

44 $a^2+(x+y)a-2x^2+5xy-2y^2$

$=a^2+(x+y)a-(2x^2-4xy-xy+2y^2)$

$=a^2+(x+y)a-\{2x(x-2y)-y(x-2y)\}$

$$=a^2+\{(2x-y)-(x-2y)\}a-(2x-y)(x-2y)$$
$$=\{a+(2x-y)\}\{a-(x-2y)\}$$
$$=(a+2x-y)(a-x+2y)$$

45 $x(x+a)-2a^2+3b(a+x)+2b^2$
$$=x^2+x(a+3b)-2a^2+3ab+2b^2$$
$$=x^2+x(a+3b)-2a^2+4ab-ab+2b^2$$
$$=x^2+x(a+3b)-2a(a-2b)-b(a-2b)$$
$$=x^2+x(a+3b)-(a-2b)(2a+b)$$
$$=x^2+x\{(2a+b)-(a-2b)\}-(2a+b)(a-2b)$$
$$=\{x+(2a+b)\}\{x-(a-2b)\}$$
$$=(x+2a+b)(x-a+2b)$$

46 $x^2+4xy+3y^2+2yz-z^2$
$$=x^2+4xy+3y^2+3yz-yz-z^2$$
$$=x^2+4xy+3y(y+z)-z(y+z)$$
$$=x^2+x\{(y+z)+(3y-z)\}+(y+z)(3y-z)$$
$$=(x+y+z)(x+3y-z)$$

47 $4a^2-4ab-3b^2+12bc-9c^2$
$$=4a^2-4ab-3(b^2-4bc+3c^2)$$
$$=4a^2-4ab-3\{b(b-3c)-c(b-3c)\}$$
$$=4a^2-4ab-3(b-3c)(b-c)$$
$$=(2a)^2+2a\{(b-3c)-3(b-c)\}-3(b-c)(b-3c)$$
$$=\{2a+(b-3c)\}\{2a-3(b-c)\}$$
$$=(2a+b-3c)(2a-3b+3c)$$

48 $x^4+6x^3+8x^2+6x-9$
$$=x^4+6x^3+8x^2+12x-6x-9$$
$$=x^4+6x^3+4x(2x+3)-3(2x+3)$$
$$=(x^2)^2+(x^2)\{(2x+3)+(4x-3)\}+(2x+3)(4x-3)$$
$$=(x^2+2x+3)(x^2+4x-3)$$

49 $a^4-4a^3b-5a^2b^2+6ab^3-b^4$
$$=a^4-4a^3b-b^2(5a^2-6ab+b^2)$$
$$=a^4-4a^3b-b^2(5a^2-5ab-ab+b^2)$$
$$=a^4-4a^3b-b^2\{5a(a-b)-b(a-b)\}$$
$$=a^4-a^2\ 4ab-b^2(a-b)(5a-b)$$
$$=(a^2)^2+a^2\{(ab-b^2)-(5ab-b^2)\}-(ab-b^2)(5ab-b^2)$$

$$=\{a^2+(ab-b^2)\}\{a^2-(5ab-b^2)\}$$
$$=(a^2+ab-b^2)(a^2-5ab+b^2)$$

50 $4x^4-20x^3+24x^2+6x-9$
$$=4x^4-20x^3+3(8x^2+2x-3)$$
$$=4x^4-20x^3+3(8x^2+6x-4x-3)$$
$$=4x^4-20x^3+3\{2x(4x+3)-(4x+3)\}$$
$$=4x^4-20x^3+3(4x+3)(2x-1)$$
$$=(2x^2)^2+2x^2\{(3-6x)-(3+4x)\}-(3-6x)(3+4x)$$
$$=(2x^2+3-6x)(2x^2-3-4x)$$
$$=(2x^2-6x+3)(2x^2-4x-3)$$

51 $x^4-2x^3+2x^2-2x+1$
$$=x^4+2x^2+1-2x(x^2+1)$$
$$=(x^2+1)^2-2x(x^2+1)$$
$$=(x^2+1)(x^2+1-2x)=(x^2+1)(x-1)^2$$

52 $a^4-9a^2+30a-25=a^4-(9a^2-30a+25)$
$$=a^4-(3a-5)^2=(a^2+3a-5)(a^2-3a+5)$$

53 $a^2-2abx-(ac-b^2)x^2+bcx^3$
$$=a^2-2abx+b^2x^2-acx^2+bcx^3$$
$$=(a-bx)^2-cx^2(a-bx)=(a-bx)(a-bx-cx^2)$$

54 $x^4y^4+x^2y^2-z^2+2xyz+1$
$$=x^4y^4+2x^2y^2+1-x^2y^2-z^2+2xyz$$
$$=(x^2y^2+1)^2-(xy-z)^2$$
$$=(x^2y^2+1+xy-z)(x^2y^2+1-xy+z)$$
$$=(x^2y^2+xy-z+1)(x^2y^2-xy+z+1)$$

55 $x^2(y^2-z^2)+4xyz-y^2+z^2$
$$=x^2y^2+2xyz+z^2-(x^2z^2-2xyz+y^2)$$
$$=(xy+z)^2-(xz-y)^2$$
$$=(xy+z+xz-y)(xy+z-xz+y)$$
$$=\{x(y+z)-y+z\}\{x(y-z)+y+z\}$$

56 $(a^2-b^2)(x^2+y^2)+2(a^2+b^2)xy$
$$=a^2(x^2+y^2+2xy)-b^2(x^2+y^2-2xy)$$
$$=a^2(x+y)^2-b^2(x-y)^2$$
$$=\{a(x+y)+b(x-y)\}\{a(x+y)-b(x-y)\}$$
$$=\{x(a+b)+y(a-b)\}\{x(a-b)+y(a+b)\}$$

57 $x^4-4x^3-x^2+10x+4$
$$=x^4-4x^3+4x^2-5x^2+10x+4$$
$$=(x^2-2x)^2-5(x^2-2x)+4$$
$$=(x^2-2x)^2-(-4-1)(x^2-2x)+(-4)(-1)$$
$$=(x^2-2x-4)(x^2-2x-1)$$

58 $a^4-6a^3+15a^2-18a+5$
$$=a^4-6a^3+9a^2+6a^2-18a+5$$
$$=(a^2-3a)^2+6(a^2-3a)+5$$
$$=(a^2-3a)^2+(5+1)(a^2-3a)+5\cdot 1$$
$$=(a^2-3a+5)(a^2-3a+1)$$

59 $4x^4+12x^3-5x^2-21x+12$
$$=4x^4+12x^3+9x^2-14x^2-21x+12$$
$$=(2x^2+3x)^2-7(2x^2+3x)+12$$
$$=(2x^2+3x)^2-(-3-4)(2x^2+3x)+(-3)(-4)$$
$$=(2x^2+3x-3)(2x^2+3x-4)$$

60 $x^4-5x^3y+6x^2y^2-5xy^3+y^4$
$$=x^4+x^2y^2+y^4-5xy(x^2-xy+y^2)$$
$$=(x^2+xy+y^2)(x^2-xy+y^2)-5xy(x^2-xy+y^2)$$
$$=(x^2-xy+y^2)(x^2+xy+y^2-5xy)$$
$$=(x^2-xy+y^2)(x^2-4xy+y^2)$$

61 $x^4-5x^3+14x^2-20x+16$
$$=x^4+8x^2+16-5x^3-20x+6x^2$$
$$=(x^2+4)^2-5x(x^2+4)+6x^2$$
$$=(x^2+4)^2+(x^2+4)(-3x-2x)+(-3x)(-2x)$$
$$=\{(x^2+4)-3x\}\{(x^2+4)-2x\}$$
$$=(x^2-3x+4)(x^2-2x+4)$$

62 $a^4-7a^3b+14a^2b^2-14ab^3+4b^4$
$$=a^4+4a^2b^2+4b^4-7a^3b-14ab^3+10a^2b^2$$
$$=(a^2+2b^2)^2-7ab(a^2+2b^2)+10a^2b^2$$
$$=(a^2+2b^2)^2+(a^2+2b^2)(-5ab-2ab)+(-5ab)(-2ab)$$
$$=\{(a^2+2b^2)-5ab\}\{(a^2+2b^2)-2ab\}$$
$$=(a^2-5ab+2b^2)(a^2-2ab+2b^2)$$

63 $x^4+4x^3-11x^2+20x+25$
$$=x^4+10x^2+25+4x^3+20x-21x^2$$

$$=(x^2+5)^2+4x(x^2+5)-21x^2$$
$$=(x^2+5)^2+(x^2+5)(7x-3x)+7x(-3x)$$
$$=\{(x^2+5)+7x\}\{(x^2+5)-3x\}$$
$$=(x^2+7x+5)(x^2-3x+5)$$

64 $a^4+4a^3b-10a^2b^2+4ab^3+b^4$
$$=a^4+2a^2b^2+b^4+4ab(a^2+b^2)-12a^2b^2$$
$$=(a^2+b^2)^2+(a^2+b^2)(6ab-2ab)+6ab(-2ab)$$
$$=\{(a^2+b^2)+6ab\}\{(a^2+b^2)-2ab\}$$
$$=(a^2+b^2+6ab)(a-b)^2$$

65 $x^4+8x^3+24x^2+32x-20$
$$=x^4+8x^3+16x^2+8x^2+32x-20$$
$$=(x^2+4x)^2+8(x^2+4x)-20$$
$$=(x^2+4x)^2+(x^2+4x)(10-2)+10\times(-2)$$
$$=(x^2+4x+10)(x^2+4x-2)$$

66 $(x+1)(x+3)(x-4)(x-6)+13$
$$=(x+1)(x-4)(x+3)(x-6)+13$$
$$=(x^2-3x-4)(x^2-3x-18)+13$$
$$=(x^2-3x)^2-22(x^2-3x)+72+13$$
$$=(x^2-3x)^2-22(x^2-3x)+85$$
$$=(x^2-3x)^2+(x^2-3x)(-17-5)+(-17)(-5)$$
$$=(x^2-3x-17)(x^2-3x-5)$$

67 $(x+2)(x+3)(x+4)(x+5)-360$
$$=(x+2)(x+5)(x+3)(x+4)-360$$
$$=(x^2+7x+10)(x^2+7x+12)-360$$
$$=(x^2+7x)^2+22(x^2+7x)+120-360$$
$$=(x^2+7x)^2+22(x^2+7x)-240$$
$$=(x^2+7x)^2+(x^2+7x)(30-8)+(30)(-8)$$
$$=(x^2+7x+30)(x^2+7x-8)$$
$$=(x^2+7x+30)(x+8)(x-1)$$

68 $x(2x+1)(x-2)(2x-3)-63$
$$=x(2x-3)(2x+1)(x-2)-63$$
$$=(2x^2-3x)(2x^2-3x-2)-63$$
$$=(2x^2-3x)^2-2(2x^2-3x)-63$$
$$=(2x^2-3x)^2+(2x^2-3x)(7-9)+7(-9)$$

$$=(2x^2-3x+7)(2x^2-3x-9)$$
$$=(2x^2-3x+7)(2x^2-6x+3x-9)$$
$$=(2x^2-3x+7)\{2x(x-3)+3(x-3)\}$$
$$=(2x^2-3x+7)(x-3)(2x+3).$$

69 $xy(x+y)+yz(y+z)+xz(z+x)+3xyz$
$$=xy(x+y)+xyz+yz(y+z)+xyz+xz(z+x)+xyz$$
$$=xy(x+y+z)+yz(y+z+x)+xz(z+x+y)$$
$$=(x+y+z)(xy+yz+xz)$$
$$=\{a(b-c)+b(c-a)+c(a-b)\}(xy+yz+zx)$$
$$=0\times(xy+yz+zx)=0$$

70 $(y-z)(y^2-z^2)-x\{(y-z)^2+x(y+z)+x^2\}$
$$=(y-z)^2(y+z)-x(y-z)^2-x^2(y+z)+x^3$$
$$=(y-z)^2(y+z-x)-x^2(y+z-x)$$
$$=(y+z-x)\{(y-z)^2-x^2\}$$
$$=(y+z-x)(y-z+x)(y-z-x)$$
$$=(y+z-x)(y-z-x)\{(a^2-b^2)+(b^2-c^2)-(a^2-c^2)\}$$
$$=(y+z-x)(y-z-x)\times 0=0$$

71 $x^2-2(y-2)x-3y^2+20y-32$
$$=x^2+xy-4x-3xy-3y^2+12y+8x+8y-32$$
$$=x(x+y-4)-3y(x+y-4)+8(x+y-4)$$
$$=(x+y-4)(x-3y+8)=0\times(x-3y+8)=0$$

72 $x^2-y^2+4x+14y-45$
$$=(x+y)(x-y)+9x+9y-5x+5y-45$$
$$=25\cdot 6+9(x+y)-5(x-y)-45$$
$$=25\cdot 6+9\cdot 25-5\cdot 6-45$$
$$=150+225-30-45=375-75=300$$

73 $x^6+y^6-z^6+3x^2y^2z^2$
$$=(x^2)^3+(y^2)^3+(-z^2)^3-3\cdot x^2\cdot y^2\cdot(-z^2)$$
$$=(x^2+y^2-z^2)(x^4+y^4+z^4-x^2y^2+x^2z^2+y^2z^2)$$
$$=\{(a^2-b^2)^2+4a^2b^2-(a^2+b^2)^2\}(x^4+y^4+z^4 \text{ etc.})$$
$$=\{(a^2-b^2)^2-(a^2+b^2)^2+4a^2b^2\}\times(\text{etc.})$$
$$=(-4a^2b^2+4a^2b^2)\times\text{etc.}=0$$

74 $8xy(x^2+y^2)=4xy(2x^2+2y^2)$
$$=\{(x+y)^2-(x-y)^2\}\{(x+y)^2+(x-y)^2\}$$
$$=(3-2)(3+2)=5$$

75 $x^5+y^5=(x+y)(x^4-x^3y+x^2y^2-xy^3+y^4)$
$$=a(x^4+2x^2y^2+y^4-x^3y-2x^2y^2-xy^3+x^2y^2)$$
$$=a\{(x^2+y^2)^2-xy(x^2+2xy+y^2)+x^2y^2\}$$
$$=a[\{(x+y)^2-2xy\}^2-xy(x+y)^2+b^4]$$
$$=a[(a^2-2b^4)^2-b^2a^2+b^4]$$
$$=a(a^4-4a^2b^2+4b^4-b^2a^2+b^4)=a(a^4-5a^2b^2+5b^4)$$

76 $x^3+y^3+z^3-3xyz$
$$=(x+y+z)(x^2+y^2+z^2-xy-yz-xz)$$
$$=(x+y+z)\tfrac{1}{2}(2x^2+2y^2+2z^2-2xy-2yz-2xz)$$
$$=\tfrac{1}{2}(x+y+z)\{(x^2+y^2-2xy)+(x^2+z^2-2xz)+(y^2+z^2-2yz)\}$$
$$=\tfrac{1}{2}(658+668+674)\{(y-x)^2+(z-x)^2+(z-y)^2\}$$
$$=\tfrac{1}{2}(2000)\{(10)^2+(6)^2+(16)^2=1000(100+36+256)$$
$$=1000\times392=392000$$

Exercise 48.

1 $a^2b^3=a^2\times b^2\times b$, $a^3b^2=a^2\times a\times b^2$,
the H C F is a^2b^2

2 $12a^3b=3\times2^2\times a^3\times b$, $20a^2c^3=5\times2^2\times a^2\times c^3$,
the H C F is $2^2\times a^2=4a^2$

3 $9xy^2z^3=3^2\times xy^2z^3$, $24x^3y^4=3\times2^3\times x^3y^4$,
the H C F is $3\times xy^2=3xy^2$

4 $20a^3x^4y^5=2^2\times5\times a^3x^4y^5$, $75a^2y^3=5^2\times3\times a^2y^3$,
the H C F is $5a^2y^3$

5 $18m^2n^4=3^2\times2\times m^2n^4$, $45m^6n^3=3^2\times5\times m^6n^3$,
the H C F is $3^2\times m^2n^3=9m^2n^3$

6 $16a^3x^4y=2^4\times a^3x^4y$, $40a^2y^3x=5\times2^3\times a^2y^3x$,
$28x^3a\ =7\times2^2\times x^3a$,
the H C F is $2^2\times ax=4ax$

7 $24m^2np^5 = 3 \times 2^3 \times m^2np^5$,
$60mn^2p = 5 \times 2^2 \times 3 \times mn^2p$,
$84m^2p^2 = 7 \times 2^2 \times 3 \times m^2p^2$,
the H C F is $2^2 \times 3 \times mp = 12mp$

8 $45x^3y^2z^4 = 3^2 \times 5 \times x^3y^2z^4$,
$75x^2y^4z^3 = 3 \times 5^2 \times x^2y^4z^3$,
$90x^4y^3z^2 = 2 \times 3^2 \times 5 \times x^4y^3z^2$,
the H C F $= 3 \times 5 \times x^2y^2z^2 = 15x^2y^2z^2$

9 $36a^2b^2c^4x^5 = 2^2 \times 3^2 \times a^2b^2c^4x^5$,
$54a^5c^2x^4 = 2 \times 3^3 \times a^5c^2x^4$,
$90a^4b^3c^5 = 2 \times 3^2 \times 5 \times a^4b^3c^5$,
the H C F is $2 \times 3^2a^2c^2 = 18a^2c^2$

10 $72a^3b^4c^5 = 2^3 \times 3^2 \times a^3b^4c^5$,
$96b^3c^4d^5 = 2^5 \times 3 \times b^3c^4d^5$,
$120c^3d^4a^5 = 2^3 \times 3 \times 5 \times a^5c^3d^4$,
the H C F is $2^3 \times 3 \times c^3 = 24c^3$

11 $48a^5x^4y^3z^2 = 2^4 \times 3 \times a^5x^4y^3z^2$,
$60x^5y^4z^3b^2 = 2^2 \times 3 \times 5 \times x^5y^4z^3b^2$,
$72y^5z^4b^3a^2 = 2^3 \times 3^2 \times a^2b^3y^5z^4$,
$84z^5b^4a^3x^2 = 2^2 \times 3 \times 7 \times a^3b^4x^2z^5$
the H C F is $2^2 \times 3 \times z^2 = 12z^2$

12 $75m^4n^3p^5q^6 = 3 \times 5^2 \times m^4n^3p^5q^6$,
$90m^3n^5p^6q^4 = 2 \times 3^2 \times 5 \times m^3n^5p^6q^4$,
$105m^6n^4p^3q^5 = 3 \times 5 \times 7 \times m^6n^4p^3q^5$,
$135m^5n^6p^4q^3 = 3^3 \times 5 \times m^5n^6p^4q^3$,
the H C F is $3 \times 5 \times m^3n^3p^3q^3 = 15m^3n^3p^3q^3$

13 $54a^2b^5c^3d^4 = 2 \times 3^3 \times a^2b^5c^3d^4$,
$72a^5b^2c^4d^3 = 2^3 \times 3^2 \times a^5b^2c^4d^3$,
$108a^3b^4c^5d^2 = 2^2 \times 3^3 \times a^3b^4c^5d^2$,
$126a^4b^3c^2d^5 = 2 \times 3^2 \times 7 \times a^4b^3c^2d^5$,
the H C F is $2 \times 3^2 \times a^2b^2c^2d^2 = 18a^2b^2c^2d^2$

14 $18a^3x^4y^5 = 2 \times 3^2 \times a^3x^4y^5$,
$42a^4y^3z^4 = 2 \times 3 \times 7 \times a^4y^3z^4$,
$60x^3y^4z^5 = 2^2 \times 3 \times 5 \times x^3y^4z^5$,
$78a^2x^4z^3 = 2 \times 3 \times 13 \times a^2x^4z^3$,

the H C F is $2 \times 3 = 6$

15 $32a^2b^3x^4y^5z^6 = 2^5 \times a^2b^3x^4y^5z^6$,
$40a^3x^5y^4z^8 = 2^3 \times 5 \times a^3x^5y^4z^8$,
$56b^3x^2y^7z^4 = 2^3 \times 7 \times b^3x^2y^7z^4$,
$72x^6a^5y^2z^3 = 2^3 \times 3^2 \times a^5x^6y^2z^3$,
$96b^4a^8x^2y^3 = 2^5 \times 3 \times a^8b^4x^2y^3$,

the H C F is $2^3 \times x^2y^2 = 8x^2y^2$

Exercise 49

1 $a^3 - ab^2 = a(a^2 - b^2) = a(a+b)(a-b)$, and
$a^4 + 2a^3b + a^2b^2 = a^2(a^2 + 2ab + b^2) = a^2(a+b)^2$,

the H C F is $a(a+b)$

2 $x^5y^3 - x^3y^5 = x^3y^3(x^2 - y^2) = x^3y^3(x+y)(x-y)$, and
$x^5y^4 + x^4y^5 = x^4y^4(x+y)$,

the H C F is $x^3y^3(x+y)$

3 $6(x^2 - 9) = 3 \times 2(x+3)(x-3)$, and
$15(x^3 + 27) = 3 \times 5(x+3)(x^2 - 3x + 9)$,

the H C F is $3(x+3)$

4 $12(a^6 - a^2b^2c^2) = 2^2 \times 3 \times a^2(a^4 - b^2c^2)$
$= 2^2 \times 3 \times a^2(a^2 + bc)(a^2 - bc)$,
and $20(a^4b^2c^2 + a^2b^3c^3) = 2^2 \times 5 \times a^2b^2c^2(a^2 + bc)$,
the H C F is $2^2 \times a^2 (a^2 + bc) = 4a^2(a^2 + bc)$

5 $m^6n^2 - 2m^5n^3 + m^4n^4 = m^4n^2(m^2 - 2mn + n^2) = m^4n^2(m-n)^2$,
and $(m^2n - mn^2)^3 = m^3n^3(m-n)^3$,
the H C F is $m^3n^2(m-n)^2$

6. $4a^4x - 9a^2x^3 = a^2x(4a^2 - 9x^2) = a^2x(2a + 3x)(2a - 3x)$,
and $4a^2x^2 + 6ax^3 = 2ax^2(2a + 3x)$,

the H C F is $ax(2a + 3x)$

7 $18a^4b^3-32a^2b^5=2a^2b^3(9a^2-16b^2)$
$=2a^2b^3(3a+4b)(3a-4b),$
and $18a^4b^3+24a^3b^4=2\times3\times a^3b^3(3a+4b)$,
the H C F is $2a^2b^3(3a+4b)$

8 $9x^4y^4-36x^2y^6=9x^2y^4(x^2-4y^2)$
$=3^2\times x^2y^4(x+2y)(x-2y),$
and $24x^4y^2-48x^3y^3=2^3\times3\times x^3y^2(x-2y)$,
the H C F is $3x^2y^2(x-2y)$

9 $6a^3b^2-24ab^4=6ab^2(a^2-4b^2)$
$=2\times3\times ab^2(a+2b)(a-2b),$
and $4a^5b+32a^2b^4=4a^2b(a^3+8b^3)$
$=2^2\times a^2b(a+2b)(a^2-2ab+4b^2)$,
the H C F is $2ab(a+2b)$

10 $48x^2a^2(x+a)^2(x^2a^2-xa^3)$
$=2^4\times3\times x^2a^2(x+a)^2\times xa^2(x-a)$
$=2^4\times3\times x^3a^4(x+a)^2(x-a)$, and
$64(x^5a^2-x^2a^5)(x^3a+x^2a^2)=2^6\times x^2a^2(x^3-a^3)\times x^2a(x+a)$
$=2^6x^4a^3(x-a)(x^2+ax+x^2)(x+a)$,
the H C F is $2^4\times x^3a^3(x+a)(x-a)=16x^3a^3(x^2-a^2)$

11 $24(x^3-a^3)=2^3\times3\times(x-a)(x^2+ax+x^2)$, and
$40(x^4+x^2a^2+a^4)=2^3\times5\times(x^2+ax+a^2)(x^2-ax+a^2)$,
the H C F is $2^3\times(x^2+ax+x^2)=8(x^2+ax+a^2)$

12 $56(x^6a^2-x^2a^6)=2^3\times7\times x^2a^2(x^4-a^4)$
$=2^3\times7\times x^2a^2(x^2+a^2)(x+a)(x-a),$
and $72(x^5a^3+3a^5x^3+2a^7x)$
$=2^3\times3^2\times a^3x(x^4+3a^2x^2+2a^4)$
$=2^3\times3^2\times a^3x(x^2+2a^2)(x^2+a^2)$,
the H C F is $2^3\times a^2x(x^2+a^2)=8a^2x(x^2+a^2)$

13 $30(a^2+4ab+3b^2)=2\times3\times5\times(a+b)(a+3b),$
and $42(a^2+ab-6b^2)=2\times3\times7\times(a-2b)(a+3b)$,
the H C F is $2\times3\,(a+3b)=6(a+3b)$

14
$$28(x^3-3x^2-10x)=2^2\times 7\times x(x^2-3x-10)$$
$$=2^2\times 7\times x(x+2)(x-5),$$
and $$52(x^4-8x^3+15x^2)=2^2\times 13\times x^2(x^2-8x+15)$$
$$=2^2\times 13\times x^2(x-3)(x-5)\ ,$$
the H C F is $2^2\times x(x-5)=4x(x-5)$

15
$$x^4y+3x^3y^2-18x^2y^3=x^2y(x^2+3xy-18y^2)$$
$$=x^2y(x-3y)(x+6y),$$
and $$x^3y^2+10x^2y^3+24xy^4=xy^2(x^2+10xy+24y^2)$$
$$=xy^2(x+4y)(x+6y)\ ,$$
the H C F is $xy(x+6y)$

16
$$a^4x^3-4a^3x^4-12a^2x^5=a^2x^3(a^2-4ax-12x^2)$$
$$=a^2x^3(a+2x)(a-6x),$$
and $$a^6x^2+8a^4x^3+12a^3x^4=a^3x^2(a^2+8ax+12x^2)$$
$$=a^3x^2(a^2+8ax+12x^2)$$
$$=a^3x^2(a+2x)(a+6x)\ ,$$
the H C F is $a^2x^2(a+2a)$

17
$$4x^3+12x^2+9x=x(4x^2+12x+9)=x(2x+3)^2,$$
and $$4x^2-2x-12=2(2x^2-x-6)=2(2x^2-4x+3x-6)$$
$$=2\{2x(x-2)+3(x-2)\}$$
$$=2(x-2)(2x+3)\ ,$$
the H C F is $2x+3$

18
$$a^2-ab-2b^2=a^2-2ab+ab-2b^2$$
$$=a(a-2b)+b(a-2b)$$
$$=(a-2b)(a+b),$$
and $$a^3-a^2b-4ab^2+4b^3=a^2(a-b)-4b^2(a-b)$$
$$=(a-b)(a^2-4b^2)$$
$$=(a-b)(a+2b)(a-2b)\ ,$$
the H C F is $(a-2b)$

19
$$x^2+3x-10=(x+5)(x-2),$$
and $$x^3-x^2-14x+24=x^3-2x^2+x^2-2x-12x+24$$
$$=x^2(x-2)+x(x-2)-12(x-2)$$
$$=(x-2)(x^2+x-12)=(x-2)(x+4)(x-3)\ ,$$
the H C F is $x-2$

20 $54(x^3+8a^3)=2\times 3^3\times(x+2a)(x^2-2ax+4a^2)$,

and $90(x^3+7ax^2+16a^2x+12a^3)$

$=2\times 3^2\times 5\times(x^3+2ax^2+5ax^2+10a^2x+6a^2x+12a^3)$

$=2\times 3^2\times 5\times\{x^2(x+2a)+5ax(x+2a)+6a^2(x+2a)\}$

$=2\times 3^2\times 5\times(x+2a)(x^2+5ax+6a^2)$

$=2\times 3^2\times 5\times(x+2a)(x+2a)(x+3a)$,

the H C F is $2\times 3^2(x+2a)=18(x+2a)$

21 $(a^3-b^3)(a+b)^2=(a-b)(a^2+ab+b^2)(a+b)^2$,

$a^4-b^4=(a^2+b^2)(a+b)(a-b)$,

and $3a^4+2a^3b-5a^2b^2=a^2(3a^2+2ab-5b^2)$

$=a^2(3a^2+5ab-3ab-5b^2)$

$=a^2\{a(3a+5b)-b(3a+5b)\}$

$=a^2(3a+5b)(a-b)$,

the H C F is $(a-b)$

22 $(2x-3)^2(x^2+x-2)=(2x-3)^2(x+2)(x-1)$,

$4x^2-x-18=4x^2+8x-9x-18$

$=4x(x+2)-9(x+2)$

$=(x+2)(4x-9)$,

and $2x^2-23x-54=2x^2+4x-27x-54$

$=2x(x+2)-27(x+2)$

$=(x+2)(2x-27)$,

the H C F is $(x+2)$

23 $8(27a^5b+a^2b^4)=2^3a^2b(27a^3+b^3)$

$=2^3a^2b(3a+b)(9a^2-3ab+b)$,

$12(6a^4b^2-7a^3b^3-3a^2b^4)=2^2\times 3\times a^2b^2(6a^2-7ab-3b^2)$

$=2^2\times 3\times a^2b^2(6a^2+2ab-9ab-3b^2)$

$=2^2\times 3\times a^2b^2\{2a(3a+b)-3b(3a+b)\}$

$=2^2\times 3\times a^2b^2(3a+b)(2a-3b)$,

and $40(3a^3b^2+13a^2b^3+4ab^4)$

$=2^3\times 5\times ab^2(3a^2+13ab+4b^2)$

$=2^3\times 5\times ab^2(3a^2+12ab+ab+4b^2)$

$=2^3\times 5\times ab^2\{3a(a+4b)+b(a+4b)\}$

$=2^3\times 5\times ab^2(a+4b)(3a+b)$,

the H C F is $2^2\times ab(3a+b)=4ab(3a+b)$

24 $x^4-13x^2+36=(x^2-4)(x^2-9)$

$$=(x+2)(x-2)(x+3)(x-3),$$

$$\begin{aligned}3x^3+13x^2+8x-12&=3x^3+9x^2+4x^2+12x-4x-12\\&=3x^2(x+3)+4x(x+3)-4(x+3)\\&=(x+3)(3x^2+4x-4)\\&=(x+3)(3x^2+6x-2x-4)\\&=(x+3)\{3x(x+2)-2(x+2)\}\\&=(x+3)(x+2)(3x-2),\end{aligned}$$

and $4x^3+17x^2+9x-18$

$$\begin{aligned}&=4x^3+8x^2+9x^2+18x-9x-18\\&=4x^2(x+2)+9x(x+2)-9(x+2)\\&=(x+2)(4x^2+9x-9)\\&=(x+2)(4x^2+12x-3x-9)\\&=(x+2)\{4x(x+3)-3(x+3)\}\\&=(x+2)(x+3)(4x-3),\end{aligned}$$

the H C F is $(x+2)(x+3)=x^2+5x+6$

Exercise 50

1

$$\begin{array}{r|l|l}
2x^2+5x-3 & 2x^3+3x^2-32x+15 & x-1 \\
 & \underline{2x^3+5x^2-3x} & \\
 & -2x^2-29x+15 & \\
 & \underline{-2x^2-5x+3} & \\
-12 & -24x+12 & \\
2x-1 & 2x^2+5x-3 & x+3 \\
 & \underline{2x^2-x} & \\
 & 6x-3 & \\
 & \underline{6x-3} & \\
\end{array}$$

the required H C F $=2x-1$

[*N B* The above process may, for convenience, be put in the manner shown below —

$$\begin{array}{r}
2x^3+3x^2-32x+15\\
\underline{-(2x^2+5x-3)x}\\
-2x^2-29x+15\\
\underline{-(2x^2+5x-3)\times(-1)}\\
-12\,|\,\underline{-24x+12}\\
2x-1
\end{array}$$

$$2x^2+5x-3$$
$$-(2x-1)x$$
$$6x-3$$
$$-(2x-1)3$$

the required H. C. F. $=2x-1$]

2. $$3x^2+16x-12)\,3x^3+4x^2-28x+16\,(x-4$$
$$3x^3+16x^2-12x$$
$$-12x^2-16x+16$$
$$-12x^2-64x+48$$
$$16\,|\,48x-32$$
$$3x-2)\,3x^2+16x-12\,(x+6$$
$$3x^2-2x$$
$$18x-12$$
$$18x-12$$

the required H. C. F. $=3x-2$

[*N. B.* The above process may, for convenience, be put in the manner shown below:—

$$3x^3+4x^2-28x+16$$
$$-(3x^2+16x-12)x$$
$$-12x^2-16x+16$$
$$-(3x^2+16x-12)\times(-4)$$
$$16\,|\,48x-32$$
$$3x-2$$

$$3x^2+16x-12$$
$$-(3x-2)x$$
$$18x-12$$
$$-(3x-2).6$$

the required H. C. F. $=3x-2$]

3. $$2x^3+3ax^2-45a^2x-100a^3$$
$$-(2x^2-3ax-20a^2)x$$
$$6ax^2-25a^2x-100a^3$$
$$-(2x^2-3ax-20a^2)3a$$
$$-8a^2\,|\,-16a^2x-40a^3$$
$$2x+5a$$

$$2x^2-3ax-20a^2$$
$$-(2x+5a)x$$
$$-8ax-20a^2$$
$$-(2x+5a)(-4a)$$

the required H. C. F. $=(2x+5a)$

4. 1st expression $= x(3x^3+7x^2-14x-24)$,
2nd expression $= 2x^2(3x^2-5x-12)$

Now to find x, the H.C.F. of

$$3x^3+7x^2-14x-24 \text{ and } 3x^2-5x-12 \text{ —}$$

$$\begin{array}{r}
3x^3+7x^2-14x-24 \\
-\underline{(3x^2-5x-12)x} \\
12x^2-2x-24 \\
-\underline{(3x^2-5x-12)4} \\
6\,|\,\underline{18x+24} \\
3x+4
\end{array}$$

$$\begin{array}{r}
3x^2-5x-12 \\
-\underline{(3x+4)x} \\
-9x-12 \\
-\underline{(3x+4)(-3)}
\end{array}$$

Thus $x = 3x+4$

$\therefore$ the required H.C.F. $= x(3x+4)$

5.

$$\begin{array}{r}
6a^3-11a^2-3a+2 \\
-\underline{(3a^3+20a^2+23a-10)2} \\
-51a^2-49a+22
\end{array}$$

$$\begin{array}{r}
3a^3+20a^2+23a-10 \\
\underline{17} \\
51a^3+340a^2+391a-170 \\
-\underline{(-51a^2-49a+22)(-a)} \\
291a^2+413a-170 \\
\underline{17} \\
4947a^2+7021a-2890 \\
-\underline{(-51a^2-49a+22)\times(-97)} \\
756\,|\,\underline{2268a-756} \\
3a-1
\end{array}$$

$$\begin{array}{r}
-51a^2-49a+22 \\
-\underline{(3a-1)\times(-17a)} \\
-66a+22 \\
-\underline{(3a-1)\times(-22)}
\end{array}$$

$\therefore$ the required H.C.F. $= 3a-1$

6
$$\begin{array}{l}
4a^3+12a^2b-7ab^2-30b^3 \\
\underline{3} \\
12a^3+36a^2b-21ab^2-90b^3 \\
\underline{-(6a^3-25a^2b+32ab^2-12b^3)\times 2} \\
b|\underline{86a^2b-85ab^2-66b^3} \\
\quad 86a^2-85ab-66b^2
\end{array}$$

$$\begin{array}{l}
6a^3-25a^2b+32ab^2-12b^3 \\
\underline{43} \\
258a^3-1075a^2b+1376ab^2-516b^3 \\
\underline{-(86a^2-85ab-66b^2)\,3a} \\
-2b|\underline{-820a^2b+1574ab^2-516b^3} \\
\quad 410a^2-787ab+258b^2 \\
\quad \underline{43} \\
\quad 17630a^2-33841ab+11094b^2 \\
\underline{-(86a^2-85ab-66b^2)\,205} \\
-6208b|\underline{-16416ab+24624b^2} \\
\qquad\qquad 2a\quad -3b
\end{array}$$

$$\begin{array}{l}
86a^2-85ab-66b^2 \\
\underline{-(2a-3b)\,43a} \\
\quad 44ab-66b^2 \\
\quad \underline{-(2a-3b)\,22b}
\end{array}$$

the required H C F $=2a-3b$

7
$$\begin{array}{l}
3x^3+5x^2+5x+2 \\
\underline{2} \\
6x^3+10x^2+10x+4 \\
\underline{-(2x^3+5x^2+5x+3)\,3} \\
-5|\underline{-5x^2-5x-5} \\
\qquad x^2+x+1
\end{array}$$

$$\begin{array}{l}
2x^3+5x^2+5x+3 \\
\underline{-(x^2+x+1),2x} \\
3x^2+3x+3 \\
\underline{-(x^2+x+1)\,3}
\end{array}$$

the required H C F $=x^2+x+1$

8
$$\begin{array}{l}
4x^3-7x^2y+7xy^2-3y^3 \\
\underline{3} \\
12x^3-21x^2y+21xy^2-9y^3 \\
\underline{-(3x^3-7x^2y+7xy^2-4y^3)\,4} \\
\quad 7y|\underline{7x^2y-7xy^2+7y^3} \\
\qquad\quad x^2-xy+y^2
\end{array}$$

$$\begin{array}{r}
3x^3-7x^2y+7xy^2-4y^3 \\
\underline{-(x^2-xy+y^2)\,3x\qquad\qquad} \\
-4x^2y+4xy^2-4y^3 \\
\underline{-(x^2-xy+y^2)\times(-4y)}
\end{array}$$

the required H C F $=x^2-xy+y^2$

9 1st expression $=x(6x^3+7x^2+5x+2)$,
2nd expression $=2x^2(2x^3-9x^2-4x-5)$

Now to find x, the H C F of

$6x^3+7x^2+5x+2$ and $2x^3-9x^2-4x-5$ —

$$\begin{array}{r}
6x^3+7x^2+5x+2 \\
\underline{-(2x^3-9x^2-4x-5)\,3} \\
17\,|\,\underline{34x^2+17x+17} \\
2x^2+x+1
\end{array}$$

$$\begin{array}{r}
2x^3-9x^2-4x-5 \\
\underline{-(2x^2+x+1)x\qquad\quad} \\
-10x^2-5x-5 \\
\underline{-(2x^2+x+1)(-5)}
\end{array}$$

Thus $x=2x^2+x+1$

the required H C F $=x(2x^2+x+1)$

10

$$\begin{array}{r}
3x^4+10x^3+7x^2+4x+1 \\
\underline{2\qquad\qquad\qquad\qquad\qquad} \\
6x^4+20x^3+14x^2+8x+2 \\
\underline{-(2x^3+3x^2-7x-3)3x\qquad} \\
11x^3+35x^2+17x+2 \\
\underline{2\qquad\qquad\qquad\qquad} \\
22x^3+70x^2+34x+4 \\
\underline{-(2x^3+\ 3x^2-\ 7x-3)\,11} \\
37\,|\,\underline{37x^2+111x+37} \\
x^2+3x+1
\end{array}$$

$$\begin{array}{r}
2x^3+3x^2-7x-3 \\
\underline{-(x^2+3x+1)\,2x\qquad} \\
-3x^2-9x-3 \\
\underline{-(x^2+3x+1)\times(-3)}
\end{array}$$

the required H C F $=x^2+3x+1$

11

$$\begin{array}{l}
3x^4+13x^3+9x^2+9x+2 \\
\underline{\quad 4 \quad} \\
12x^4+52x^3+36x^2+36x+8 \\
\underline{-(4x^3+13x^2-8x-3)\,3x} \\
\qquad 13x^3+60x^2+45x+8 \\
\qquad \underline{\quad 4 \quad} \\
\qquad 52x^3+240x^2+180x+32 \\
\qquad \underline{-(4x^3+13x^2-8x-3)\,13} \\
\qquad\quad 71\,|\,\underline{71x^2+284x+71} \\
\qquad\qquad\quad x^2+4x+1
\end{array}$$

$$\begin{array}{l}
4x^3+13x^2-8x-3 \\
\underline{-(x^2+4x+1)\,4x} \\
\quad -3x^2-12x-3 \\
\quad \underline{-(x^2+4x+1)\times(-3)}
\end{array}$$

the required H C F $=x^2+4x+1$

12

$$\begin{array}{l}
21a^3+17a^2x+\ 9ax^2+\ x^3 \\
\underline{\quad 4 \quad} \\
84a^3+68a^2x+36ax^2+4x^3 \\
\underline{-(12a^3+11a^2x+\ 6ax^2+\ x^3)\,7} \\
\quad -3x\,|\,\underline{-\ 9a^2x-\ 6ax^2-3x^3} \\
\qquad\quad 3a^2\ +\ 2ax\ +\ x^2
\end{array}$$

$$\begin{array}{l}
12a^3+11a^2x+6ax^2+x^3 \\
\underline{-(\ 3a^2+\ 2ax\ +\ \ x^2)\,4a} \\
\qquad 3a^2x+2ax^2+x^3 \\
\qquad \underline{-(3a^2+2ax\ +x^2)\,x}
\end{array}$$

the required H C F $=3a^2+2ax+x^2$.

13

$$\begin{array}{l}
65a^3+54a^2x+22ax^2+3x^3 \\
\underline{\quad 7 \quad} \\
455a^3+378a^2x+154ax^2+21x^3 \\
\underline{-(\ 35a^3+\ 31a^2x+\ 13ax^2+\ 2x^3)\,13} \\
\quad -5x\,|\,\underline{-\ 25a^2x-\ 15ax^2-\ 5x^3} \\
\qquad\quad 5a^2\ +\ \ 3ax\ +\ \ x^2
\end{array}$$

$$\begin{array}{l}
35a^3+31a^2x+13ax^2+2x^3 \\
\underline{-(\ 5a^2+\ 3ax\ +x^2)\,7a} \\
\qquad 10a^2x+\ 6ax^2+2x^3 \\
\qquad \underline{-(\ 5a^2\ +\ 3ax\ +\ x^2)\,2x}
\end{array}$$

the required H C F $=5a^2+3ax+x^2$

14

$$91x^3 - 25ax^2 + 20a^2x + 4a^3$$
$$10$$
$$910x^3 - 250ax^2 + 200a^2x + 40a^3$$
$$-(70x^3 - 9ax^2 + 11a^2x + 6a^3)\ 13$$
$$-19a \mid -133ax^2 + 57a^2x - 38a^3$$
$$7x^2 - 3ax + 2a^2$$

$$70x^3 - 9ax^2 + 11a^2x + 6a^3$$
$$-(7x^2 - 3ax + 2a^2)\ 10x$$
$$21ax^2 - 9a^2x + 6a^3$$

the required H C F $= 7x^2 - 3ax + 2a^2$

15

$$85x^3 - 36x^2 + 25x + 6$$
$$15$$
$$1275x^3 - 540x^2 + 375x + 90$$
$$-(75x^3 - 35x^2 + 24x + 4)\ 17$$
$$11 \mid 55x^2 - 33x + 22$$
$$5x^2 - 3x + 2$$

$$75x^3 - 35x^2 + 24x + 4$$
$$-(5x^2 - 3x + 2)\ 15x$$
$$10x^2 - 6x + 4$$
$$-(5x^2 - 3x + 2)\ 2$$

the required H C F $= 5x^2 - 3x + 2$

16

$$49x^3 - 49x^2 + 5x + 3$$
$$5$$
$$245x^3 - 245x^2 + 25x + 15$$
$$-(35x^3 - 34x^2 + 3x + 2)\ 7$$
$$-7x^2 + 4x + 1$$

$$35x^3 - 34x^2 + 3x + 2$$
$$-(-7x^2 + 4x + 1)(-5x)$$
$$-14x^2 + 8x + 2$$
$$-(-7x^2 + 4x + 1)\ 2$$

the required H C F $= -7x^2 + 4x + 1$

17 1st expression $= x^3(6x^3 + 21ax^2 + 30a^2x + 24a^3)$,

2nd expression $= 2x^2(2x^4 + ax^3 + 7a^2x^2 + 5a^3x + 12a^4)$

$$2x^4 + ax^3 + 7a^2x^2 + 5a^3x + 12a^4$$
$$3$$
$$6x^4 + 3ax^3 + 21a^2x^2 + 15a^3x + 36a^4$$
$$-(6x^3 + 21ax^2 + 30a^2x + 24a^3)\,x$$
$$-18ax^3 - 9a^2x^2 - 9a^3x + 36a^4$$
$$-(6x^3 + 21ax^2 + 30a^2x + 24a^3) \times (-3a)$$
$$27a^2 \mid 54a^2x^2 + 81a^3x + 108a^4$$
$$2x^2 + 3ax + 4a^2$$

$6x^3+21ax^2+30a^2x+24a^3$

$-(2x^2+3ax+4a^2)\,3x$

$12ax^2+18a^2x+24a^3$

$-(2x^2+3ax+4a^2)\,6a$

the required H C F $=x^2(2x^2+3ax+4a^2)$

18 1st expression $=2\times2\times(a^4+8a^3+18a^2+11a+2)$,

2nd expression $=2\times3(a^4+9a^3+23a^2+13a+2)$

$a^4+9a^3+23a^2+13a+2$

$-(a^4+8a^3+18a^2+11a+2)\,1$

$a|a^3+5a^2+2a$

a^2+5a+2

$a^4+8a^3+18a^2+11a+2$

$-(a^2+5a+2)\,a^2$

$3a^3+16a^2+11a+2$

$-(a^2+5a+2)\,3a$

a^2+5a+2

$-(a^2+5a+2)$

the reqd H C F $=2(a^2+5a+2)$

19 $6x^4+21x^3+3x-6$

$-(2x^4-19x^2+31x-6)\,3$

$3|21x^3+57x^2-60x+12$

$7x^3+19x^2-20x+4$

$2x^4-19x^2+21x-6$

7

$14x^4-133x^2+147x-42$

$-(7x^3+19x^2-20x+4)\,2x$

$-38x^3-93x^2+139x-42$

7

$-266x^3-651x^2+973x-294$

$-(7x^3+19x^2-20x+4)\times(-38)$

$71|71x^2+213x-142$

x^2+3x-2

$7x^3+19x^2-20x+4$

$-(x^2+3x-2)\,7x$

$-2x^2-6x+4$

$-(x^2+3x-2)\times(-2)$

∴ the reqd H C F $=x^2+3x-2$

20 1st expression $=6(2x^4-5x^2+21x+15)$,
2nd expression $=5(3x^4-5x^3+29x-15)$

$$
\begin{array}{l}
-15+29x \qquad -5x^3+3x^4 \\
\underline{-(15+21x \quad -5x^2 \quad +2x^4)\times(-1)} \\
5x\,|\,\underline{50x-5x^2-5x^3+5x^4} \\
\qquad 10-x-x^2+x^3 \\[1ex]
\qquad 15+21x-5x^2+2x^4 \\
\qquad \underline{2} \\
\qquad 30+42x-10x^2+4x^4 \\
\qquad \underline{-(10-x-x^2+x^3)\,3} \\
\qquad 45x-7x^2-3x^3+4x^4 \\
\qquad \underline{2} \\
\qquad 90x-14x^2-6x^3+8x^4 \\
\qquad \underline{-(10- \ x - x^2+ x^3)\,9x} \\
\qquad -x\,|\,\underline{-5x^2+3x^3-x^4} \\
\qquad \qquad 5-3x+x^2 \\[1ex]
\qquad 10-x-x^2+x^3 \\
\qquad \underline{-(5-3x+x^2)\,2} \\
\qquad 5x-3x^2+x^3 \\
\qquad \underline{-(5-3x+x^2)\,x}
\end{array}
$$

the required H C F $=5-3x+x^2$

21 1st expression $=9(2x^4+13x^3+18x^2+8x+1)$,
2nd expression $=12x^4+68x^3+72x^2+108x+20$

$$
\begin{array}{l}
12x^4+68x^3+72x^2+108x+20 \\
\underline{-(2x^4+13x^3+18x^2+8x+1)\times 6} \\
-2\,|\,\underline{-10x^3-36x^2+60x+14} \\
\qquad 5x^3+18x^2-30x-7 \\[1ex]
2x^4+13x^3+18x^2+8x+1 \\
\underline{5} \\
10x^4+65x^3+90x^2+40x+5 \\
\underline{-(5x^3+18x^2-30x-7)\,2x} \\
\qquad 29x^3+150x^2+54x+5 \\
\qquad \underline{10} \\
\qquad 290x^3+1500x^2+540x+50 \\
\qquad \underline{-(5x^3+18x^2-30x-7)\,58} \\
\qquad 456\,|\,\underline{456x^2+2280x+456} \\
\qquad \qquad x^2+5x+1
\end{array}
$$

$$\begin{array}{r}
5x^3+18x^2-30x-7 \\
-(x^2+5x+1)\,5x \\
\hline
-7x^2-35x-7 \\
-(x^2+5x+1)\times(-7) \\
\hline
\end{array}$$

the required H C F $=x^2+5x+1$

22

$$\begin{array}{r}
x^5 \qquad -5x^2+6x+12 \\
-(x^4-8x^2-24x-32)\,x \\
\hline
8x^3+19x^2+38x+12
\end{array}$$

$$\begin{array}{r}
x^4-8x^2-24x-32 \\
8 \\
\hline
8x^4-64x^2-192x-256 \\
-(8x^3+19x^2+38x+12)\,x \\
\hline
-19x^3-102x^2-204x-256 \\
8 \\
\hline
-152x^3-816x^2-1632x-2048 \\
-(8x^3+19x^2+38x+12)(-19) \\
\hline
-455 \,|\, -455x^2-910x-1820 \\
x^2+2x+4
\end{array}$$

$$\begin{array}{r}
8x^3+19x^2+38x+12 \\
-(x^2+2x+4)\,8x \\
\hline
3x^2+6x+12 \\
-(x^2+2x+4)\,3 \\
\hline
\end{array}$$

the reqd H C F $=x^2+2x+4$

23

$$\begin{array}{r}
x^5 \qquad -4x^3 \qquad +45x \quad +75 \\
-(x^4+5x^3+3x^2-14x-40)\,x \\
\hline
-5x^4-7x^3+14x^2+85x+75 \\
-(x^4+5x^3+3x^2-14x-40)\times(-5) \\
\hline
18x^3+29x^2+15x-125 \\
x^4+5x^3+3x^2-14x-40 \\
18 \\
\hline
18x^4+90x^3+54x^2-252x-720 \\
-(18x^3+29x^2+15x-125)\,x \\
\hline
61x^3+39x^2-127x-720 \\
18 \\
\hline
1098x^3+702x^2-2286x-12960 \\
-(18x^3+29x^2+15x-125)\,61 \\
\hline
-1067 \,|\, -1067x^2-3201x-5335 \\
x^2+3x+5
\end{array}$$

$$18x^3+29x^2+15x-125$$
$$-(x^2+\ 3x\ +5)\,18$$
$$-25x^2-75x-125$$
$$-(x^2+3x+5)\times(-25)$$

the required H C F $=x^2+3x+5$

24 1st expression $=2\times 2\times(x^5-2x^3a^2+7x^2a^3-6xa^4+6a^5)$,
2nd expression $=2\times 3\times(x^4+4x^3a-2x^2a^2-4xa^3+16a^4)$

$$x^5 \qquad -2x^3a^2+7x^2a^3-6xa^4+6a^5$$
$$-(x^4+4x^3a-2x^2a^2-4xa^3\ +16a^4)\,x$$
$$-4x^4a+11x^2a^3-22xa^4+6a^5$$
$$-(x^4+4x^3a-2x^2a^2-4xa^3+16a^4)\times(-4a)$$
$$a^2\,|\,16x^3a^2+3x^2a^3-38xa^4+70a^5$$
$$16x^3\ \ +3x^2a\ -38xa^2\ +70a^3$$

$$x^4+4x^3a-2x^2a^2-4xa^3+16a^4$$
$$16$$
$$16x^4+64x^3a-32x^2a^2-64xa^3+256a^4$$
$$-(16x^3+3x^2a-38xa^2+70a^3)\,x$$
$$61x^3a+6x^2a^2-134xa^3+256a^4$$
$$16$$
$$976x^3a+96a^2x^2-2144xa^3+4096a^4$$
$$-(16x^3+3x^2a-38xa^2+70a^3)\,61a$$
$$-87a^2\ \ -87x^2a^2+174xa^3-174a^4$$
$$x^2-2xa+2a^2$$

$$16x^3+3x^2a-38xa^2+70a^3$$
$$-(x^2-2xa+2a^2)\,16x$$
$$35x^2a-70xa^2+70a^3$$
$$-(x^2-2xa+2a^2)\,35a$$

the reqd H C F $=2(x^2-2xa+2a^2)$

25 $6x^5+4x^4y+5x^3y^2+4x^2y^3 \qquad +8y^5$

$$3$$
$$18x^5+12x^4y+15x^3y^2+12x^2y^3 \qquad +24y^5$$
$$-(9x^4+18x^3y-13x^2y^2-38xy^3-12y^4)\,2x$$
$$48x^4y+41x^3y^2+88x^2y^3+24xy^4+24y^5$$
$$-(9x^4+18x^3y-13x^2y^2-38xy^3-12y^4)\times(-2y)$$
$$xy\,|\,66x^4y+5x^3y^2+62x^2y^3-52xy^4$$
$$66x^3\ +5x^2y\ +62xy^2-52y^3$$

$9x^4 - 18x^3y - 13x^2y^2 - 38xy^3 - 12y^4$
13
$117x^4 - 234x^3y - 169x^2y^2 - 494xy^3 - 156y^4$
$-(66x^3 + 5x^2y + 62xy^2 - 52y^3)\,3y$
$117x^4 - 432x^3y - 184x^2y^2 - 680xy^3$
22
$2574x^4 - 9504x^3y - 4048x^2y^2 - 14960xy^3$
$-(66x^3 + 5x^2y + 62xy^2 - 52y^3)\,39x$
$-3233xy \mid -9699x^3y - 6466x^2y^2 - 12932xy^3$
$3x^2 + 2xy + 4y^2$
$66x^3 + 5x^2y + 62xy^2 - 52y^3$
$-(3x^2 + 2xy + 4y^2)\,22x$
$-39x^2y - 26xy^2 - 52y^3$
$-(3x^2 + 2xy + 4y^2) \times (-13y)$

the required H C F $= 3x^2 + 2xy + 4y^2$

26 $4x^5 + 11x^4 \qquad + 81$
$-(2x^5 \qquad - 11x^2 \qquad - 9)2$
$11 \mid 11x^4 + 22x^2 + 99$
$x^4 + 2x^2 + 9$
$2x^5 \qquad - 11x^2 - 9$
$-(x^4 + 2x^2 \qquad + 9)\,2x$
$-1 \mid -4x^3 - 11x^2 - 18x - 9$
$4x^3 + 11x^2 + 18x + 9$
$x^4 \qquad + 2x^2 + 9$
4
$4x^4 \qquad + 8x^2 \qquad + 36$
$-(4x^3 + 11x^2 + 18x + 9)\,x$
$-11x^3 - 10x^2 - 9x + 36$
4
$-44x^3 - 40x^2 - 36x + 144$
$-(4x^3 + 11x^2 + 18x + 9) \times (-11)$
$81 \mid 81x^2 + 162x + 243$
$x^2 + 2x + 3$
$4x^3 + 11x^2 + 18x + 9$
$-(x^2 + 2x + 3)\,4x$
$3x^2 + 6x + 9$
$-(x^2 + 2x + 3)\,3$

the required H C F $= x^2 + 2x + 3$

27 1st expression $=5a(12a^4+2a^3-9a^2+9a-10)$,
2nd expression $=2(16a^4+52a^3-10a^2-61a+15)$

$$16a^4+52a^3-10a^2-61a+15$$
$$\underline{3\qquad\qquad\qquad\qquad\qquad}$$
$$48a^4+156a^3-30a^2-183a+45$$
$$\underline{-(12a^4+2a^3-9a^2+9a-10)4}$$
$$148a^3+6a^2-219a+85$$

$$12a^4+2a^3-9a^2+9a-10$$
$$\underline{37\qquad\qquad\qquad\qquad\qquad}$$
$$444a^4+74a^3-333a^2+333a-370$$
$$\underline{-(148a^3+6a^2-219a+85)\,3a}$$
$$2\,|\,\underline{56a^3+324a^2+78a-370}$$
$$28a^3+162a^2+39a-185$$
$$\underline{37\qquad\qquad\qquad\qquad\qquad}$$
$$1036a^3+5994a^2+1443a-6845$$
$$\underline{-(148a^3+6a^2-219a+85)\,7}$$
$$1488\,|\,\underline{5952a^2+2976a-7440}$$
$$4a^2+\quad 2a-\quad 5$$

$$148a^3+6a^2-219a+85$$
$$\underline{-(\ 4a^2+2a-5)\,37a\qquad}$$
$$-68a^2-34a+85$$
$$\underline{-(4a^2+2a-5)\times(-17)}$$

the required H C F $=4a^2+2a-5$

28

$$3x^6-3x^4-18x^3+x^2+2x+3$$
$$\underline{-(x^5+2x^4-5x^2-7x+3)3x}$$
$$-6x^5-3x^4-3x^3+22x^2-7x+3$$
$$\underline{-(x^5+2x^4\qquad-5x^2-7x+3)\times(-6)}$$
$$9x^4-3x^3-8x^2-49x+21$$

$$x^5+2x^4\qquad-5x^2-7x+3$$
$$\underline{9\qquad\qquad\qquad\qquad\qquad}$$
$$9x^5+18x^4\quad-45x^2-63x+27$$
$$\underline{-(9x^4-3x^3-8x^2-49x+21)x}$$
$$21x^4+8x^3+4x^2-84x+27$$
$$\underline{3\qquad\qquad\qquad\qquad\qquad}$$
$$63x^4+24x^3+12x^2-252x+81$$
$$\underline{-(9x^4-3x^3-8x^2-49x+21)\,7}$$
$$45x^3+68x^2+91x-66$$

$$9x^4-3x^3-8x^2-49x+21$$
$$\underline{\quad 5 \quad}$$
$$45x^4-15x^3-40x^2-245x+105$$
$$\underline{-(45x^3+68x^2+91x-66)\ x}$$
$$-83x^3-131x^2-179x+105$$
$$\underline{\quad 22 \quad}$$
$$-1826x^3-2882x^2-3938x+2310$$
$$\underline{-(45x^3+68x^2+91x-66)\times(-35)}$$
$$-251x\,|\,\underline{-251x^3-502x^2-753x}$$
$$x^2+2x+3$$
$$45x^3+68x^2+91x-66$$
$$\underline{-(x^2+2x+3)\,45x}$$
$$-22x^2-44x-66$$
$$\underline{-(x^2+2x+3)\times(-22)}$$

the required H C F $=x^2+2x+3$

Exercise 51.

1 Let $A=x^3-3x^2-4x+12$,
and $B=x^3-7x^2+16x-12$,
$A+B=2x^3-10x^2+12x=2x(x^2-5x+6)$
and $A-B=4x^2-20x+24=4(x^2-5x+6)$,
the required H C F $=x^2-5x+6$

2 Let $A=2x^3-17x+12$,
and $B=4x^4-2x^3-34x^2+41x-12$,
$A+B=4x^4-34x^2+24x$
$=2x(2x^3-17x+12)$,
and $2A-B=2x^3-17x+12$,
the required H C F $=2x^3-17x+12$

3 Let $A=4x^3+13x^2+19x+4$,
and $B=2x^3+5x^2+5x-4$,
$A+B=6x^3+18x^2+24x=6x(x^2+3x+4)$,
and $A-2B=3x^2+9x+12=3(x^2+3x+4)$,
the required H C F $=x^2+3x+4$

4 Let $A=3x^3-5x^2+7$,
and $B=6x^4-7x^3-5x^2+14x+7$,

$B-A=6x^4-10x^3+14x$
$=2x(3x^3-5x^2+7)$,
the required H C F $=3x^3-5x^2+7$

5 Let $A=6x^4-11x^3+16x^2-22x+8$,
and $B=6x^4-11x^3-8x^2+22x-8$,
$A+B=12x^4-22x^3+8x^2=2x^2(6x^2-11x+4)$,
and $A-B=24x^2-44x+16=4(6x^2-11x+4)$,
the required H C F $=6x^2-11x+4$

6 Let $A=2x^4+19x^3+20x^2-31x+8$,
and $B=2x^4+7x^3-64x^2+62x+6$,
$A-B=12x^3+84x^2-93x+24$
$=3(4x^3+28x^2-31x+8)$
Again let $C=4x^3+28x^2-31x+8$,
$A-C=2x^4+15x^3-8x^2$
$=x^2(2x^2+15x-8)$
Let $C'=2x^2+15x-8$,
$C+C'=4x^3+30x^2-16x$
$=2x(2x^2+15x-8)$,
the required H C F $=2x^2+15x-8$

7 Let $A=3x^4-7x^3-27x^2-6x+2$,
and $B=3x^4-13x^3-40x^2-9x+3$,
$A-B=6x^3+13x^2+3x-1=C$ (suppose),
and $3A-2B=3x^4+5x^3-x^2=x^2(3x^2+5x-1)$
Let $C'=3x^2+5x-1$,
$C-C'=6x^3+10x^2-2x=2x(3x^2+5x-1)$,
the required H C F $=3x^2+5x-1$

8 Let $A=5x^4-18x^3-7x^2+12x+3$,
and $B=5x^4-23x^3-9x^2+16x+4$,
$A-B=5x^3+2x^2-4x-1=C$ (suppose),
and $4A-3B=5x^4-3x^3-x^2=x^2(5x^2-3x-1)$
Let $C'=5x^2-3x-1$,
$C-C'=5x^3-3x^2-x=x(5x^2-3x-1)$,
the required H C F $=5x^2-3x-1$

9 Let $A=2x^4-5x^3-17x^2-2x+2$,
and $B=6x^5+23x^4+34x^3+21x^2-2x-2$,
$A+B=6x^5+25x^4+29x^3+4x^2-4x$
$=x(6x^4+25x^3+29x^2+4x-4)$

Also let $6x^4+25x^3+29x^2+4x-4=C$,
$2A-C=10x^4+15x^3-5x^2=5x^2(2x^2+3x-1)$

Let $B'=2x^2+3x-1$,
$C-4B'=6x^4+25x^3+21x^2-8x$
$=x(6x^3+25x^2+21x-8)$

Again let $C'=6x^3+25x^2+21x-8$,
$C'-8B'=6x^3+9x^2-3x=3x(2x^2+3x-1)$,
the required H C F $=2x^2+3x+1$

10 Let $A=6x^5+9x^4-13x^3-4x^2+9x-3$,
and $B=9x^5+12x^4-18x^3-5x^2+12x-4$,
$3A-2B=3x^4-3x^3-2x^2+3x-1=C$ (suppose),
$A-3C=6x^5-4x^3+2x^2=2x^2(3x^3-2x+1)$

Also let $3x^3-2x+1=C'$,
$C+C'=3x^4-2x^2+x=x(3x^3-2x+1)$,
the required H C F $=3x^3-2x+1$

11 Let $A=x^5-x^2+8$, and $B=x^5-x^3+4$,
$A-B=-x^3+x^2+4=C$ (suppose),
and $2B-A=x^5+x^3-2x^2=x^2(x^3+x-2)$

Let $C'=x^3+x-2$,
$C+C'=x^2+x+2=A'$ (suppose),
$A'+C'=x^3+x^2+2x=x(x^2+x+2)$,
the required H C F $=x^2+x+2$

12 Let $A=3x^6+139x^2-44$,
and $B=39x^6+139x^4-16$,
$B-13A=139x^4-1807x^2+556$
$=139(x^4-13x^2+4)$

Let $C=x^4-13x^2+4$,
$B+4C=39x^6+143x^4-52x^2$
$=13x^2(3x^4+11x^2-4)$

Also let $A' = 3x^3 + 11x^2 - 4$,

$$C + A' = x^4 + 3x^3 - 2x^2$$
$$= x^2(x^2 + 3x - 2)$$

Let $x^2 + 3x - 2 = B'$,

$$A' - 2B' = 3x^3 + 9x^2 - 6x$$
$$= 3x(x^2 + 3x - 2) \text{ ,}$$

the required H C F $= x^2 + 3x - 2$

Exercise 52.

1

$$\begin{array}{r}
2x^3 + 7x^2 - 5x - 4 \\
\underline{-(x^3 + 8x^2 + 11x - 20)\,2} \\
-9\,|\,\underline{-9x^2 - 27x + 36} \\
x^2 + 3x - 4 \\
x^3 + 8x^2 + 11x - 20 \\
\underline{-(x^2 + 3x - 4)\,x} \\
5x^2 + 15x - 20 \\
\underline{-(x^2 + 3x - 4)\,5} \\
2x^3 + 19x^2 + 49x + 20 \\
\underline{-(x^2 + 3x - 4)\,2x} \\
13x^2 + 57x + 20 \\
\underline{-(x^2 + 3x - 4)\,13} \\
18\,|\,\underline{18x + 72} \\
x + 4
\end{array}$$

$$\begin{array}{r}
x^2 + 3x - 4 \\
\underline{-(x + 4)x} \\
-x - 4 \\
\underline{-(x + 4) \times (-1)}
\end{array}$$

the reqd H C F $= x + 4$

2 Let $A = 2x^4 + 3x^3 + 8x^2 + 15x - 10$,

and $B = 2x^4 - 5x^3 + 12x^2 - 25x + 10$,

$$A + B = 4x^4 - 2x^3 + 20x^2 - 10x$$
$$= 2x(2x^3 - x^2 + 10x - 5),$$

and $A - B = 8x^3 - 4x^2 + 40x - 20$

$$= 4(2x^3 - x^2 + 10x - 5) \text{ ,}$$

the H C F of A and $B = 2x^3 - x^2 + 10x - 5$

$$\begin{array}{r}
2x^4 - 5x^3 + 10x^2 - 20x + 8 \\
\underline{-(2x^3 - x^2 + 10x - 5)x} \\
-4x^3 - 15x + 8 \\
\underline{-(2x^3 - x^2 + 10x - 5) \times (-2)} \\
-2x^2 + 5x - 2
\end{array}$$

$$\begin{array}{r}
2x^3-\ x^2+10x-5 \\
-(-2x^2+5x-2)\times(-x) \\ \hline
4x^2+8x-5 \\
-(-2x^2+5x-2)\times(-2) \\ \hline
9\,|\,\underline{18x-9} \\
2x-1
\end{array}
\qquad
\begin{array}{r}
-2x^2+5x-2 \\
-(2x-1)\times(-x) \\ \hline
4x-2 \\
-(2x-1)\,2 \\ \hline
\end{array}$$

the reqd H C F $=2x-1$

3 Let $A=2x^4+5x^3-16x^2-10x+24$,

and $B=2x^4+5x^3-10x^2+5x-12$,

$A-B=-6x^2-15x+36=-3(2x^2+5x-12)$,

and $A+2B=6x^4+15x^3-36x^2=3x^2(2x^2+5x-12)$,

the H C F of A and $B=2x^2+5x-12$

$$\begin{array}{r}
2x^4+7x^3-19x^2-14x+30 \\
-(2x^2+5x-12)\ x^2 \\ \hline
2x^3-7x^2\ \ -14x+30 \\
-(2x^2+5x\ \ -12)\,x \\ \hline
-12x^2-2x+30 \\
-(2x^2+5x-12)\times(-6) \\ \hline
14\,|\,\underline{28x-42} \\
2x-3
\end{array}
\qquad
\begin{array}{r}
2x^2+5x-12 \\
-(2x-3)\,x \\ \hline
8x-12 \\
-(2x-3)\,4 \\ \hline
\end{array}$$

the required H C F $=2x-3$

4 Let $A=2x^4-4x^3-69x^2-2x-35$,

and $B=2x^4-6x^3-55x^2-3x-28$,

$A-B=2x^3-14x^2+x-7$,

and $5B-4A=2x^4-14x^3+x^2-7x$

$=x(2x^3-14x^2+x-7)$,

the H C F of A and $B=2x^3-14x^2+x-7$

$$\begin{array}{r}
2x^4+18x^3+41x^2+9x+20 \\
-(2x^3-14x^2+x-7)\,x \\ \hline
4\,|\,\underline{32x^3+40x^2+16x+20} \\
8x^3+10x^2+4x+5 \\
-(2x^3-14x^2+x-7)\,4 \\ \hline
33\,|\,\underline{66x^2+33} \\
2x^2+1
\end{array}
\qquad
\begin{array}{r}
2x^3-14x^2+x-7 \\
-(2x^2+1)\,x \\ \hline
-14x^2-7 \\
-(2x^2+1)\times(-7) \\ \hline
\end{array}$$

the required H C F $=2x^2+1$

5 Let $A=3a^3+28a^2b+52ab^2-48b^3$,

and $B=3a^3+4a^2b-28ab^2+16b^3$,

$$A-B=24a^2b+80ab^2-64b^3$$
$$=8b(3a^2+10ab-8b^2),$$
and $A+3B=12a^3+40a^2b-32ab^2$
$$=4a(3a^2+10ab-8b^2)\ ,$$
the H C F of A and $B=3a^2+10ab-8b^2$

$$\begin{array}{l}
3a^3+10a^2b-44ab^2+24b^3 \\
\underline{-(3a^2+10ab-8b^2)\,a} \\
-12b^2 \,|\, \underline{-36ab^2+24b^3} \\
\qquad 3a \quad -2b
\end{array}
\qquad\qquad
\begin{array}{l}
3a^2+10ab-8b^2 \\
\underline{-(3a-2b)\,a} \\
12ab-8b^2 \\
\underline{-(3a-2b)\,4b}
\end{array}$$

the required H C F $=3a-2b$

6 Let $A=6a^3-37a^2b+57ab^2-20b^3$,
and $B=3a^3-8a^2b-31ab^2+60b^3$,
$2B-A=21a^2b-119ab^2+140b^3$
$$=7b(3a^2-17ab+20b^2),$$
and $3A+B=21a^3-119a^2b+140ab^2$
$$=7a(3a^2-17ab+20b^2)\ ,$$
the H C F of A and $B=3a^2-17ab+20b^2$

$$\begin{array}{l}
6a^3+5a^2b-34ab^2+15b^3 \\
\underline{-(3a^2-17ab+20b^2)\,2a} \\
39a^2b-74ab^2+15b^3 \\
\underline{-(3a^2-17ab+20b^2)\ 13b} \\
49b^2 \,|\, \underline{147ab^2-245b^3} \\
\qquad 3a-5b
\end{array}
\qquad\qquad
\begin{array}{l}
3a^2-17ab+20b^2 \\
\underline{-(3a-5b)a} \\
-12ab+20b^2 \\
\underline{-(3a-5b)\times(-4b)}
\end{array}$$

the required H C F $=3a-5b$

7 Let $A=3x^4+11x^3-32x^2-44x+80$,
and $B=3x^4-x^3-52x^2+124x-80$,
$A+B=6x^4+10x^3-84x^2+80x$
$$=2x(3x^3+5x^2-42x+40),$$
and $A-B=12x^3+20x^2-168x+160$
$$=4(3x^3+5x^2-42x+40)\ ,$$
the H C F of A and $B=3x^3+5x^2-42x+40$
$=C'$ (suppose)
Let $C=3x^4+2x^3-83x^2-50x+200$,
$C-5C'=3x^4-13x^3-108x^2+160x$
$=x(3x^3-13x^2-108x+160)$
$=x\ D$ (suppose) ,

$$C-D=18x^2+66x-120$$
$$=6(3x^2+11x-20),$$
and $$4C-D=9x^3+33x^2-60x$$
$$=3x(3x^2+11x+20)\ ,$$
the H C F of A, B and C
$$=3x^2+11x-20$$

$$\begin{array}{l}
3x^4+2x^3-20x^2-8x+32 \\
\underline{-(3x^2+11x-20)x^2} \\
-9x^3-8x+32 \\
\underline{-(3x^2+11x-20)\times(-3x)} \\
33x^2-68x+32 \\
\underline{-(3x^2+11x-20)\,11} \\
-63\,|\,\underline{-189x+252} \\
3x-4
\end{array}
\qquad
\begin{array}{l}
3x^2+11x-20 \\
\underline{-(3x-4)\ x} \\
15x-20 \\
\underline{-(3x-4)\,5}
\end{array}$$

the required H C F $=3x-4$

8 Let $A=6x^5-28x^4+17x^3+54x^2-39x-18$,
and $B=6x^5+8x^4-79x^3-36x^2+105x+36$,

$$B-A=36x^4-96x^3-90x^2+144x+54$$
$$=6(6x^4-16x^3-15x^2+24x+9),$$
and $$2A+B=18x^5-48x^4-45x^3+72x^2+27x$$
$$=3x(6x^4-16x^3-15x^2+24x+9)\ ,$$
the H C F of A & $B=6x^4-16x^3-15x^2+24x+9$
$=C'$ (suppose)

Let also $C=6x^5+14x^4-53x^3-37x^2+66x+24$,
and $D=2x^5-2x^4-31x^3+51x^2+42x-72$,

$$C-3D=20x^4+40x^3-190x^2-60x+240$$
$=10(2x^4+4x^3-19x^2-6x+24)=10\ E$ (suppose) ,
$$C-E=6x^5+12x^4-57x^3-18x^2+72x$$
$$=3x(2x^4+4x^3-19x^2-6x+24)\ ,$$
the H C F of C and D
$=2x^4+4x^3-19x^2-6x+24=D'$ (suppose) ,
$$3D'-C'=28x^3-42x^2-42x+63$$
$$=7(4x^3-6x^2-6x+9)$$
$=7\ E'$ (suppose) ,

$C - E' = 6x^4 - 20x^3 - 9x^2 + 30x$
$\qquad = x(6x^3 - 20x^2 - 9x + 30)$
$\qquad = x\,G$ (suppose) ,
$3E' - 2G = 22x^2 - 33 = 11(2x^2 - 3)$,
and $10E' - 3G = 22x^3 - 33x = 11x(2x^2 - 3)$,
the required H C F $= 2x^2 - 3$

Exercise 53

1 1st expression $= a^2 \times b$,
2nd „ $= a \times b^2$,
the required L C M $= a^2 \times b^2 = a^2b^2$

2 1st expression $= a^3 \times b^2$
2nd „ $= a^2 \times b \times c$,
the required L C M $= a^3 \times b^2 \times c = a^3b^2c$

3 1st expression $= 2 \times 3 \times x^2 \times y^4$,
2nd „ $= 2 \times 5 \times x \times y^2$,
the required L C M $= 2 \times 3 \times 5 \times x^2 \times y^4 = 30x^2y^4$

4 1st expression $= 2^2 \times m^2 \times n^3$,
2nd „ $= 2^2 \times 7 \times m^4 \times n^2 \times p$,
the required L C M $= 2^2 \times 7 \times m^4 \times n^3 \times p = 28m^4n^3p$

5 1st expression $= 2^3 \times x^2 \times y^3 \times z$,
2nd „ $= 2^2 \times 3 \times x^3 \times y^2 \times z^2$,
the required L C M $= 2^3 \times 3 \times x^3 \times y^3 \times z^2 = 24x^3y^3z^2$

6 1st expression $= 2^2 \times a^2 \times b \times c$,
2nd „ $= 2 \times 5 \times a \times b^2 \times c$,
3rd „ $= 2 \times 7 \times a \times b \times c^2$,
the reqd L C M $= 2^2 \times 5 \times 7 \times a^2 \times b^2 \times c^2$
$\qquad = 140a^2b^2c^2$

7 1st expression $= 2^3 \times a^3 \times b^2 \times c$,
2nd „ $= 2^2 \times 3 \times a \times b^3 \times c^2$,
3rd „ $= 2^2 \times 5 \times a^3 \times b \times c^3$,
the reqd L C M $= 2^3 \times 3 \times 5 \times a^3 \times b^3 \times c^3 = 120a^3b^3c^3$

8 1st expression $=2\times3\times x^4\times y$,
2nd " $=3^2\times x^2\times y^3\times z$,
3rd " $=2^2\times3\times a^2\times x\times y^2$,
4th " $=3\times5\times a\times x\times z^2$,
the reqd L C M $=3^2\times2^2\times5\times x^4\times y^3\times z^2\times a^2$
$=180x^4y^3z^2a^2$

9 1st expression $=ab(a^2-b^2)=ab(a+b)(a-b)$,
2nd " $=a^2b^2(a+b)$,
the reqd L C M $=a^2b^2(a+b)(a-b)=a^2b^2(a^2-b^2)$

10 1st expression $=2^2\times(x-y)^2$,
2nd " $=2\times3\times(x-y)(x+y)$,
3rd " $=2^3\times(x+y)^2$,
the reqd L C M $=2^3\times3\times(x-y)^2(x+y)^2=24(x^2-y^2)^2$

11 1st expression $=(x-3)(x-1)$,
2nd " $=(x-2)(x-3)$,
the reqd L C M $=(x-1)(x-2)(x-3)$

12 1st expression $=a(a^2+2ax-3x^2)=a(a+3x)(a-x)$,
2nd " $=a^2(a^2+ax-6x^2)=a^2(a+3x)(a-x)$,
the reqd L C M $=a^2(a-x)(a-2x)(a+3x)$

13 1st expression $=a^2(a-2)(a+2)$,
2nd " $=a^2(a^2+2a-8)=a^2(a+4)(a-2)$,
the reqd L C M $=a^2(a-2)(a+2)(a+4)$

14 1st expression $=2^2\times a^2\times x^2$,
2nd " $=2\times x\times(x-a)(x+a)$,
3rd " $=2\times3\times a^3\times x(x+a)(x^2-ax+a^2)$,
the required L C M
$=2^2\times3\times a^3\times x^2\times(x-a)(x+a)(x^2-ax+a^2)$
$=12a^3x^2(x-a)(x+a)(x^2-ax+a^2)$

15 1st expression $=2^2\times3\times(x+5)(x-2)$,
2nd " $=2^4\times(x+6)(x-2)$,
the reqd L C M $=2^4\times3\times(x+6)(x+5)(x-2)$
$=48(x+6)(x+5)(x-2)$

16 1st expression $=(x+5)(x-3)$,
2nd " $=(x+5)(x+4)$,
3rd " $=(x+7)(x-3)$,
the reqd L C M $=(x+7)(x+5)(x+4)(x-3)$

17 1st expression $=3a^2(4a^2-9b^2)=3a^2(2a+3b)(2a-3b)$,
2nd " $=(2a+3b)(a-b)$,
3rd " $=(2a-3b)(a+b)$,
the reqd L C M $=3a^2(2a+3b)(2a-3b)(a+b)(a-b)$
$=3a^2(4a^2-9b^2)(a^2-b^2)$

18 1st expression $=(2a+3b)(4a^2-6ab+9b^2)$,
2nd " $=(2a-3b)(4a^2+6ab+9b^2)$,
3rd " $=16a^4+72a^2b^2+81b^4-36a^2b^2$
$=(4a^2+9b^2)^2-6ab)^2$
$=(4a^2+6ab+9b^2)(4a^2-6ab+9b^2)$,
the reqd L C M
$=(2a+3b)(2a-3b)(4a^2+6ab+9b^2)$
$(4a^2-6ab+9b^2)$
$=(8a^3+27b^3)(8a^3-27b^3)$

19 1st expression $=2x^2(4x^2-25y^2)=2x^2(2x+5y)(2x-5y)$,
2nd " $=3x(4x^2+8xy-5y^2)=3x(2x+5y)(2x-y)$,
3rd " $=4(4x^2-12xy+5y^2)=4(2x-5y)(2x-y)$,
the reqd L C M $=3\times 4\times x^2(2x+5y)(2x-5y)(2x-y)$
$=12x^2(4x^2-25y^2)(2x-y)$

20 1st expression $=(2x-3a)^2$,
2nd " $=(3x+a)(2x-3a)$,
3rd " $=(3x-a)(2x-3a)$,
the reqd L. C M $=(2x-3a)^2(3x+a)(3x-a)$
$=(2x-3a)^2(9x^2-a^2)$

21 1st expression $=2x^2+6x+9$,
2nd " $=2x^2(2x^2-6x+9)$,
3rd " $=4x^4+36x^2+81-36x^2$
$=(2x^2+9)^2-(6x)^2$
$=(2x^2+6x+9)(2x^2-6x+9)$,
the reqd L C M $=2x(2x^2+6x+9)(2x^2-6x+9)$
$=2x(4x^4+81)$

22 1st expression $=(3a-x)^2$,

2nd " $=2(3a^2+5ax-2x^2)=2(3a-x)(a+2x)$,

3rd " $=3(3a^2-7ax+2x^2)=3(3a-x)(a-2x)$,

$\therefore$ the reqd L C M $=2\times 3(3a-x)^2(a+2x)(a-2x)$

$=6(3a-x)^2(a^2-4x^2)$

23 1st expression $=(2x)^3-3(2x)^2+3(2x)-1=(2x-1)^3$,

2nd " $=(2x-1)(4x^2-1)=(2x-1)^2(2x+1)$,

3rd " $=(2x-1)(x+3)$,

$\therefore$ the reqd L C M $=(2x-1)^3(2x+1)(x+3)$

$=(2x-1)^2(4x^2-1)(x+3)$

24 1st expression $=(x-4y)(x-2y)$,

2nd " $=(x-3y)(x-4y)$,

3rd " $=(x-3y)(x+5y)$,

$\therefore$ the reqd L C M $=(x-2y)(x-3y)(x-4y)(x+5y)$

25 1st expression $=(2x-1)(3x+1)$,

2nd " $=(3x+1)(x+2)$,

3rd " $=(2x-1)(x+2)$,

$\therefore$ the reqd L C M $=(2x-1)(3x+1)(x+2)$

26 1st expression $=(1+2x)^2-(4x^2)^2$

$=(1+2x+4x^2)(1+2x-4x^2)$,

2nd " $=(1+2x)(1-8x^3)$

$=(1+2x)(1-2x)(1+2x+4x^2)$,

$\therefore$ the reqd L C M $=(1-4x^2)(1+2x+4x^2)(1+2x-4x^2)$

27 1st expression $=(9x^2-1)(x^2-3)=(3x+1)(3x-1)(x^2-3)$,

2nd " $=(9x^2-1)(3x^2-1)=(3x+1)(3x-1)(3x^2-1)$,

3rd " $=(9x^2-1)(3x^2+1)=(3x+1)(3x-1)(3x^2+1)$,

4th " $=(x^2-3)^2$,

$\therefore$ the reqd L C M $=(x^2-3)^2(9x^4-1)(9x^2-1)$

Exercise 54

1

$$
\begin{array}{r}
3x^3+14x^2+13x-8 \\
-(3x^3+\ \ 2x^2-11x+4) \\
\hline
12\ \big|\ 12x^2+24x-12 \\
\hline
x^2+2x\ \ -1
\end{array}
$$

$$\begin{array}{r}3x^3+2x^2-11x+4\\ \underline{-(x^2+2x-1)\,3x\qquad\qquad}\\ -4x^2-8x+4\\ \underline{-(x^2+2x-1)\times(-4)}\end{array}$$

Thus the H C F $=x^2+2x-1$

The L. C M

$$=\frac{3x^3+2x^2-11x+4}{x^2+2x-1}(3x^3+14x^2+13x-8)$$

$$=(3x-4)(3x^3+14x^2+13x-8)$$

$$=9x^4+30x^3-17x^2-76x+32$$

2

$$\begin{array}{r}6x^3+17x^2+9x-4\\ \underline{-(6x^3-7x^2-27x+8)}\\ 12\,\big|\,\underline{24x^2+36x-12}\\ 2x^2+3x-1\end{array}$$

$$\begin{array}{r}6x^3-7x^2-27x+8\\ \underline{-(2x^2+3x-1)\,3x\qquad\qquad}\\ -16x^2-24x+8\\ \underline{-(2x^2+3x-1)\times(-8)}\end{array}$$

Thus the H C F $=2x^2+3x-1$

The L C M

$$=\frac{6x^3-7x^2-27x+8}{2x^2+3x-1}(6x^3+17x^2+9x-4)$$

$$=(3x-8)(6x^3+17x^2+9x-4)$$

$$=18x^4+3x^3-109x^2-84x+32$$

3

$$\begin{array}{r}12x^3-4x^2-25x+12\\ \underline{-(12x^3-28x^2-7x-12)}\\ 8x\,\big|\,\underline{24x^2-32x}\\ 3x-4\end{array}$$

$$\begin{array}{r}12x^3-28x^2-7x-12\\ \underline{-(3x-4)\,4x^2\qquad\qquad}\\ -12x^2-7x-12\\ \underline{-(3x-4)(-4x)}\\ -9x+12\\ \underline{-(3x-4)(-3)}\end{array}$$

Thus the H C F $=3x-4$

$$\text{The L C M} = \frac{12x^3 - 28x^2 + 7x + 12}{3x - 4}(12x^3 - 4x^2 - 25x + 12)$$
$$= (4x^2 - 4x - 3)(12x^3 - 4x^2 - 25x + 12)$$
$$= 48x^5 - 64x^4 - 120x^3 + 160x^2 + 27x - 36$$

4 Let $A = 9x^3 - 12x^2 - 15x + 20$,
and $B = 15x^3 + 12x^2 - 25x - 20$,
$A + B = 24x^3 - 40x = 8x(3x^2 - 5) = 8x\ C$ (suppose),
$A + 4C = 9x^3 - 15x = 3x(3x^2 - 5)$

Thus, the H C F of A and $B = 3x^2 - 5$

$$\text{The L C M} = \frac{9x^3 - 12x^2 - 15x + 20}{3x^2 - 5}(15x^3 + 12x^2 - 25x - 20)$$
$$= (3x - 4)(15x^3 + 12x^2 - 25x - 20)$$
$$= 45x^4 - 24x^3 - 123x^2 + 40x + 80$$

5 Let $A = 4x^3 - 10x^2 - 18x + 45$,
and $B = 6x^3 + 8x^2 - 27x - 36$,
$2B - 3A = 46x^2 - 207 = 23(2x^2 - 9)$,
and $B - 4C = 6x^3 - 27x = 3x(2x^2 - 9)$

Thus, the H C F of A and $B = 2x^2 - 9$

$$\text{The L C M} = \frac{4x^3 - 10x^2 - 18x + 45}{2x^2 - 9}(6x^3 + 8x^2 - 27x - 36)$$
$$= (2x - 5)(6x^3 + 8x^2 - 27x - 36)$$
$$= 12x^4 - 14x^3 - 94x^2 + 63x + 180$$

6 Let $A = 4x^4 + 4x^3 + 7x^2 + 11x + 4$,
and $B = 6x^4 + 7x^3 + 4x^2 + 5x + 2$,
$2B - A = 8x^4 + 10x^3 + x^2 - x$
$= x(8x^3 + 10x^2 + x - 1) = x\ C$ (suppose),
$B + 2C = 6x^4 + 23x^3 + 24x^2 + 7x$
$= x(6x^3 + 23x^2 + 24x + 7) = x\ D$ (suppose),
$7C + D = 62x^3 + 93x^2 + 31x$
$= 31x(2x^2 + 3x + 1) = 31x\ C'$ (suppose),
$C + C' = 8x^3 + 12x^2 + 4x = 4x(2x^2 + 3x + 1)$

Thus, the H C F of A and $B = 2x^2 + 3x + 1$

$$\text{The L C M} = \frac{4x^4+4x^3+7x^2+11x+4}{2x^2+3x+1}(6x^4+7x^3+4x^2+5x+2)$$
$$=(2x^2-x+4)(6x^4+7x^3+4x^2+5x+2)$$
$$=12x^6+8x^5+25x^4+34x^3+15x^2+18x+18$$

7 Let $A=8x^4-6x^3-8x^2+9x-6$,
and $B=16x^4-12x^3+20x^2-9x+6$,
$A+B=24x^4-18x^3+12x^2$
$=6x^2(4x^2-3x+2)=6x^2\,C$ (suppose),
$A+3C=8x^4-6x^3+4x^2=2x^2(4x^2-3x+2)$

Thus, the H C F of A and $B=4x^2-3x+2$
∴ The L C M

$$=\frac{8x^4-6x^3-8x^2+9x-6}{4x^2-3x+2}(16x^4-12x^3+20x^2-9x+6)$$
$$=(2x^2-3)(16x^4-12x^3+20x^2-9x+6)$$
$$=32x^6-24x^5-8x^4+18x^3-48x^2+27x-18$$

8 Let $A=4x^4+8x^3+21x^2+18x+27$,
and $B=3x^4+6x^3+17x^2+16x+24$,
$4B-3A=5x^2+10x+15$
$=5(x^2+2x+3)=5\,C$ (suppose),
$B-8C=3x^4+6x^3+9x^2=3x^2(x^2+2x+3)$

Thus, the H C F of A and B
$=x^2+2x+3$
The L C M

$$=\frac{4x^4+8x^3+21x^2+18x+27}{x^2+2x+3}(3x^4+6x^3+17x^2+16x+24)$$
$$=(4x^2+9)(3x^4+6x^3+17x^2+16x+24)$$
$$=12x^6+24x^5+95x^4+118x^3+249x^2+144x+216$$

9 We have $l=\frac{xy}{h}$ or $xy=hl$,
and $h+l=x+y$,
$$h^3+l^3=(h+l)^3-3hl(h+l)$$
$$=(x+y)^3-3xy(x+y)$$
$$=x^3+y^3.$$

Exercise 55

1

$$
\begin{array}{l}
6x^2+x-2 \\
\underline{-(3x^2-10x-8)\,2} \\
7\,|\,\underline{21x+14} \\
\quad 3x+2
\end{array}
$$

$$
\begin{array}{l}
3x^2-10x-8 \\
\underline{-(3x+2)x} \\
-12x-8 \\
\underline{-(3x+2)(-4)}
\end{array}
$$

Thus, the L C M of the 1st and 3rd expressions

$$=\frac{3x^2-10x-8}{3x+2}(6x^4+x-2)$$

$$=(x-4)(6x^2+x-2)=6x^3-23x^2-6x+8$$

$$
\begin{array}{l}
6x^3-23x^2-6x+8 \\
\underline{2} \\
12x^3-46x^2-12x+16 \\
\underline{-(4x^2-20x+9)\,3x} \\
14x^2-39x+16 \\
\underline{2} \\
28x^2-78x+32 \\
\underline{-(4x^2-20x+9)\,7} \\
31\,|\,\underline{62x-31} \\
\quad 2x-1
\end{array}
\qquad
\begin{array}{l}
4x^2-20x+9 \\
\underline{-(2x-1)\,2x} \\
-18x+9 \\
\underline{-(2x-1)\times(-9)}
\end{array}
$$

The reqd L C M $=\dfrac{4x^2-20x+9}{2x-1}(6x^3-23x^2-6x+8)$

$$=(2x-9)(6x^3-23x^2-6x+8)$$

$$=12x^4-100x^3+195x^2+70x-72$$

2

$$
\begin{array}{l}
6x^2-7x-3 \\
\underline{-(3x^2-23x-8)\,2} \\
13\,|\,\underline{39x+13} \\
\quad 3x+1
\end{array}
\qquad
\begin{array}{l}
3x^2-23x-8 \\
\underline{-(3x+1)x} \\
-24x-8 \\
\underline{-(3x+1)\times(-8)}
\end{array}
$$

Thus, the L C M of the 1st two expressions

$$=\frac{3x^2-23x-8}{3x+1}(6x^2-7x-3)$$

$$=(x-8)(6x^2-7x-3)$$

$$=6x^3-55x^2+53x+24$$

$$\begin{array}{r}
6x^3-55x^2+53x+24 \\
-(2x^2-11x+12)3x \\
\hline
-22x^2+17x+24 \\
-(2x^2-11x+12)\times(-11) \\
\hline
-52 \,|\, -104x+156 \\
\hline
2x-3
\end{array}
\qquad
\begin{array}{r}
2x^2-11x+12 \\
-(2x-3)x \\
\hline
-8x+12 \\
-(2x-3)\times(-4) \\
\hline
\end{array}$$

The reqd L C M

$$=\frac{2x^2-11x+12}{2x-3}(6x^3-55x^2+53x+24)$$
$$=(x-4)(6x^3-55x^2+53x+24)$$
$$=6x^4-79x^3+273x^2-188x-96$$

3
$$\begin{array}{r}
12x^2-11x+2 \\
-(6x^2-19x+10)2 \\
\hline
9 \,|\, 27x-18 \\
\hline
3x-2
\end{array}
\qquad
\begin{array}{r}
6x^2-19x+10 \\
-(3x-2)2x \\
\hline
-15x+10 \\
-(3x-2)\times(-5) \\
\hline
\end{array}$$

Thus, the L C M of the 1st two expressions

$$=\frac{6x^2-19x+10}{3x-2}(12x^2-11x+2)$$
$$=(2x-5)(12x^2-11x+2)$$
$$=24x^3-82x^2+59x-10$$

$$\begin{array}{r}
24x^3-82x^2+59x-10 \\
-(8x^2+10x-3)3x \\
\hline
-112x^2+68x-10 \\
-(8x^2+10x-3)\times(-14) \\
\hline
52 \,|\, 208x-52 \\
\hline
4x-1
\end{array}
\qquad
\begin{array}{r}
8x^2+10x-3 \\
-(4x-1)2x \\
\hline
12x-3 \\
-(4x-1)3 \\
\hline
\end{array}$$

The required L C M

$$=\frac{8x^2+10x-3}{4x-1}(24x^3-82x^2+59x-10)$$
$$=(2x+3)(24x^3-82x^2+59x-10)$$
$$=48x^4-92x^3-128x^2+157x-30$$

4 Let $A=4x^4+8x^3-7x^2-6x+3$,
and $B=2x^4+4x^3+x^2+6x-3$,
$A+B=6x^4+12x^3-6x^2$
$=6x^2(x^2+2x-1)=6x^2C$ (suppose),
$B-3C=2x^4+4x^3-2x^2=2x^2(x^2+2x-1)$

Thus, the H C F of A and $B = x^2+2x-1$,

and the L C M of the 1st two expressions

$$=\frac{2x^4+4x^3+x^2+6x-3}{x^2+2x-1}(4x^4+8x^3-7x^2-6x+3)$$

$$=(2x^2+3)(4x^4+8x^3-7x^2-6x+3)$$

$$=8x^6+16x^5-2x^4+12x^3-15x^2-18x+9$$

$$\begin{array}{l}
8x^6+16x^5-2x^4+12x^3-15x^2-18x+9 \\
-(8x^4+4x^3-2x^2-3x-3)x^2 \\
\hline
12x^5 \quad +15x^3-12x^2-18x+9 \\
\quad 2 \\
\hline
24x^5 \quad +30x^3-24x^2-36x+18 \\
-(8x^4+4x^3-2x^2-3x-3)3x \\
\hline
-12x^4+36x^3-15x^2-27x+18 \\
\quad 2 \\
\hline
-24x^4+72x^3-30x^2-54x+36 \\
-(8x^4+4x^3-2x^2-3x-3)(-3) \\
\hline
3\,|\,84x^3-36x^2-63x+27 \\
\quad 28x^3-12x^2-21x+9
\end{array}$$

$$\begin{array}{l}
8x^4+4x^3-2x^2-3x-3 \\
7 \\
\hline
56x^4+28x^3-14x^2-21x-21 \\
-(28x^3-12x^2-21x+9)2x \\
\hline
52x^3+28x^2-39x-21 \\
7 \\
\hline
364x^3+196x^2-273x-147 \\
-(28x^3-12x^2-21x+9)13 \\
\hline
88\,|\,352x^2-264 \\
\quad 4x^2-3
\end{array}$$

$$\begin{array}{l}
28x^3-12x^2-21x+9 \\
-(4x^2 \quad -3)7x \\
\hline
-12x^2 \quad +9 \\
-(4x^2 \quad -3)\times(-3) \\
\hline
\end{array}$$

The reqd L C M

$$=\frac{8x^4+4x^3-2x^2-3x-3}{4x^2-3}(8x^6+16x^5-2x^4+12x^3-15x^2-18x+9)$$

$$=(2x^2+x+1)(8x^6+16x^5-2x^4+12x^3-15x^2-18x+9)$$

$$=16x^8+40x^7+20x^6+38x^5-20x^4-39x^3-15x^2-9x+9$$

Miscellaneous Exercise (2).

I.

1 See Art I, Chap V Page 45

Since $-4 = -1-1-1-1$,

$$(-4)\times(-8) = -(-8)-(-8)-(-8)-(-8)$$
$$= 8+8+8+8 = 32$$

2 (i) $y^3(x-2z) - y^2xz + y(xz^2 - x^3 - 2z^3) + (x^3z - xz^3)$

(ii) $(xy^3 - x^3y) + z(x^3 - xy^2 - 2y^3) + z^2xy - z^3(x+2y)$

3 $$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3x\,\frac{1}{x}\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$
$$= p^3 + 3p$$

4 $$x^5 - y^5 = x^4(x-y) + x^3y(x-y) + x^2y^2(x-y) + xy^3(x-y) + y^4(x-y),$$

$$\frac{x^5-y^5}{x-y} = x^4 + x^3y + x^2y^2 + xy^3 + y^4$$

5 The given expression

$$= \{(a+b+c)+(a-b+c)\}\{(a+b+c)-(a-b+c)\} + \{(a+b-c)+(b+c-a)\}\{(a+b-c)-(b+c-a)\}$$
$$= (2a+2c)\,2b + 2b(2a-2c) = 4b\{(a+c)+(a-c)\}$$
$$= 4b\,2a = 8ab$$

Again $8ab = 8\,a\,b = 8\times(-4)\times(-4) = 128$

6 $(a-b+c+d)(a+b+c-d)$

$$= \{(a+c)-(b-d)\}\{(a+c)+(b-d)\} = (a+c)^2 - (b-d)^2$$

7 $$4x^2 + 12xy + 9y^2 - 8x - 12y = (2x+3y)^2 - 4(2x+3y)$$
$$= (2x+3y)(2x+3y-4)$$

8 Let $A = 2x^4 - 12x^3 + 19x^2 - 6x + 9$,

and $B = 4x^3 - 18x^2 + 19x - 3$,

$A + 3B = 2x^4 - 35x^2 + 51x$

$= x(2x^3 - 35x + 51) = x\,C$ (suppose),

$2C-B=18x^2-89x+105=D$ (suppose),

and $17B+C=70x^3-306x^2+288x$

$=2x(35x^2-153x+144)=2x\ E$ (suppose),

$35D-18E=-361x+1083$

$=-361(x-3)=-361\ F$ (suppose),

$D+35F=18x^2-54x=18x(x-3)$

The reqd H C F $=x-3$

II

1 See Art 3 (ii) Ch V, Page 49

2
$$
\begin{array}{l}
x^{\frac{2}{3}}+2x^{\frac{1}{2}}+3x^{\frac{1}{3}}+2x^{\frac{1}{6}}+1 \\
\underline{x^{\frac{1}{3}}-2x^{\frac{1}{6}}+1} \\
x+2x^{\frac{5}{6}}+3x^{\frac{2}{3}}+2x^{\frac{1}{2}}+x^{\frac{1}{3}} \\
\quad -2x^{\frac{5}{6}}-4x^{\frac{2}{3}}-6x^{\frac{1}{2}}-4x^{\frac{1}{3}}-2x^{\frac{1}{6}} \\
\qquad\qquad \underline{x^{\frac{2}{3}}+2x^{\frac{1}{2}}+3x^{\frac{1}{3}}+2x^{\frac{1}{6}}+1} \\
x \qquad\qquad -2x^{\frac{1}{2}} \qquad\qquad +1
\end{array}
$$

3 Putting m for $ac+bd$ and n for $ad-bc$, the left-hand expression

$$
\begin{aligned}
&=(mx+ny)^2+(my-nx)^2 \\
&=m^2x^2+n^2y^2+2mxny+m^2y^2+n^2x^2-2mynx \\
&=m^2x^2+m^2y^2+n^2y^2+n^2x^2 \\
&=m^2(x^2+y^2)+n^2(x^2+y^2) \\
&=(m^2+n^2)(x^2+y^2) \\
&=(a^2c^2+b^2d^2+2acbd+a^2d^2+b^2c^2-2acbd)(x^2+y^2) \\
&=\{a^2(c^2+d^2)+b^2(c^2+d^2)\}(x^2+y^2) \\
&=(a^2+b^2)(c^2+d^2)(x^2+y^2)
\end{aligned}
$$

4 $x^2-2x+1=(x-1)^2$, the required cube

$$
\begin{aligned}
&=\{(x-1)^2\}^3=(x-1)^6 \\
&=x^6-6x^5+15x^4-20x^3+15x^2-6x+1
\end{aligned}
$$

5

$$
\begin{array}{r|l}
x-1 & x^5-px^4+qx^3-qx^2+px-1 \\
 & x^5-x^4 \\ \hline
 & -x^4(p-1)+qx^3-qx^2+px-1 \\
 & -x^4(p-1)+(p-1)x^3 \\ \hline
 & (q-p+1)x^3-\ qx^2+px-1 \\
 & (q-p+1)x^3-(q-p+1)x^2 \\ \hline
 & -(p-1)x^2+\ px-1 \\
 & -(p-1)x^2+(p-1)x \\ \hline
 & x-1 \\
 & x-1 \\ \hline
\end{array}
$$

Quotient $=x^4-(p-1)x^3+(q-p+1)x^2-(p-1)x+1$

6 (i) $ab-ac-b^2+bc=a(b-c)-b(b-c)=(b-c)(a-b)$

(ii) $b^2-12ac-4a^2-9c^2=b^2-(4a^2+12ac+9c^2)$

$=b^2-(2a+3c)^2=(b+2a+3c)(b-2a-3c)$

7 1st. expression $=2x^3-16-15x+30$

$=2(x^3-8)-15(x-2)$

$=2(x-2)(x^2+2x+4)-15(x-2)$

$=(x-2)(2x^2+4x+8-15)$

$=(x-2)(2x^2+4x-7)$.

2nd expression $=x^4-15x^2+28x-12$

$=x^4-15x^2+44+28x-56$

$=(x^2-11)(x^2-4)+28(x-2)$

$=(x-2)(x^3+2x^2-11x-22+28)$

$=(x-2)(x^3+2x^2-11x+6)$,

the reqd H. C. F. $=x-2$

8 1st. expression $=x^2(x+b)+a(x+b)$

$=(x+b)(x^2+a)$

2nd expression $=x^2-ax+bx-ab$

$=x(x-a)+b(x-a)$

$=(x-a)(x+b)$

the reqd L. C. M.

$=(x+b)(x-a)(x^2+a)$

$=\{x^2+(b-a)x-ab\}(x^2+a)$

$=x^4+(b-a)x^3+(a-ab)x^2+(ab-a^2)x-a^2b$

III.

1 $a^3+b^3=(a+b)^3-3ab(a+b)$

$$=(8)^3-3\cdot5\cdot8=512-120=392$$

2 The given expression

$$=(7c)^2-2\{3(a+b)\cdot7c\}+\{3(a+b)\}^2$$
$$=\{7c-3(a+b)\}^2$$
$$=\{7\cdot8-3(89-69)\}^2$$
$$=(56-3\cdot20)^2=(-4)^2=16$$

3 The dividend $=x^3(y-z)+y^3z-y^3x+z^3x-z^3y$

$$=x^3(y-z)+yz(y^2-z^2)-x(y^3-z^3)$$
$$=(y-z)\{x^3+yz(y+z)-x(y^2+yz+z^2)\}$$
$$=(y-z)\{x^3+x^2y+x^2z-x^2(y+z)-x(y+z)^2$$
$$+xyz+yz(y+z)\}$$
$$=(y-z)\{x^2(x+y+z)-x(y+z)(x+y+z)$$
$$+yz(x+y+z)\}$$
$$=(y-z)(x+y+z)(x^2-xy-xz+yz),$$

and the divisor $=xy-xz+y^2-z^2$

$$=x(y-z)+(y+z)(y-z)=(y-z)(x+y+z)\,;$$

$\therefore$ the quotient $=x^2-xy-xz+yz$

4 $4a-3+16a^2+64a^3$

$$=(4a-1)+\{(4a)^2-1\}+\{(4a)^3-1\}$$
$$=(4a-1)\{1+(4a+1)+(16a^2+4a+1)\}$$
$$=(4a-1)(16a^2+8a+3)$$

5 $(1+x+x^2)^2-(1-x+x^2)^2$

$$=\{(1+x+x^2)+(1-x+x^2)\}\{(1+x+x^2)-(1-x+x^2)\}$$
$$=(2+2x^2)\,2x=4x(1+x^2)$$

6 The left-hand expression

$$=ns^2-2(a_1+a_2+a_3+\cdots+a_n)s$$
$$+a_1^2+a_2^2+a_3^2+\cdots+a_n^2$$
$$=ns^2-2\,\frac{ns}{2}\,s+(a_1^2+a_2^2+a_3^2+\cdots+a_r^2)$$
$$=ns^2-ns^2+(a_1^2+a_2^2+a_3^2+\cdots+a_r^2)$$
$$=a_1^2+a_2^2+a_3^2+\cdots+a_r^2$$

7 The 1st expression

$$=(x^4-x^2)-(px^3-px)+(qx^2-q)$$
$$=(x^2-1)(x^2-px+q),$$

the 2nd expression

$$=(x^4-x^2)-(qx^3-qx)+(px^2-p)$$
$$=(x^2-1)(x^2-qx+p),$$

the reqd H C F $=x^2-1$

8 1st expression $=x^3-7x^2+10x+6x-12$

$$=x(x-2)(x-5)+6(x-2)$$
$$=(x-2)(x^2-5x+6)$$
$$=(x-2)(x-2)(x-3),$$

2nd expression $=x(3x^2-14x+16)$

$$=x(3x^2-6x-8x-16)$$
$$=x(3x-8)(x-2),$$

the reqd L C M

$$=x(x-2)(x-2)(x-3)(3x-8)$$
$$=x(x^3-7x^2+16x-12)(3x-8)$$
$$=3x^5-29x^4+104x^3-164x^2+96x$$

IV

1

$$\begin{array}{l}
a^{\frac{5}{2}}-2a^2b^{\frac{1}{3}}+4a^{\frac{3}{2}}b^{\frac{2}{3}}-8ab+16a^{\frac{1}{2}}b^{\frac{4}{3}}-32b^{\frac{5}{3}} \\
a^{\frac{1}{2}}+2b^{\frac{1}{3}} \\
\hline
a^3-2a^{\frac{5}{2}}b^{\frac{1}{3}}+4a^2b^{\frac{2}{3}}-8a^{\frac{3}{2}}b+16ab^{\frac{4}{3}}-32a^{\frac{1}{2}}b^{\frac{5}{3}} \\
\qquad +2a^{\frac{5}{2}}b^{\frac{1}{3}}-4a^2b^{\frac{2}{3}}+8a^{\frac{3}{2}}b-16ab^{\frac{4}{3}}+32a^{\frac{1}{2}}b^{\frac{5}{3}}-64b^2 \\
\hline
a^3 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -64b^2
\end{array}$$

2 (i) $a^3-3abc+(b^3+c^3)$,

(ii) $a^2(b-c)-a(b^2-c^2)+(b^2c-bc^2)$,

(iii) $a^4(b-c)-a(b^4-c^4)+(b^4c-bc^4)$

3 $(x+a)(x+b)(x+c)=\{x^2+(a+b)x+ab\}(x+c)$

$$=x^3+(a+b+c)x^2+(ab+ac+bc)x+abc$$

Hence $(x-7)(x+8)(x-12)$

$$=x^3+(-7+8-12)x^2+\{(-7\cdot 8)+(-7)(-12)+8(-12)\}x+7(-8)(-12)$$

$$=x^3-11x^2-68x+672\,,$$

$\therefore$ the co-efficients of x^2 and x are respectively -11 and -68

4 The left-hand expression

$$=\{(ab+cd)+(ac+bd)\}\{(ab+cd)-(ac+bd)\}$$
$$=(ab+cd)^2-(ac+bd)^2$$
$$=a^2b^2+2abcd+c^2d^2-(a^2c^2+2abcd+b^2d^2)$$
$$=a^2b^2+c^2d^2-a^2c^2-b^2d^2$$

5 The given expression

$$=(a-b-c)^2=\{(q+r+s)-(r+s-p)-(p+q-r)\}^2$$
$$=r^2$$

6 The dividend $=a^3+(2b)^3+(3c)^3-3\,a\,(2b)\,(3c)$

$$=(a+2b+3c)\{a^2+(2b)^2+(3c)^2-a\cdot 2b-a\cdot 3c-2b\cdot 3c\}$$
$$=(a+2b+3c)(a^2+4b^2+9c^2-2ab-3ac-6bc)\,,$$

$\therefore$ the reqd quotient $=a+2b+3c$

7 The Highest Common Factor of two or more algebraical expressions is that Common factor which is resolvable into the greatest number of elementary factors

1st expression $=2\cdot 2\cdot 3\cdot 3\cdot x^2a^4c^2$,

2nd expression $=2\cdot 2\cdot 2\cdot 3\cdot xy^2a^3b^4$,

3rd expression $=2\cdot 2\cdot 2\cdot 2\cdot 3\cdot 5\,y^3a^4b^2c$,

$\therefore$ the reqd H C F $=2\cdot 2\cdot 3\,a^3=12a^3$

8 1st expression $=(x-y)(x+y)$,

2nd expression $=(x-y)(x-y)$,

3rd expression $=(x-y)(x^2+xy+y^2)$,

$\therefore$ the reqd L C M

$$=(x-y)^2(x+y)(x^2+xy+y^2)$$
$$=(x-y)(x+y)(x^3-y^3)$$
$$=(x^2-y^2)(x^3-y^3)=x^5-x^3y^2-x^2y^3+y^5$$

V.

1 The given expression

$$=(7a)^2+2(7a)(9b)+(9b)^2$$
$$=(7a+9b)^2=(322-333)^2=(-11)^2=121$$

2 The left-hand expression

$$=s^2-2as+a^2+s^2-2sb+b^2+s^2-2sc+c^2+s^2$$
$$=4s^2-2s(a+b+c)+(a^2+b^2+c^2)$$
$$=4s^2-2s\cdot 2s+a^2+b^2+c^2$$
$$=4s^2-4s^2+(a^2+b^2+c^2)=a^2+b^2+c^2$$

3 Putting x for $5a-7c$ and y for $8c-3a$, we get $x+y=2a+c$, the given expression becomes

$$=x^3+y^3+3(x+y)xy=(x+y)^3=(2a+c)^3$$
$$=8a^3+12a^2c+6ac^2+c^3$$

4
$$a-b=\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)$$
$$=\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}\right)\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}\right)\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right),$$
$$\frac{a-b}{a^{\frac{1}{4}}-b^{\frac{1}{4}}}=\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}\right)\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)$$
$$=a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{4}}+a^{\frac{1}{4}}b^{\frac{1}{2}}+b^{\frac{3}{4}}$$

5 (i) The given expression $=6a^4x^2-6a^3x^3+a^3x-a^2x^2$

$$=6a^3x^2(a-x)+a^2x(a-x)$$
$$=(6a^3x^2+a^2x)(a-x)$$
$$=a^2x(6ax+1)(a-x)$$

(ii) The given expression $=xyz^2+x^2z+y^2z+xy$

$$=xz(yz+x)+y(yz+x)$$
$$=(yz+x)(xz+y)$$

6 1st. expression $=2a^2+a(b-c)-(3b^2+3bc+bc+c^2)$

$$=2a^2+a(b-c)-(b+c)(3b+c)$$
$$=2a^2-2a(b+c)+a(3b+c)-(b+c)(3b+c)$$
$$=2a(a-b-c)+(3b+c)(a-b-c)$$
$$=(a-b-c)(2a+3b+c),$$

2nd expression $=2a^2-a(5b-9c)-4(3b^2-2bc-c^2)$

$$=2a^2-a(5b-9c)-4(b-c)(3b+c)$$

$$=2a^2-8a(b-c)+a(3b+c)-4(b-c)(3b+c)$$
$$=2a(a-4b+4c)+(3b+c)(a-4b+4c)$$
$$=(2a+3b+c)(a-4b+4c),$$

the reqd H C F $=2a+3b+c$

7 See Art 3 (cor) Chap X Page 155

8 1st expression $=6x^3-9x^2-2x^2+5x-3$
$$=3x^2(2x-3)-(2x-3)(x-1)$$
$$=(2x-3)(3x^2-x+1),$$

2nd expression $=9x^3-6x^2-3x^2+5x-2$
$$=3x^2(3x-2)-(3x-2)(x-1)$$
$$=(3x-2)(3x^2-x+1),$$

the reqd L C M $=(2x-3)(3x-2)(3x^2-x+1)$
$$=(6x^2-13x+6)(3x^2-x+1)$$
$$=18x^4-45x^3+37x^2-19x+6$$

VI

1 The given expression
$$=(8765943)^2-(8765938)^2$$
$$=(8765943+8765938)\times(8765943-8765938)$$
$$=17531881\times 5=87659405$$

2 The given expression
$$=(3a)^3+3(3a)^2(4b)+3(3a)(4b)^2+(4b)^3$$
$$=(3a+4b)^3=(87-92)^3=(-5)^3=-125$$

3 $a+(b+c)\,\big)\, a^3-3abc \quad +(b^3+c^3)$ Quotient $=a^2-a(b+c)+(b^2+c^2-bc)$

$$\begin{array}{l}
\underline{a^3+a^2b+a^2c} \\
-a^2(b+c)-3abc+(b^3+c^3) \\
\underline{-a^2(b+c)-a(b^2+c^2+2bc)} \\
a(b^2+c^2-bc)+(b^3+c^3) \\
\underline{a(b^2+c^2-bc)+(b^3+c^3)}
\end{array}$$

$a^3+b^3+c^3-3abc$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=\tfrac{1}{2}(a+b+c)(2a^2+2b^2+2c^2-2ab-2ac-2bc)$$
$$=\tfrac{1}{2}(a+b+c)\{(a^2+b^2-2ab)+(a^2+c^2-2ac)+(b^2+c^2-2bc)\}$$
$$=\tfrac{1}{2}(a+b+c)\{(a-b)^2+(c-a)^2+(b-c)^2\}$$

4 $(a-b)(a-c)(b-c)$

$$=\{a^2-a(b+c)+bc\}(b-c)$$
$$=a^2(b-c)-a(b^2-c^2)+b^2c-bc^2$$
$$=a^2(b-c)+b^2(c-a)+c^2(a-b)$$

Putting a for $(x+2y+3z)$, b for $(y+2z+3x)$ and c for $(z+2x+3y)$ we have

$$a-b=z-2x+y,$$
$$b-c=x-2y+z,$$
$$\text{and } c-a=y-2z+x\ ,$$

the given expression becomes

$$=a^2(b-c)+b^2(c-a)+c^2(a-b)+(b-c)(c-a)(a-b)$$
$$=(a-b)(a-c)(b-c)-(b-c)(a-c)(a-b)=0$$

5 $(a^2-b^2-c^2+d^2)^2-4(ad-bc)^2$

$$=(a^2-b^2-c^2+d^2)^2-(2ad-2bc)^2$$
$$=\{(a^2-b^2-c^2+d^2)+(2ad-2bc)\}\{(a^2-b^2-c^2+d^2)-(2ad-2bc)\}$$
$$=\{(a^2+d^2+2ad)-(b^2+c^2+2bc)\}\{(a^2+d^2-2ad)-(b^2+c^2-2bc)\}$$
$$=\{(a+d)^2-(b+c)^2\}\{(a-d)^2-(b-c)^2\}$$
$$=(a+d+b+c)(a+d-b-c)(a-d+b-c)(a-d-b+c)$$
$$=(a+b+c+d)(a-b-c+d)(a+b-d-c)(a-b+c-d)$$

6 (i) $a^2-2ab+b^2+2a-2b=(a-b)^2+2(a-b)=(a-b)(a-b+2)$

(ii) $6a^2-ab-b^2+6a-3b=(6a^2-3ab+2ab-b^2)+6a-3b$

$$=(3a+b)(2a-b)+3(2a-b)$$
$$=(2a-b)(3a+b+3)$$

(iii) $15x^2-4xy-4y^2+10x+4y$

$$=15x^2+6xy-10xy-4y^2+10x+4y$$
$$=3x(5x+2y)-2y(5x+2y)+2(5x+2y)$$
$$=(5x+2y)(3x-2y+2)$$

7 The dividend $=(2x-y)^2a^4-\{(x+y)^2a^2x^2-2(x+y)ax\,x^3+x^6\}$

$$=(2x-y)^2a^4-\{(x+y)ax-x^3\}^2$$
$$=\{(2x-y)a^2+(x+y)ax-x^3\}\{(2x-y)a^2-(x+y)ax+x^3\},$$

the required quotient $=(2x-y)a^2+(x+y)ax-x^3$

8 1st expression $=x^4-a^2x^2-b^2x^2+a^2b^2$

$$=(x^2-a^2)(x^2-b^2)=(x+a)(x-a)(x+b)(x-b)$$

2nd expression $=x^4-\{(a+b)^2x^2+2ab(a+b)x-a^2b^2\}$

$$=x^4-\{(a+b)x-ab\}^2$$
$$=\{x^2+(a+b)x-ab\}\{x^2-(a+b)x+ab\}$$
$$=\{x^2+(a+b)x-ab\}(x-a)(x-b)\ ,$$

the required H C F $=(x-a)(x-b)$

$$=x^2-x(a+b)+ab$$

VII

1 $2(xy+yz+zx)=(x+y+z)^2-(x^2+y^2+z^2)$

$$=(8)^2-50=64-50=14\ ,$$

, $xy+yz+zx=7$

2 Putting x for $(2a-3b)$, y for $(3b-5c)$ and z for $(5c-2a)$, we get $x+y+z=0$,

and the left-hand expression

$$=x^2+y^2+z^2=(x+y+z)^2-2xy-2yz-2zx$$
$$=-2xy-2yz-2zx$$
$$=-2(2a-3b)(3b-5c)-2(3b-5c)(5c-2a)-2(5c-2a)(2a-3b)$$
$$=2(3b-5c)(3b-2a)+2(5c-2a)(5c-3b)+2(2a-3b(2a-5c)$$

3. The dividend $=(x+a)\{(x-a)a^2-ab(x+a)+b(x^2-xa+a^2)\}$

$$=(x+a)\{a^2(x-a)-abx-a^2b+bx^2-abx+a^2b\}$$
$$=(x+a)\{a^2(x-a)+bx(x-2a)\}\ ,$$

the quotient $=x+a$

4 (i) $6x^2+x-15=6x^2+10x-9x-15$

$$=2x(3x+5)-3(3x+5)=(3x+5)(2x-3)$$

(ii) $35(x-y)^2-41(x-y)+12$

$$=35(x-y)^2-20(x-y)-21(x-y)+12$$
$$=5(x-y)\{7(x-y)-4\}-3\{7(x-y)-4\}$$
$$=(7x-7y-4)(5x-5y-3)$$

(iii) $11x^2 - 54xy^2 + 63y^4$

$$= 11x^2 - 33xy^2 - 21xy^2 + 63y^4$$
$$= 11x(x - 3y^2) - 21y^2(x - 3y^2)$$
$$= (x - 3y^2)(11x - 21y^2)$$

5 $(x+y)(y+z)(z+x) = \{y^2 + y(x+z) + xz\}(z+x)$
$$= \{y(y+x+z) + xz\}(z+x)$$
$$= y(z+x)(y+x+z) + xz(z+x)$$
$$= y(z+x)(x+y+z) + xz(x+y+z) - xyz$$
$$= (x+y+z)(yz+xy+xz) - xyz$$

6 (a) $(x+y)(y+z)(z+x) = (x+y+z)(xy+xz+yz) - xyz$
$$= 0 \times (xy+xz+yz) - xyz = -xyz$$

(b) $x^3+y^3+z^3 = (x+y+z)(x^2+y^2+z^2-xy-xz-yz) + 3xyz$
$$= 0 \times (x^2+y^2+z^2-xy-xz-yz) + 3xyz = 3xyz$$

7 The given expression
$$= (a-b)\{c^2 + c(a+b) + ab\} + (b-c)\{a^2 + a(b+c) + bc\} + (c-a)\{b^2 + b(c+a) + ca\}$$
$$= \{c^2(a-b) + a^2(b-c) + b^2(c-a)\}$$
$$+ \{c(a^2-b^2) + a(b^2-c^2) + b(c^2-a^2)\}$$
$$+ \{ab(a-b) + bc(b-c) + ca(c-a)\}$$
$$= (a-b)(a-c)(b-c) + (a-b)(b-c)(c-a)$$
$$+ (a-b)(a-c)(b-c)$$
$$= 2(a-b)(a-c)(b-c) - (a-b)(b-c)(a-c)$$
$$= (a-b)(a-c)(b-c)$$

8 1st expression $= 35x^2 - 21x + 10x - 6$
$$= 7x(5x-3) + 2(5x-3)$$
$$= (5x-3)(7x+2),$$

2nd expression $= 40x^2 - 24x - 5x + 3$
$$= 8x(5x-3) - (5x-3)$$
$$= (8x-1)(5x-3),$$

reqd L C M $= (5x-3)(7x+2)(8x-1)$
$$= (7x+2)(40x^2 - 29x + 3)$$
$$= 280x^3 - 123x^2 - 37x + 6$$

VIII

1 $x^2+y^2+z^2=(x+y+z)^2-2(xy+yz+xz)$

$=(15)^2-2(85)=225-170=55$

2 Putting x for $(a+b-2c)$, y for $(b+c-2a)$ and z for $(c+a-2b)$, we get $x+y+z=0$,

and the left-hand expression

$=x^3+y^3+z^3$

$=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)+3xyz$

$=0\times(x^2+y^2+z^2-xy-xz-yz)+3xyz$

$=3xyz=3(a+b-2c)(b+c-2a)(c+a-2b)$

3 $(ad-bc)(ad+bc)=a^2d^2-b^2c^2$

$=a^2d^2+a^2c^2-a^2c^2-b^2c^2$

$=a^2(d^2+c^2)-c^2(a^2+b^2)$

$=a^2\cdot 1-c^2\cdot 1=a^2-c^2=(a+c)(a-c)$

4 1st expression $=3x^2-9x-2x+6=3x(x-3)-2(x-3)$

$=(x-3)(3x-2)$,

2nd expression $=2x^2-x-6x+3$

$=x(2x-1)-3(2x-1)=(x-3)(2x-1)$,

3rd expression $=6x^2-3x-4x+2$

$=3x(2x-1)-2(2x-1)$

$=(2x-1)(3x-2)$,

∴ the reqd L.C.M. $=(x-3)(3x-2)(2x-1)$

5 1st expression $=a^2x^3+a^5-2abx^3-2a^4b+b^2x^3+a^3b^2$

$=a^2(x^3+a^3)-2ab(x^3+a^3)+b^2(x^3+a^3)$

$=(x^3+a^3)(a^2-2ab+b^2)$

$=(x+a)(x^2-xa+a^2)(a-b)^2$,

2nd expression $=2a^2x^4-2b^2x^4-5a^4x^2+5a^2b^2x^2+3a^6-3a^4b^2$

$=2x^4(a^2-b^2)-5a^2x^2(a^2-b^2)+3a^4(a^2-b^2)$

$=(a^2-b^2)(2x^4-5a^2x^2+3a^4)$

$=(a-b)(a+b)(x^2-a^2)(2x^2-3a^2)$

$=(a-b)(a+b)(x+a)(x-a)(2x^2-3a^2)$,

The reqd H.C.F. $=(x+a)(a-b)$

6 The dividend

$$=\{(ax+by)+(ax-by)\}\{(ax+by)^2-(ax+by)(ax-by)+(ax-by)^2\}+\{(bx-ay)+(bx+ay)\}\{(bx-ay)^2-(bx-ay)(bx+ay)+(bx+ay)^2\}$$
$$=2ax\{2(a^2x^2+b^2y^2)-(a^2x^2-b^2y^2)\}+2bx\{2(b^2x^2+a^2y^2)-(b^2x^2-a^2y^2)\}$$
$$=2ax(a^2x^2+3b^2y^2)+2bx(b^2x^2+3a^2y^2)$$
$$=2x(a^3x^2+3ab^2y^2+b^3x^2+3a^2by^2)$$
$$=2x\{x^2(a^3+b^3)+3aby^2(a+b)\}$$
$$=2x(a+b)\{x^2(a^2+b^2-ab)+3aby^2\}$$
$$=2x(a+b)\{x^2(a+b)^2-3abx^2+3aby^2\}$$
$$=2x(a+b)\{x^2(a+b)^2-3ab(x^2-y^2)\},$$

The reqd quotient $=2x(a+b)$

7 (a) $x^2+\dfrac{1}{x^2}=\left(x+\dfrac{1}{x}\right)^2-2x\dfrac{1}{x}=a^2-2$

(b) $x^3+\dfrac{1}{x^3}=\left(x+\dfrac{1}{x}\right)^3-3x\dfrac{1}{x}\left(x+\dfrac{1}{x}\right)=a^3-3a$

(c) $x^4+\dfrac{1}{x^4}=\left(x^2+\dfrac{1}{x^2}\right)^2-2x^2\dfrac{1}{x^2}$
$$=(a^2-2))^2-2=a^4-4a^2+4-2=a^2-4a^2+2$$

8 The left-hand expression

$$=x^4+x^2(y^2+z^2)+y^2z^2+2x(x^2+yz)(y+z)+4x^2yz$$
$$=(x^4+2x^2yz+y^2z^2)+x^2(y^2+z^2+2yz)+2x(x^2+yz)(y+z)$$
$$=(x^2+yz)^2+\{x(y+z)\}^2+2(x^2+yz)(y+z)$$
$$=(x^2+yz+xy+xz)^2$$

IX

1 The left hand expression

$$=\{(a+b)^2-c^2\}\{(a-b)^2+c^2\}$$
$$=(a^2-b^2)^2+c^2\{(a+b)^2-(a-b)^2\}-c^4$$
$$=(a^2-b^2)^2+c^2(4ab)-c^4$$
$$=(a^2-b^2)^2+c^2(4ab-c^2)$$

2 $\left(x+\frac{2}{x}\right)^5=x^5+5x^4\left(\frac{2}{x}\right)+10x^3\left(\frac{2}{x}\right)^2$

$$+10x^2\left(\frac{2}{x}\right)^3+5x\left(\frac{2}{x}\right)^4+\left(\frac{2}{x}\right)^5$$

$$=x^5+10x^3+40x+\frac{80}{x}+\frac{80}{x^3}+\frac{32}{x^5}$$

3 Let $A=x^5+11x-12$ and $B=x^5+11x^3+54$,

$B-A=11x^3-11x+66=11(x^3-x+6)=11\,C$ (suppose),

$A+2C=x^5+2x^3+9x=x(x^4+2x^2+9)=x\,D$ (suppose),

$2D-3C=2x^4-3x^3+4x^2+3x$

$=x(2x^3-3x^2+4x+3)=x\,E$ (suppose),

$2E-C=3x^3-6x^2+9x=3x(x^2-2x+3)=3x\,F$ (suppose),

$C-2F=x^3-2x^2+3x=x(x^2-2x+3)$,

the reqd H C F $=x^2-2x+3$

4 $a(b+c)+bc\,)\,a^3(b^2-c^2)-a^2(b^3-c^3)+b^2c^2(b-c)$

$a^3(b^2-c^2)+a^2bc(b-c)$

$-a^2(b-c)(b^2+c^2+2bc)+b^2c^2(b-c)$

$-a^2(b-c)(b^2+c^2+2bc)-abc(b^2-c^2)$

$abc(b^2-c^2)+b^2c^2(b-c)$

$abc(b^2-c^2)+b^2c^2(b-c)$

$a^2(b-c)-a(b^2-c^2)+bc(b-c)$ is the Quotient

5 1st expression $=x^2-5xy+2xy-10y^2$

$=x(x-5y)+2y(x-5y)=(x+2y)(x-5y)$,

2nd expression $=x^2+7xy-5xy-35y^2$

$=x(x+7y)-5y(x+7y)=(x+7y)(x-5y)$,

3rd expression $=x^2-5xy-3xy+15y^2$

$=x(x-5y)-3y(x-5y)=(x-3y)(x-5y)$,

the reqd L C M $=(x-5y)(x+2y)(x+7y)(x-3y)$

6 The left-hand expression

$=(x^2-2ax+a^2)(b-c)+(x^2-2bx+b^2)(c-a)$

$+(x^2-2cx+c^2)(a-b)$

$=x^2\{(b-c)+(c-a)+(a-b)\}-2x\{a(b-c)+b(c-a)$

$+c(a-b)\}+a^2(b-c)+b^2(c-a)+c^2(a-b)$

$=x^2\times 0-2x\times 0+(a-b)(a-c)(b-c)=(a-b)(a-c)(b-c)$

7 $(a+b+c)^3-a^3-b^3-c^3$

$$=(a+b)^3+c^3+3c(a+b)(a+b+c)-a^3-b^3-c^3$$
$$=a^3+b^3+3ab(a+b)+3c(a+b)(a+b+c)-a^3-b^3$$
$$=3(a+b)\{ab+c(a+b+c)\}$$
$$=3(a+b)\{(ab+ac)+c(b+c)\}=3(a+b)(b+c)(a+c)$$

Putting a for $y+z-x$, b for $z+x-y$ and c for $x+y-z$, we get $a+b+c=x+y+z$

The left-hand expression

$$=(a+b+c)^3-a^3-b^3-c^3$$
$$=3(a+b)(b+c)(c+a)$$
$$=3\{(y+z-x)+(z+x-y)\}\{(z+x-y)+(x+y-z)\}$$
$$\{(x+y-z)+(y+z-x)\}$$
$$=3(2z)(2x)(2y)=24xyz$$

8 $a^3-b^3+c^3+3abc$

$$=a^3+(-b)^3+c^3-3a(-b)(c)$$
$$=(a-b+c)(a^2+b^2+c^2+ab-ac+bc)$$
$$=(\cdot4278-1\cdot2345+\cdot8067)(a^2+b^2+c^2+ab-ac+bc)$$
$$=(1\cdot2345-1\cdot2345)(a^2+b^2+c^2+ab-ac+bc)=0$$

X

1 The left-hand expression

$$=\{(1+a^2)+2a\}(1+c^2)-\{(1+c^2)+2c\}(1+a^2)$$
$$=(1+a^2)(1+c^2)+2a(1+c^2)-(1+c^2)(1+a^2)-2c(1+a^2)$$
$$=2a(1+c^2)-2c(1+a^2)=2(a+ac^2-c-a^2c)$$
$$=2\{(a-c)-ac(a-c)\}$$
$$=2(a-c)(1-ac)$$

2 $a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$

$$=2\{(a^4+2a^2b^2+b^4)-ab(a^2+ab+b^2)\}$$
$$=2[(a^2+b^2)^2-7\{(a+b)^2-ab\}]$$
$$=2[\{(a+b)^2-2ab\}^2-7(4-7)]$$
$$=2\{(4-14)^2-7(-3)\}$$
$$=2\{(-10)^2+21\}=2(100+21)=242$$

3 The dividend $= a - b - c - 3(b^{\frac{1}{3}} + c^{\frac{1}{3}})\{a^{\frac{2}{3}} - a^{\frac{1}{3}}(b^{\frac{1}{3}} + c^{\frac{1}{3}}) + b^{\frac{1}{3}}c^{\frac{1}{3}}\}$

$$= (a^{\frac{1}{3}})^3 - (b^{\frac{1}{3}})^3 - (c^{\frac{1}{3}})^3 - 3a^{\frac{2}{3}}(b^{\frac{1}{3}} + c^{\frac{1}{3}})$$

$$+ 3a^{\frac{1}{3}}(b^{\frac{1}{3}} + c^{\frac{1}{3}})^2 - 3b^{\frac{1}{3}}c^{\frac{1}{3}}(b^{\frac{1}{3}} + c^{\frac{1}{3}})$$

$$= (a^{\frac{1}{3}})^3 - \{(b^{\frac{1}{3}})^3 + (c^{\frac{1}{3}})^3 + 3b^{\frac{1}{3}}c^{\frac{1}{3}}(b^{\frac{1}{3}} + c^{\frac{1}{3}})\}$$

$$- 3a^{\frac{1}{3}}(b^{\frac{1}{3}} + c^{\frac{1}{3}})\{a^{\frac{1}{3}} - (b^{\frac{1}{3}} + c^{\frac{1}{3}})\}$$

$$= (a^{\frac{1}{3}})^3 - (b^{\frac{1}{3}} + c^{\frac{1}{3}})^3 - 3a^{\frac{1}{3}}(b^{\frac{1}{3}} + c^{\frac{1}{3}})$$

$$\{a^{\frac{1}{3}} - (b^{\frac{1}{3}} + c^{\frac{1}{3}})\}$$

$$= \{(a^{\frac{1}{3}}) - (b^{\frac{1}{3}} + c^{\frac{1}{3}})\}^3 = (a^{\frac{1}{3}} - b^{\frac{1}{3}} - c^{\frac{1}{3}})^3;$$

$\therefore$ the required quotient

$$= (a^{\frac{1}{3}} - b^{\frac{1}{3}} - c^{\frac{1}{3}})^2 = a^{\frac{2}{3}} + b^{\frac{2}{3}} + c^{\frac{2}{3}} - 2a^{\frac{1}{3}}b^{\frac{1}{3}} - 2a^{\frac{1}{3}}c^{\frac{1}{3}} + 2b^{\frac{1}{3}}c^{\frac{1}{3}}$$

4. The given expression

$$= 2(a^2 - b^2)(a^4 - a^2b^2 + b^4) - ab(a^2 - b^2)(2ab + 3a^2 - 3b^2)$$

$$= (a^2 - b^2)(2a^4 - 2a^2b^2 + 2b^4 - 2a^2b^2 - 3a^3b + 3ab^3)$$

$$= (a^2 - b^2)\{2(a^4 - 2a^2b^2 + b^4) - 3ab(a^2 - b^2)\}$$

$$= (a^2 - b^2)\{2(a^2 - b^2)^2 - 3ab(a^2 - b^2)\}$$

$$= (a^2 - b^2)(a^2 - b^2)(2a^2 - 2b^2 - 3ab)$$

$$= (a^2 - b^2)(a + b)(a - b)(2a^2 - 4ab + ab - 2b^2)$$

$$= (a^2 - b^2)(a + b)(a - b)(a - 2b)(2a + b)$$

5 (i) The given expression

$$= 6(a^4 - 2a^2b^2 + b^4) - 59ab(a^2 - b^2) + 68a^2b^2$$

$$= 6(a^2 - b^2)^2 - 59ab(a^2 - b^2) + 68a^2b^2$$

$$= 6(a^2 - b^2)^2 - 8ab(a^2 - b^2) - 51ab(a^2 - b^2) + 68a^2b^2$$

$$= 2(a^2 - b^2)\{3(a^2 - b^2) - 4ab\} - 17ab\{3(a^2 - b^2) - 4ab\}$$

$$= (3a^2 - 3b^2 - 4ab)(2a^2 - 2b^2 - 17ab)$$

$$= (3a^2 - 4ab - 3b^2)(2a^2 - 17ab - 2b^2).$$

(ii) The given expression

$$= 12(x^4 - 2x^2 + 1) - 37x(x^2 - 1) + 21x^2$$

$$= 12(x^2 - 1)^2 - 37x(x^2 - 1) + 21x^2$$

$$= 12(x^2 - 1)^2 - 9x(x^2 - 1) - 28x(x^2 - 1) + 21x^2$$

$$= 3(x^2 - 1)\{4(x^2 - 1) - 3x\} - 7x\{4(x^2 - 1) - 3x\}$$

$$= \{4(x^2 - 1) - 3x\}\{3(x^2 - 1) - 7x\}$$

$$= (4x^2 - 3x - 4)(3x^2 - 7x - 3)$$

(iii) The given expression

$$=ab(x^4+2x^2+1)+(ac+b^2)x(x^2+1)+bcx^2$$
$$=ab(x^2+1)^2+(ac+b^2)x(x^2+1)+bcx^2$$
$$=ab(x^2+1)^2+acx(x^2+1)+b^2x(x^2+1)+bcx^2$$
$$=a(x^2+1)\{b(x^2+1)+cx\}+bx\{b(x^2+1)+cx\}$$
$$=\{b(x^2+1)+cx\}\{a(x^2+1)+bx\}$$
$$=(bx^2+cx+b)(ax^2+bx+a)$$

6 Putting x for $a+b-2c$, y for $c+a-2b$ and z for $b+c-2a$, we get $x-y=3(b-c)$, $y-z=3(a-b)$ and $z-x=3(c-a)$,

$$3\{(b-c)(b+c-2a)^2+(c-a)(c+a-2b)^2+(a-b)(a+b-2c)^2\}$$
$$=z^2(x-y)+y^2(z-x)+x^2(y-z)$$
$$=(x-y)(x-z)(y-z)$$
$$=3(b-c)\ 3(a-c)\ 3(a-b)$$
$$=3\times 9(a-b)(a-c)(b-c)\ ,$$

the left-hand expression $=9(a-b)(a-c)(b-c)$

7 $a^3+b^3+c^3-3abc$

$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=\tfrac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$
$$=\tfrac{1}{2}(2658+2664+2678)\{(2658-2664)^2+(2664-2678)^2+(2678-2658)^2\}$$
$$=\tfrac{1}{2}(8000)\{(-6)^2+(-14)^2+(20)^2\}$$
$$=4000(36+196+400)=4000\times 632=2528000$$

8 $x^3+y^3+z^3-3xyz$

$$=\tfrac{1}{2}(x+y+z)\{(x-y)^2+(y-z)^2+(z-x)^2\}$$
$$=\tfrac{1}{2}(a+b+c)\{(2b-2a)^2+(2c-2b)^2+(2a-2c)^2\}$$
$$=\tfrac{1}{2}(a+b+c)\{4(a-b)^2+4(b-c)^2+4(c-a)^2\}$$
$$=4\ \tfrac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$
$$=4(a^3+b^3+c^3-3abc)$$

Exercise 56

1 $\dfrac{2a^2b^3}{4a^2b^4}=\dfrac{2a^2b^3}{2a^2b^3\ 2b}=\dfrac{1}{2b}$ 2 $\dfrac{6x^2y^3}{8xy^4}=\dfrac{2xy^3\ 3x}{2xy^3\ 4y}=\dfrac{3x}{4y}$

3 $\dfrac{4a^2xy^2}{10ax^2y^2}=\dfrac{2axy^2\,2a}{2axy^2\,5x}=\dfrac{2a}{5x}$ 4 $\dfrac{15x^3y^2z^4}{25x^2y^5z^3}=\dfrac{5x^2y^2z^3\,3xz}{5x^2y^2z^3\,5y^3}=\dfrac{3xz}{5y^3}$

5 $\dfrac{18a^2bc^4d^5}{27a^3b^2c^4d^4}=\dfrac{9a^2bc^4d^4\,2d}{9a^2bc^4d^4\,3ab}=\dfrac{2d}{3ab}$

6 $\dfrac{16x^2a^4y^3z^5}{40a^3z^4x^3y^4}=\dfrac{8x^2a^3y^3z^4\,2az}{8x^2a^3y^3z^4\,5xy}=\dfrac{2az}{5xy}$

7 $\dfrac{70a^2b^3c^4d^7}{105c^4d^2a^3b^3}=\dfrac{35a^2b^3c^4d^2\,2d^5}{35a^2b^3c^4d^2\,5a}=\dfrac{2d^5}{5a}$

8 $\dfrac{39m^3n^5p^3q^6}{65p^2m^4n^4q^6}=\dfrac{13m^3n^4p^2q^6\,3npq}{13m^3n^4p^2q^6\,5m}=\dfrac{3npq}{5m}$

9 $\dfrac{x^2-a^2}{x^2+ax}=\dfrac{(x-a)(x+a)}{x(x+a)}=\dfrac{x-a}{x}$

10 $\dfrac{x^2-3x}{9x-x^3}=\dfrac{x(x-3)}{x(9-x^2)}=\dfrac{x-3}{(3-x)(3+x)}=\dfrac{-(3-x)}{(3-x)(3+x)}=-\dfrac{1}{3+x}$

11 $\dfrac{4x^2-9a^2}{4x^2+6ax}=\dfrac{(2x+3a)(2x-3a)}{2x(2x+3a)}=\dfrac{2x-3a}{2x}$

12 $\dfrac{3a^2-12ab}{48b^2-3a^2}=\dfrac{3a(a-4b)}{3(16b^2-a^2)}=\dfrac{a(a-4b)}{(4b-a)(4b+a)}$

$$=\frac{-a(4b-a)}{(4b-a)(4b+a)}=-\frac{a}{4b+a}$$

13 $\dfrac{3ax-12a^2}{x^2-16a^2}=\dfrac{3a(x-4a)}{(x-4a)(x+4a)}=\dfrac{3a}{x+4a}$

14 $\dfrac{2x^4-4a^2x^2}{x^4-4a^2x^2+4a^4}=\dfrac{2x^2(x^2-2a^2)}{(x^2-2a^2)^2}=\dfrac{2x^2}{x^2-2a^2}$

15 $\dfrac{4x^2+8x}{x^2+5x+6}=\dfrac{4x(x+2)}{(x+2)(x+3)}=\dfrac{4x}{x+3}$

16 $\dfrac{x^2+2x-8}{x^2+x-12}=\dfrac{(x+4)(x-2)}{(x+4)(x-3)}=\dfrac{x-2}{x-3}$

17 $\dfrac{x^2+2x-15}{x^2+9x+20}=\dfrac{(x+5)(x-3)}{(x+5)(x+4)}=\dfrac{x-3}{x+4}$

18 $\dfrac{a^2-3ab-4b^2}{a^2-4ab-5b^2}=\dfrac{(a-4b)(a+b)}{(a-5b)(a+b)}=\dfrac{a-4b}{a-5b}$

19 $\frac{a^4-a^3b+a^2b^2}{a^3+b^3}=\frac{a^2(a^2-ab+b^2)}{(a+b)(a^2-ab+b^2)}=\frac{a^2}{a+b}$

20 $\frac{1-7x+12x^2}{1-8x+15x^2}=\frac{(1-3x)(1-4x)}{(1-3x)(1-5x)}=\frac{1-4x}{1-5x}$

21 $\frac{x^2-6xy+5y^2}{x^2+2xy-35y^2}=\frac{(x-5y)(x-y)}{(x-5y)(x+7y)}=\frac{x-y}{x+7}$

22 $\frac{1-9a^2+14a^4}{1-4a^2-21a^4}=\frac{(1-7a^2)(1-2a^2)}{(1-7a^2)(1+3a^2)}=\frac{1-2a^2}{1+3a^2}$

23 $\frac{x^4-8x^2-65}{x^4+x^2-20}=\frac{(x^2-13)(x^2+5)}{(x^2-4)(x^2+5)}=\frac{(x^2-13)}{(x^2-4)}$

24 $\frac{3a^3x+9a^2x^2+27ax^3}{a^3-27x^3}=\frac{3ax(a^2+3ax+9x^2)}{(a-3x)(a^2+3ax+9x^2)}=\frac{3ax}{a-3x}$

25 $\frac{2x^2-x-6}{3x^2-2x-8}=\frac{2x^2-4x+3x-6}{3x^2-6x+4x-8}=\frac{(x-2)(2x+3)}{(x-2)(3x+4)}=\frac{2x+3}{3x+4}$

26 $\frac{3x^2-5ax+2a^2}{3x^2+ax-2a^2}=\frac{3x^2-3ax-2ax+2a^2}{3x^2+3ax-2ax-2a^2}=\frac{(x-a)(3x-2a)}{(x+a)(3x-2a)}$

$=\frac{x-a}{x+a}$

27 $\frac{3x^2+16ax+5a^2}{3x^2+22ax+7a^2}=\frac{3x^2+15ax+ax+5a^2}{3x^2+21ax+ax+7a^2}$

$=\frac{(x+5a)(3x+a)}{(x+7a)(3x+a)}=\frac{x+5a}{x+7a}$

28 $\frac{6x^2-7x-20}{9x^2+6x-8}=\frac{6x^2-15x+8x-20}{9x^2+12x-6x-8}=\frac{(2x-5)(3x+4)}{(3x+4)(3x-2)}=\frac{2x-5}{3x-2}$

29 $\frac{2x^2+3ax-20a^2}{3x^2+5ax-28a^2}=\frac{2x^2+8ax-5ax-20a^2}{3x^2+12ax-7ax-28a^2}$

$=\frac{(x+4a)(2x-5a)}{(x+4a)(3x-7a)}=\frac{2x-5a}{3x-7a}$

30 $\frac{10-17ax+3a^2x^2}{5-26ax+5a^2x^2}=\frac{10-2ax-15ax+3a^2x^2}{5-ax-25ax+5a^2x^2}$

$=\frac{(5-ax)(2-3ax)}{(5-ax)(1-5ax)}=\frac{2-3ax}{1-5ax}$

31 $\frac{x^2-(a-b)x-ab}{x^3+bx^2+ax+ab}=\frac{(x-a)(x+b)}{(x+b)(x^2+a)}=\frac{x-a}{x^2+a}$

32 $$\frac{6ac+10bc+9ax+15bx}{6c^2+9cx-2c-3x}=\frac{2c(3a+5b)+3x(3a+5b)}{3c(2c+3x)-(2c+3x)}$$

$$=\frac{(3a+5b)(2c+3x)}{(2c+3x)(3c-1)}=\frac{3a+5b}{3c-1}$$

33 $$\frac{8bx+12ab+6xy+9ay}{12bx+8ab+9xy+6ay}=\frac{4b(2x+3a)+3y(2x+3a)}{4b(3x+2a)+3y(3x+2a)}$$

$$=\frac{(2x+3a)(4b+3y)}{(3x+2a)(4b+3y)}=\frac{2x+3a}{3x+2a}$$

34 $$\frac{2a^2+ab-b^2}{a^3+a^2b-a-b}=\frac{2a^2+2ab-ab-b^2}{a^2(a+b)-(a+b)}$$

$$=\frac{(a+b)(2a-b)}{(a+b)(a^2-1)}=\frac{2a-b}{a^2-1}$$

35 $$\frac{x^3+4x^2+x-6}{x^2+x-2}=\frac{x^3+3x^2+x^2+3x-2x-6}{x^2+x-2}$$

$$=\frac{x^2(x+3)+x(x+3)-2(x+3)}{x^2+x-2}$$

$$=\frac{(x+3)(x^2+x-2)}{x^2+x-2}=x+3$$

36 $$\frac{a^2-b^2-2bc-c^2}{a^2+2ab+b^2-c^2}=\frac{a^2-(b^2+2bc+c^2)}{(a^2+2ab+b^2)-c^2}$$

$$=\frac{a^2-(b+c)^2}{(a+b)^2-c^2}$$

$$=\frac{(a+b+c)(a-b-c)}{(a+b+c)(a+b-c)}=\frac{a-b-c}{a+b-c}$$

37 $$\frac{x^3-7x+6}{x^3+2x^2-13x+10}=\frac{x^3-x-6x+6}{x^3-x^2+3x^2-3x-10x+10}$$

$$=\frac{x(x^2-1)-6(x-1)}{x^2(x-1)+3x(x-1)-10(x-1)}$$

$$=\frac{(x-1)(x^2+x-6)}{(x-1)(x^2+3x-10)}=\frac{x^2+3x-2x-6}{x^2+5x-2x-10}$$

$$=\frac{(x+3)(x-2)}{(x+5)(x-2)}=\frac{x+3}{x+5}$$

38 $$\frac{a^3+2a^2b-2ab^2+3b^3}{a^3-5a^2b+5ab^2-4b^3}=\frac{(a^3+b^3)+(2a^2b-2ab^2+2b^3)}{(a^3+b^3)-(5a^2b-5ab^2+5b^3)}$$

$$=\frac{(a+b)(a^2-ab+b^2)+2b(a^2-ab+b^2)}{(a+b)(a^2-ab+b^2)-5b(a^2-ab+b^2)}$$

$$=\frac{(a^2-ab+b^2)(a+b+2b)}{(a^2-ab+b^2)(a+b-5b)}=\frac{a+3b}{a-4b}$$

39 $$\frac{x^4+(2b^2-a^2)x^2+b^4}{x^4+2ax^3+a^2x^2-b^4}=\frac{x^4+2b^2x^2+b^4-a^2x^2}{x^2(x^2+2ax+a^2)-b^4}$$

$$=\frac{(x^2+b^2)^2-(ax)^2}{x^2(x+a)^2-b^4}=\frac{(x^2+b^2+ax)(x^2+b^2-ax}{(x^2+ax+b^2)(x^2+ax-b^2)}$$

$$=\frac{x^2+ax+b^2}{x^2-b^2+ax}$$

40 $$\frac{3x^3+4x^2y-7xy^2+2y^3}{2x^3+9x^2y+8xy^2-5y^3}$$

$$=\frac{3x^3-2x^2y+6x^2y^2-4xy^2-3xy^2+2y^3}{2x^3+5x^2y+4x^2y+10xy^2-2xy^2-5y^3}$$

$$=\frac{x^2(3x-2y)+2xy(3x-2y)-y^2(3x-2y)}{x^2(2x+5y)+2xy(2x+5y)-y^2(2x+5y)}$$

$$=\frac{(3x-2y)(x^2+2xy-y^2)}{(2x+5y)(x^2+2xy-y^2)}=\frac{3x-2y}{2x+5y}$$

41 $$\frac{1+3x-x^3-3x^4}{1-x+2x^2+x^3+3x^4}$$

$$=\frac{(1+3x)-x^3(1+3x)}{1+x+x^2-2x-2x^2-2x^3+3x^2+3x^3+3x^4}$$

$$=\frac{(1+3x)(1-x^3)}{(1+x+x^2)(1-2x+3x^2)}=\frac{(1+3x)(1-x)}{1-2x+3x^2}=\frac{1+2x-3x^2}{1-2x+3x^2}$$

42 $$\frac{x^4-x^3-x+1}{x^4-2x^3-x^2-2x+1}=\frac{x^3(x-1)-(x-1)}{x^4+x^3+x^2-3x^3-3x^2-3x+x^2+x+1}$$

$$=\frac{(x-1)(x^3-1)}{x^2(x^2+x+1)-3x(x^2+x+1)+(x^2+x+1)}$$

$$=\frac{(x-1)(x-1)(x^2+x+1)}{(x^2+x+1)(x^2-3x+1)}=\frac{(x-1)^2}{x^2-3x+1}$$

43 The H C F of the numerator and denominator of the given fraction can be found as follows —

$$\begin{array}{r}
x^4-9x^2+30x-25 \\
-(x^4+x^2 \qquad\quad +25) \\ \hline
-10\;|\;-10x^2+30x-50 \\ \hline
x^2-3x+5
\end{array}$$

$$\begin{array}{r}
x^4 \qquad +x^2 \;+25 \\
-(x^2-3x+5)x^2 \qquad \\ \hline
3x^3-4x^2+25 \\
-(x^2-3x+5)\,3x \\ \hline
5x^2-15x+25 \\
-(x^2-3x+5)\,5 \\ \hline
\end{array}$$

Thus the H C F required $=x^2-3x+5$

Hence, the required result $=\dfrac{(x^4+x^2+25)\div(x^2-3x+5)}{(x^4-9x^2+30x-25)\div(x^2-3x+5)}$

$$=\frac{x^2+3x+5}{x^2+3x-5}.$$

44

$$\begin{array}{r}
4x^3-12ax^2+19a^2x-15a^3 \\
-(2x^3+3ax^2+5a^2x-21a^3)\,2 \\ \hline
-9a\,|\,-18ax^2+9a^2x+27a^3 \\ \hline
2x^2-ax-3a^2
\end{array}$$

$$\begin{array}{r}
2x^3+3ax^2+5a^2x-21a^3 \\
-(2x^2-ax-3a^2)x \qquad\qquad \\ \hline
4ax^2+8a^2x-21a^3 \\
(2x^2-ax-3a^2)\,2a \\ \hline
5a^2\;|\;10a^2x-15a^3 \\ \hline
2x-3a
\end{array}$$

$$\begin{array}{r}
2x^2-ax-3a^2 \\
(2x-3a)x \qquad \\ \hline
2ax-3a^2 \\
(2x-3a)\,a \\ \hline
\end{array}$$

Thus the H C F of the numerator and denominator $=2x-3a$.

The required result $=\dfrac{(2x^3+3ax^2+5a^2x-21a^3)\div(2x-3a)}{(4x^3-12ax^2+19a^2x-15a^3)\div(2x-3a)}$

$$=\frac{x^2+3ax+7a^2}{2x^2-3ax+5a^2}.$$

45 $$\frac{2x^4+x^3-3x^2+2x+3}{3x^4+x^3-4x^2+3x+4}=\frac{(2x^4+3x^3-2x^3-3x^2)+(2x+3)}{(3x^4+4x^3-3x^3-4x^2)+(3x+4)}$$
$$=\frac{(2x+3)(x^3-x^2)+(2x+3)}{(3x+4)(x^3-x^2)+(3x+4)}$$
$$=\frac{(2x+3)(x^3-x^2+1)}{(3x+4)(x^3-x^2+1)}=\frac{2x+3}{3x+4}$$

46 $$\frac{9x^3-7a^2x-2a^3}{9x^3+6ax^2-5a^2x-2a^3}=\frac{9x^3-9a^2x+2a^2x-2a^3}{(9x^3+6ax^2+a^2x)-(6a^2x+2a^3)}$$
$$=\frac{9x(x^2-a^2)+2a^2(x-a)}{x(3x+a)^2-2a^2(3x+a)}$$
$$=\frac{(x-a)(9x^2+9ax+2a^2)}{(3x+a)(3x^2+ax-2a^2)}$$
$$=\frac{(x-a)(9x^2+3ax+6ax+2a^2)}{(3x+a)(3x^2+3ax-2ax-2a^2)}$$
$$=\frac{(x-a)(3x+a)(3x+2a)}{(3x+a)(x+a)(3x-2a)}$$
$$=\frac{(x-a)(3x+2a)}{(x+a)(3x-2a)}=\frac{3x^2-ax-2a^2}{3x^2+ax-2a^2}$$

47 $$\begin{array}{r|l} 2 & 2a^3-16a^2b+44ab^2-42b^3 \\ \hline & a^3-8a^2b+22ab^2-21b^3 \\ 3 & 3a^3+6a^2b-24ab^2-63b^3 \\ \hline & a^3+2a^2b-8ab^2-21b^3 \\ & -(a^3-8a^2b+22ab^2-21b^3) \\ \hline 10ab & 10a^2b-30ab^2 \\ \hline & a-3b \end{array} \qquad \begin{array}{r} a^3-8a^2b+22ab^2-21b^3 \\ (a-3b)\,a^2 \quad\quad\quad\quad \\ \hline -5a^2b+22ab^2-21b^3 \\ -(a-3b)(-5ab) \\ \hline 7ab^2-21b^3 \\ -(a-3b)\,7b \\ \hline \end{array}$$

The H C F of the numerator and denominator $=a-3b$

The required result

$$=\frac{(2a^3-16a^2b+44ab^2-42b^3)\div(a-3b)}{(3a^3+6a^2b-24ab^2-63b^3)\div(a-3b)}=\frac{2(a^2-5ab+7b^2)}{3(a^2+5ab+7b^2)}$$

48 $$\begin{array}{r|l} 3 & 9x^4+30x^3+12x^2-6x-45 \\ \hline & 3x^4+10x^3+4x^2-2x-15 \\ 4 & 8x^4+28x^3+16x^2-4x-48 \\ \hline & 2x^4+7x^3+4x^2-x-12 \\ & 3 \\ \hline & 6x^4+21x^3+12x^2-3x-36 \\ & -(3x^4+10x^3+4x^2-2x-15)\,2 \\ \hline & x^3+4x^2-x-6 \end{array}$$

$$\begin{array}{l} 3x^4+10x^3+4x^2-2x-15 \\ -(x^3+4x^2+x-6)\,3x \\ \hline -2x^3+x^2+16x-15 \\ -(x^3+4x^2+x-6)\times(-2) \\ \hline 9\,|\,9x^2+18x-27 \\ \quad x^2+2x-3 \end{array} \qquad \begin{array}{l} x^3+4x^2+x-6 \\ -(x^2+2x-3)\,x \\ \hline 2x^2+4x-6 \\ -(x^2+2x-3)\,2 \\ \hline \end{array}$$

Thus the H C F of the numerator and denominator $=x^2+2x-3$

The required result

$$=\frac{3(3x^4+10x^3+4x^2-2x-15)\div(x^2+2x-3)}{4(2x^4+7x^3+4x^2-x-12)\div(x^2+2x-3)}=\frac{3(3x^2+4x+5)}{4(2x^2+3x+4)}$$

49
$$\begin{array}{l} a\,|\,4a^6-6a^4b+3a^3b^3-ab^4 \\ \quad 4a^4-6a^3b+3ab^3-b^4 \\ a^2\,|\,6a^6-9a^5b+a^4b^2+3a^3b^3-a^2b^4 \\ \quad 6a^4-9a^3b+a^2b^2+3ab^3-b^4 \\ \quad 2 \\ \hline 12a^4-18a^3b+2a^2b^2+6ab^3-2b^4 \\ -(4a^4-6a^3b+3ab^3-b^4)\,3 \\ \hline b^2\,|\,2a^2b^2-3ab^3+b^4 \\ \quad 2a^2-3ab+b^2 \end{array}$$

$$\begin{array}{l} 4a^4-6a^3b+3ab^3-b^4 \\ -(2a^2-3ab+b^2)\,2a^2 \\ \hline -2a^2b^2+3ab^3-b^4 \\ -(2a^2-3ab+b^2)\times(-b^2) \\ \hline \end{array}$$

Thus the H C F of the numerator and denominator
$=a(2a^2-3ab+b^2)$,

the required result

$$=\frac{a^2(6a^4-9a^3b+a^2b^2+3ab^3-b^4)\div a(2a^2-3ab+b^2)}{a(4a^4-6a^3b+3ab^3-b^4)\div a(2a^2-3ab+b^2)}$$

$$=\frac{a(3a^2-b^2)}{2a^2-b^2}$$

50
$$\begin{array}{l} 4x\,|\,24x^5+16x^4y-28x^3y^2-24x^2y^3-12xy^4 \\ \quad 6x^4+4x^3y-7x^2y^2-6xy^3-3y^4 \end{array}$$

$$
\begin{array}{l}
5y\ \underline{|\ 45x^4y+30x^3y^2-15x^2y^3-20xy^4-10y^5} \\
\qquad 9x^4+6x^3y-3x^2y^2-4xy^3-2y^4 \\
\qquad 2 \\
\hline
\qquad 18x^4+12x^3y-6x^2y^2-8xy^3-4y^4 \\
\qquad -(6x^4+4x^3y-7x^2y^2-6xy^3-3y^4)\,3 \\
\hline
\qquad\qquad 5y^2\ \underline{|\ 15x^2y^2+10xy^3+5y^4} \\
\qquad\qquad\qquad 3x^2+2xy+y^2 \\
\qquad 6x^4+4x^3y-7x^2y^2-6xy^3-3y^4 \\
\qquad -(3x^2+2xy+y^2)\,2x^2 \\
\hline
\qquad\qquad -9x^2y^2-6xy^3-3y^4 \\
\qquad\qquad \underline{-(3x^2+2xy+y^2)\times(-3y^2)}
\end{array}
$$

Thus the H C F of the numerator and denominator

$$=3x^2+2xy+y^2,$$

the reqd result

$$=\frac{4x(6x^4+4x^3y-7x^2y^2-6xy^3-3y^4)-(3x^2+2xy+y^2)}{5y(9x^4+6x^3y-3x^2y^2-4xy^3-2y^4)-(3x^2+2xy+y^2)}$$

$$=\frac{4x(2x^2-3y^2)}{5y(3x^2-2y^2)}$$

Exercise 57

1 The L C M of the denominators $=4bdf$

$$\frac{a}{2b}=\frac{a\times 2df}{2b\times 2df}=\frac{2adf}{4bdf},\qquad \frac{3c}{4d}=\frac{3c\times bf}{4d\times bf}=\frac{3bcf}{4bdf},$$

$$\text{and } \frac{c}{f}=\frac{c\times 4bd}{f\times 4bd}=\frac{4bdc}{4bdf}$$

2 The L C M of the denominators $=12abc$,

$$\frac{x^2}{2bc}=\frac{x^2\times 6a}{2bc\times 6a}=\frac{6ax^2}{12abc},\qquad \frac{y^2}{3ca}=\frac{y^2\times 4b}{3ca\times 4b}=\frac{4by^2}{12abc},$$

$$\text{and } \frac{z^2}{4ab}=\frac{z^2\times 3c}{4ab\times 3c}=\frac{3cz^2}{12abc}$$

3 The L C M of the denominators $=60x^3y^2$,

$$\frac{ab}{4xy^2}=\frac{ab\times 15x^2}{4xy^2\times 15x^2}=\frac{15abx^2}{60x^3y^2},$$

$$\frac{bc}{6x^2y}=\frac{bc\times 10xy}{6x^2y\times 10xy}=\frac{10bcxy}{60x^3y^2},$$

$$\frac{ca}{10x^3}=\frac{ca\times 6y^2}{10x^3\times 6y^2}=\frac{6acy^2}{60x^3y^2}$$

4 The L C M of the denominators $= a(a^2-b^2)$,

$$\frac{a}{a-b} = \frac{a \times a(a+b)}{(a-b) \times a(a+b)} = \frac{a^2(a+b)}{a(a^2-b^2)},$$

$$\frac{a}{a+b} = \frac{a \times a(a-b)}{(a+b) \times a(a-b)} = \frac{a^2(a-b)}{a(a^2-b^2)},$$

and $$\frac{c}{a(a+b)} = \frac{c \times (a-b)}{a(a+b) \times (a-b)} = \frac{c(a-b)}{a(a^2-b^2)}$$

5 The L C M of the denominators $= a(a^2-4b^2)$,

$$\frac{x^2}{a^2+2ab} = \frac{x^2 \times (a-2b)}{a(a+2b) \times (a-2b)} = \frac{x^2(a-2b)}{a(a^2-4b^2)},$$

and $$\frac{y^2}{a-2b} = \frac{y^2 \times a(a+2b)}{(a-2b) \times a(a+2b)} = \frac{ay^2(a+2b)}{a(a^2-4b^2)}$$

6 The L C M of the denominators $= a(a-b)$,

$$\frac{2a}{a-b} = \frac{2a \times a}{(a-b) \times a} = \frac{2a^2}{a(a-b)},$$

and $$\frac{a-c}{ab-a^2} = \frac{(a-c) \times (-1)}{a(b-a) \times (-1)} = \frac{c-a}{a(a-b)}$$

7 The L C M of the denominators $= a^2-b^2$,

$$\frac{2a}{a-b} = \frac{2a(a+b)}{(a-b) \times (a+b)} = \frac{2a(a+b)}{a^2-b^2},$$

$$\frac{3b}{b-a} = \frac{3b \times \{-(b+a)\}}{(b-a) \times -(b+a)} = \frac{-3b(a+b)}{a^2-b^2},$$

and $$\frac{4c}{a+b} = \frac{4c \times (a-b)}{(a+b) \times (a-b)} = \frac{4c(a-b)}{a^2-b^2}.$$

8 The L C M of the denominators $= a^2b^2c^2(a^2-x^2)$,

$$\frac{2x}{a^2(a+x)} = \frac{2x \times b^2c^2(a-x)}{a^2(a+x) \times b^2c^2(a-x)} = \frac{2b^2c^2x(a-x)}{a^2b^2c^2(a^2-x^2)},$$

$$\frac{3y}{b^2(a-x)} = \frac{3y \times a^2c^2(a+x)}{b^2(a-x) \times a^2c^2(a+x)} = \frac{3a^2c^2y(a+x)}{a^2b^2c^2(a^2-x^2)},$$

and $$\frac{4z}{c^2(a^2-x^2)} = \frac{4z \times a^2b^2}{c^2(a^2-x^2) \times a^2b^2} = \frac{4a^2b^2z}{a^2b^2c^2(a^2-x^2)}$$

9 The L C M of the denominators $= xy(4x^2-9y^2)$,

$$\frac{a^2}{2xy-3y^2} = \frac{a^2 \times x(2x+3y)}{y(2x-3y) \times x(2x+3y)} = \frac{a^2x(2x+3y)}{xy(4x^2-9y^2)},$$

$$\frac{b^2}{2x^2+3xy}=\frac{b^2\times y(2x-3y)}{x(2x+3y)\times y(2x-3y)}=\frac{b^2y(2x-3y)}{xy(4x^2-9y^2)},$$

$$\text{and } \frac{c^2}{4x^2y-9xy^3}=\frac{c^2}{xy(4x^2-9y^2)}=\frac{c^2}{xy(4x^2-9y^2)}$$

10 The L C M of the denominators

$$=(x^2+x+1)(x^2-x+1)=x^4+x^2+1,$$

$$\frac{a^2}{x^2+x+1}=\frac{a^2\times(x^2-x+1)}{(x^2+x+1)\times(x^2-x+1)}=\frac{a^2(x^2-x+1)}{x^4+x^2+1},$$

$$\text{and } \frac{b^2}{x^2-x+1}=\frac{b^2\times(x^2+x+1)}{(x^2-x+1)\times(x^2+x+1)}=\frac{b^2(x^2+x+1)}{x^4+x^2+1}$$

11 $x^2-x-2=(x-2)(x+1)$, and $x^2+x-6=(x-2)(x+3)$

Thus the L C M of the denominators

$$=(x-2)(x+1)(x+3)=x^3+2x^2-5x-6,$$

$$\frac{3}{x^2-x-2}=\frac{3(x+3)}{(x^2-x-2)\times(x+3)}=\frac{3(x+3)}{x^3+2x^2-5x-6},$$

$$\text{and } \frac{4}{x^2+x-6}=\frac{4(x+1)}{(x^2+x-6)\times(x+1)}=\frac{4(x+1)}{x^3+2x^2-5x-6}$$

12 $a^3+8b^3=(a+2b)(a^2-2ab+4b^2)$

The L C M of the denominators

$$=a(a^3+8b^3)=a^4+8ab^3,$$

$$\frac{a-2b}{a(a^2-2ab+4b^2)}=\frac{(a-2b)\times(a+2b)}{a(a^2-2ab+4b^2)\times(a+2b)}$$

$$=\frac{a^2-4b^2}{a^4+8ab^3},$$

$$\text{and } \frac{bc}{a^3+8b^3}=\frac{bc\,a}{(a^3+8b^3)\times a}=\frac{abc}{a^4+8ab^3}$$

13 $a^3-27b^3=(a-3b)(a^2+3ab+9b^2)$,

the L C M of the denominators $=a^3-27b^3$,

$$\frac{a}{a-3b}=\frac{a(a^2+3ab+9b^2)}{(a-3b)\times(a^2+3ab+9b^2)}=\frac{a(a^2+3ab+9b^2)}{a^3-27b^3},$$

$$\frac{b}{a^2+3ab+9b^2}=\frac{b(a-3b)}{(a^2+3ab+9b^2)\times(a-3b)}=\frac{b(a-3b)}{a^3-27b^3},$$

$$\text{and } \frac{c}{a^3-27b^3}=\frac{c}{a^3-27b^3}$$

14 $a^2+b^2-c^2-2ab=a^2+b^2-2ab-c^2$

$$=(a-b)^2-c^2=(a-b+c)(a-b-c),$$

the L C M of the denominators $=ab(a^2+b^2-c^2-2ab)$,

$$\frac{a}{b(a-b-c)}=\frac{a\times a(a-b+c)}{b(a-b-c)\times a(a-b+c)}=\frac{a^2(a-b+c)}{ab(a^2+b^2-c^2-2ab)},$$

$$\frac{b}{a(a-b+c)}=\frac{b\times b(a-b-c)}{a(a-b+c)\times b(a-b-c)}=\frac{b^2(a-b-c)}{ab(a^2+b^2-c^2-2ab)},$$

and $$\frac{c}{a^2+b^2-c^2-2ab}=\frac{c\times ab}{(a^2+b^2-c^2-2ab)\times ab}=\frac{abc}{ab(a^2+b^2-c^2-2ab)}$$

15 The L C M of the denominators $=(a-b)(b-c)(c-a)$,

$$\frac{c-a}{(a-b)(b-c)}=\frac{(c-a)\times(c-a)}{(a-b)(b-c)\times(c-a)}=\frac{(c-a)^2}{(a-b)(b-c)(c-a)},$$

$$\frac{b-a}{(a-c)(b-c)}=\frac{(b-a)\times\{-(a-b)\}}{(a-c)(b-c)\times\{-(a-b)\}}=\frac{(a-b)^2}{(a-b)(b-c)(c-a)},$$

and $$\frac{b-c}{(c-a)(a-b)}=\frac{(b-c)\times(b-c)}{(c-a)(a-b)\times(b-c)}=\frac{(b-c)^2}{(a-b)(b-c)(c-a)}$$

Exercise 58

1 The given expression $=\frac{(a+b)b}{ab}+\frac{(a-b)a}{ab}$

$$=\frac{b(a+b)+a(a-b)}{ab}=\frac{a^2+b^2}{ab}$$

2 The given expression $=\frac{z(x-y)}{xyz}+\frac{x(y-z)}{xyz}+\frac{y(z-x)}{xyz}$

$$=\frac{z(x-y)+x(y-z)+y(z-x)}{xyz}=\frac{0}{xyz}=0$$

3 The given expression $=\frac{a}{a-x}+\frac{x(-1)}{(x-a)(-1)}=\frac{a-x}{a-x}=1$

4 The given expression $=\frac{(a+b)(a+b)}{(a-b)(a+b)}-\frac{(a-b)(a-b)}{(a+b)(a-b)}$

$$=\frac{(a+b)^2-(a-b)^2}{a^2-b^2}=\frac{4ab}{a^2-b^2}$$

5 The given expression $= \dfrac{2(a^2+b^2)}{2(a^2-b^2)} - \dfrac{(a-b)(a-b)}{2(a+b)(a-b)}$

$$= \frac{2(a^2+b^2)-(a-b)^2}{2(a^2-b^2)} = \frac{(a+b)^2}{2(a^2-b^2)}$$

$$= \frac{a+b}{2(a-b)}$$

6 The given expression $= \dfrac{4x^2+9y^2}{4x^2-9y^2} - \dfrac{(2x-3y)(2x-3y)}{(2x+3y)(2x-3y)}$

$$= \frac{4x^2+9y^2-(2x-3y)^2}{4x^2-9y^2} = \frac{12xy}{4x^2-9y^2}$$

7 The given expression $= \dfrac{a(a-b)}{(a+b)^2(a-b)} - \dfrac{b(a+b)}{(a+b)(a-b)\times(a+b)}$

$$= \frac{a(a-b)-b(a+b)}{(a+b)^2(a-b)} = \frac{a^2-2ab-b^2}{(a+b)^2(a-b)}$$

8 The given expression

$$= \frac{(a^2+ab+b^2)(a-b)}{(a+b)(a-b)} + \frac{(a^2-ab+b^2)(a+b)}{(a-b)(a+b)}$$

$$= \frac{(a^3-b^3)+(a^3+b^3)}{a^2-b^2} = \frac{2a^3}{a^2-b^2}$$

9 The given expression

$$= \frac{b-c}{(a-b)(a-c)\times(b-c)} + \frac{a-b}{(a-c)(b-c)\times(a-b)}$$

$$= \frac{(b-c)+(a-b)}{(a-b)(a-c)(b-c)} = \frac{a-c}{(a-b)(a-c)(b-c)} = \frac{1}{(a-b)(b-c)}$$

10 The given expression

$$= \frac{1}{(x-2)(x-1)} + \frac{1}{(x-2)(x-3)}$$

$$= \frac{x-3}{(x-2)(x-1)\times(x-3)} + \frac{x-1}{(x-2)(x-3)\times(x-1)}$$

$$= \frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)} = \frac{2(x-2)}{(x-1)(x-2)(x-3)}$$

$$= \frac{2}{x^2-4x+3}$$

11 The given expression

$$=\frac{1}{(x+5)(x+2)}+\frac{1}{(x+5)(x+8)}$$

$$=\frac{x+8}{(x+5)(x+2)\times(x+8)}+\frac{x+2}{(x+5)(x+8)\times(x+2)}$$

$$=\frac{(x+8)+(x+2)}{(x+2)(x+5)(x+8)}=\frac{2(x+5)}{(x+2)(x+5)(x+8)}$$

$$=\frac{2}{x^2+10x+16}$$

12 The given expression

$$=\frac{4x^2-6xy+9y^2}{(2x+3y)(4x^2-6xy+9y^2)}-\frac{(2x-3y)^2}{8x^3+27y^3}$$

$$=\frac{4x^2-6xy+9y^2-(4x^2-12xy+9y^2)}{8x^3+27y^3}=\frac{6xy}{8x^3+27y^3}$$

13 The given expression

$$=\frac{(a+b)(a+b)}{(a-b)(a+b)}-\frac{(a-b)(a-b)}{(a+b)(a-b)}+\frac{2ab(-1)}{(b^2-a^2)\times(-1)}$$

$$=\frac{(a+b)^2-(a-b)^2-2ab}{a^2-b^2}=\frac{4ab-2ab}{a^2-b^2}=\frac{2ab}{a^2-b^2}$$

14 The given expression

$$=\frac{a-2b}{(a+2b)(a-2b)}+\frac{a+2b}{(a-2b)(a+2b)}+\frac{2a\times(-1)}{(4b^2-a^2)\times(-1)}$$

$$=\frac{(a-2b)+(a+2b)-2a}{a^2-4b^2}=\frac{0}{a^2-4b^2}=0$$

15 We have $\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}=\dfrac{(x+y)^2+(x-y)^2}{x^2-y^2}=\dfrac{2(x^2+y^2)}{x^2-y^2}$

Hence the required value $=\dfrac{2(x^2+y^2)}{x^2-y^2}-\dfrac{2(x^2-y^2)}{x^2+y^2}$

$$=\frac{2(x^2+y^2)^2-2(x^2-y^2)^2}{x^4-y^4}=\frac{8x^2y^2}{x^4-y^4}$$

16 We have $\dfrac{a-2x}{a+2x}-\dfrac{a+2x}{a-2x}=\dfrac{(a-2x)^2-(a+2x)^2}{a^2-4x^2}=\dfrac{-8ax}{a^2-4x^2}$.

Hence the required value $= \dfrac{-8ax}{a^2-4x^2} + \dfrac{8ax}{a^2+4x^2}$

$$= 8ax\left(\frac{1}{a^2+4x^2} - \frac{1}{a^2-4x^2}\right)$$

$$= 8ax\left(\frac{(a^2-4x^2)-(a^2+4x^2)}{a^4-16}\right)$$

$$= \frac{(8ax)\times(-8x^2)}{a^4-16x^4} = \frac{-64ax^3}{a^4-16x^4}$$

17 We have $\dfrac{3x+1}{x-3} - \dfrac{x-3}{3x+9} = \dfrac{3(3x+1)(x+3)-(x-3)^2}{3(x^2-9)}$

$$= \frac{8x^2+36}{3(x^2-9)}$$

Hence the required value $= \dfrac{8x^2+36}{3(x^2-9)} - \dfrac{5x^2+24x}{2x^2-18}$

$$= \frac{8x^2+36x}{3(x^2-9)} - \frac{5x^2+24x}{2(x^2-9)}$$

$$= \frac{16x^2+72x-15x^2-72x}{6(x^2-9)}$$

$$= \frac{x^2}{6(x^2-9)}$$

18 The given expression

$$= \frac{4a-b}{1-4ab} - \frac{4a+b}{1+4ab} + \frac{4b(1-8a^2)}{1-16a^2b^2}$$

$$= \frac{(4a-b)(1+4ab)-(4a+b)(1-4ab)+4b(1-8a^2)}{1-16a^2b^2}$$

$$= \frac{(4a+16a^2b-b-4ab^2)-(4a-16a^2b+b-4ab^2)+4b-32a^2b}{1-16a^2b^2}$$

$$= \frac{2b}{1-16a^2b^2}$$

19 We have $\dfrac{x}{x-2a} + \dfrac{x}{x+2a} = \dfrac{x(x+2a+x-2a)}{x^2-4a^2} = \dfrac{2x^2}{x^2-4a^2}$

Hence the required value $= 2x^2\left(\dfrac{1}{x^2-4a^2} + \dfrac{1}{x^2+4a^2}\right)$

$$= 2x^2\left(\frac{x^2+4a^2+x^2-4a^2}{x^4-16a^4}\right)$$

$$= \frac{4x^4}{x^4-16a^4}$$

20 We have $\frac{b}{a-b}+\frac{b}{a+b}=b\left(\frac{a+b+a-b}{a^2-b^2}\right)=\frac{2ab}{a^2-b^2}$

Again $\frac{2ab}{a^2-b^2}+\frac{2ab}{a^2+b^2}=2ab\left(\frac{a^2+b^2+a^2-b^2}{a^4-b^4}\right)=\frac{4a^2b}{a^4-b^4}$

Hence the required value $=\frac{4a^2b}{a^4-b^4}+\frac{4a^2b}{a^4+b^4}$

$$=4a^2b\left(\frac{a^4+b^4+a^4-b^4}{a^8-b^8}\right)=\frac{8a^6b}{a^8-b^8}$$

21 We have $\frac{x}{3x-y}+\frac{x}{3x+y}=x\left(\frac{3x+y+3x-y}{9x^2-y^2}\right)=\frac{6x^2}{9x^2-y^2}$

Hence the required value $=\frac{6x^2}{9x^2-y^2}+\frac{6x^2}{9x^2+y^2}$

$$=6x^2\left(\frac{9x^2+y^2+9x^2-y^2}{81x^4-y^4}\right)$$

$$=\frac{6x^2\cdot 18x^2}{81x^4-y^4}=\frac{108x^4}{81x^4-y^4}$$

22 We have $\frac{1}{x-3a}-\frac{1}{2x+6a}=\frac{2(x+3a)-(x-3a)}{2(x^2-9a^2)}=\frac{x+9a}{2(x^2-9a^2)}$

Hence the required value

$$=\frac{x+9a}{2(x^2-9a^2)}-\frac{x-9a}{2(x^2+9a^2)}$$

$$=\frac{(x+9a)(x^2+9a^2)-(x-9a)(x^2-9a^2)}{2(x^4-81a^4)}$$

$$=\frac{18ax(x+a)}{2(x^4-81a^4)}=\frac{9ax(x+a)}{x^4-81a^4}.$$

23 We have $\frac{a}{b}+\frac{b}{a}+2=\frac{a^2+b^2+2ab}{ab}=\frac{(a+b)^2}{ab}$

Hence the given expression

$$=\frac{(a^2+b^2)^2}{ab(a-b)^2}-\left(\frac{a}{b}+\frac{b}{a}+2\right)$$

$$=\frac{(a^2+b^2)^2}{ab(a-b)^2}-\frac{(a+b)^2}{ab}$$

$$=\frac{(a^2+b^2)^2-(a^2-b^2)^2}{ab(a-b)^2}=\frac{4a^2b^2}{ab(a-b)^2}=\frac{4ab}{(a-b)^2}$$

24 The given expression

$$=\frac{x+1-(x-1)}{x^2-1}+\frac{x+2-(x-2)}{x^2-4}$$

$$=\frac{2}{x^2-1}+\frac{4}{x^2-4}=\frac{2x^2-8+4x^2-4}{x^4-5x^2+4}=\frac{6x^2-12}{x^4-5x^2+4}$$

25 The given expression

$$=\left(\frac{1}{x-a}+\frac{1}{x+a}\right)-2\left(\frac{1}{2x+a}+\frac{1}{2x-a}\right)$$

$$=\left(\frac{x+a+x-a}{x^2-a^2}\right)-2\left(\frac{2x-a+2x+a}{4x^2-a^2}\right)$$

$$=\frac{2x}{x^2-a^2}-\frac{8x}{4x^2-a^2}=\frac{8x^3-2a^2x-8x^3+8a^2x}{(x^2-a^2)(4x^2-a^2)}$$

$$=\frac{6a^2x}{4x^4-5a^2x^2+a^4}$$

26 The given expression

$$=3\left(\frac{1}{a-x}+\frac{1}{a+x}\right)-\left(\frac{1}{x+3a}-\frac{1}{x-3a}\right)$$

$$=3\left(\frac{a+x+a-x}{a^2-x^2}\right)-\left(\frac{x-3a-x-3a}{x^2-9a^2}\right)$$

$$=\frac{6a}{a^2-x^2}+\frac{6a}{x^2-9a^2}$$

$$=6a\left(\frac{x^2-9a^2+a^2-x^2}{(a^2-x^2)(x^2-9a^2)}\right)$$

$$=6a\left(\frac{-8a^2}{(a^2-x^2)(x^2-9a^2)}\right)=\frac{48a^3}{(x^2-a^2)(x^2-9a^2)}$$

$$=\frac{48a^3}{x^4-10a^2x^2+9a^4}$$

27 The given expression

$$=\left(\frac{2}{x-1}-\frac{1}{x+1}-\frac{3}{x^2-1}\right)-\frac{x}{x^2+1}$$

$$=\frac{2x+2-x+1-3}{x^2-1}-\frac{x}{x^2+1}$$

$$=\frac{x}{x^2-1}-\frac{x}{x^2+1}=\frac{x(x^2+1-x^2+1)}{x^4-1}=\frac{2x}{x^4-1}$$

28 The given expression

$$=\frac{a-c}{(a-b)(x-a)}-\frac{b-c}{(a-b)(x-b)}$$

$$=\frac{(a-c)(x-b)-(b-c)(x-a)}{(a-b)(x-a)(x-b)}$$

$$=\frac{ax-ab-cx+bc-(bx-ab-cx+ac)}{(a-b)(x-a)(x-b)}$$

$$=\frac{x(a-b)-c(a-b)}{(a-b)(x-a)(x-b)}$$

$$=\frac{(x-c)(a-b)}{(a-b)(x-a)(x-b)}=\frac{x-c}{(x-a)(x-b)}$$

29 The given expression

$$=\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{2}{(x-3)(x-5)}$$

$$=\frac{x-3+x-1}{(x-1)(x-2)(x-3)}+\frac{2}{(x-3)(x-5)}$$

$$=\frac{2(x-2)}{(x-1)(x-2)(x-3)}+\frac{2}{(x-3)(x-5)}$$

$$=2\left\{\frac{1}{(x-1)(x-3)}+\frac{1}{(x-3)(x-5)}\right\}$$

$$=2\left(\frac{x-5+x-1}{(x-1)(x-3)(x-5)}\right)$$

$$=\frac{2\cdot 2(x-3)}{(x-1)(x-3)(x-5)}=\frac{4}{(x-1)(x-5)}=\frac{4}{x^2-6x+5}$$

30 The given expression

$$=\frac{1}{(x+a)(x+4a)}+\frac{1}{(x+4a)(x+7a)}+\frac{2}{(x+7a)(x+13a)}$$

$$=\frac{x+7a+x+a}{(x+a)(x+4a)(x+7a)}+\frac{2}{(x+7a)(x+13a)}$$

$$=\frac{2(x+4a)}{(x+a)(x+4a)(x+7a)}+\frac{2}{(x+7a)(x+13a)}$$

$$=2\left\{\frac{1}{(x+a)(x+7a)}+\frac{1}{(x+7a)(x+13a)}\right\}$$

$$=2\left\{\frac{x+13a+x+a}{(x+a)(x+7a)(x+13a)}\right\}$$

$$=\frac{2\cdot 2(x+7a)}{(x+a)(x+7a)(x+13a)}=\frac{4}{(x+a)(x+13a)}$$

$$=\frac{4}{x^2+14ax+13a^2}$$

31 The given expression

$$=\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}+\frac{2x}{(x+1)(x+3)}$$

$$=\frac{(x+3)+(x+1)+2x(x+2)}{(x+1)(x+2)(x+3)}$$

$$=\frac{2(x+2)+2x(x+2)}{(x+1)(x+2)(x+3)}$$

$$=\frac{2(x+2)(1+x)}{(x+1)(x+2)(x+3)}=\frac{2}{x+3}$$

32 The given expression

$$=\frac{(1+x+x^2)-(1-x+x^2)-2x}{1+x^2+x^4}$$

$$=\frac{2x-2x}{1+x^2+x^4}=0$$

33 The given expression

$$=\frac{(1-x+x^2)-(1+x+x^2)}{1+x^2+x^4}+\frac{2x}{1-x^2+x^4}$$

$$=\frac{-2x}{1+x^2+x^4}+\frac{2x}{1-x^2+x^4}$$

$$=2x\left\{\frac{-(1-x^2+x^4)+(1+x^2+x^4)}{1+x^4+x^8}\right\}$$

$$=\frac{4x^3}{1+x^4+x^8}$$

34 The given expression

$$=\frac{(x^2+2x+4)-(x-2)^2}{x^3-8}+\frac{6x}{x^3+8}$$

$$=\frac{(x^2+2x+4)-(x^2-4x+4)}{x^3-8}+\frac{6x}{x^3+8}$$

$$=6x\left(\frac{1}{x^3-8}+\frac{1}{x^3+8}\right)$$

$$=6x\,\frac{x^3+8+x^3-8}{x^6-64}=\frac{12x^4}{x^6-64}$$

35 The given expression

$$=\frac{(2x^2+6ax+9a^2)-(2x^2-6ax+9a^2)}{4x^4+81a^4}+\frac{12ax}{4x^4-81a^4}$$

$$=12ax\left(\frac{1}{4x^4+81a^4}+\frac{1}{4x^4-81a^4}\right)$$

$$=12ax\,\frac{4x^4-81a^4+4x^4+81a^4}{16x^8-6561a^8}=\frac{96ax^5}{16x^8-6561a^8}$$

Exercise 59

1 The required product

$$=\frac{2\times a^2\times 9\times b^2\times 8\times c^2}{3\times a\times b\times 16\times a\times c\times 9\times b\times c}=\frac{2\times 9\times 8\times a^2b^2c^2}{3\times 16\times 9\times a^2b^2c^2}=\frac{1}{3}$$

2 The required product

$$=\frac{4\times a^2\times b^2\times 9\times c^2\times 4\times d^2}{3\times c^2\times 16\times d^2\times 27\times b^2}=\frac{4\times 9\times 4\times a^2b^2c^2d^2}{3\times 16\times 27\times b^2c^2d^2}=\frac{a^2}{9}$$

3 The required product

$$=\frac{x^3\times y^3\times z^3}{y\times z\times z\times x\times x\times y}=\frac{x^3y^3z^3}{x^2y^2z^2}=xyz$$

4 The required product

$$=\frac{7\times 4\times a^2b^2c^2\times x^3y^3z^3}{12\times 21\times a^4b^4c^4\times xyz}=\frac{x^2y^2z^2}{9a^2b^2c^2}$$

5 The required product

$$=\frac{12\times 35\times m^2\times n^3\times x^3\times y\times z}{7\times 96\times m^3\times n\times x\times y^2\times z}=\frac{5n^2x^2}{8my}$$

6 The given expression

$$=\frac{x+1}{x-1}\times\frac{(x+2)(x-1)}{x(x+1)}=\frac{x+2}{x}$$

7 The given expression

$$=\frac{(a+3b)(a-3b)}{a(a+3b)}\times\frac{3a^2}{a(a-3b)}=3$$

8 The given expression

$$=\frac{(a-b)(a^2+ab+b^2)}{a(a+b)}\times\frac{(a+b)^2}{a^2+ab+b^2}=\frac{(a-b)(a+b)}{a}=\frac{a^2-b^2}{a}$$

9 The given expression

$$=\frac{(a+2x)(a^2-2ax+4x^2)}{a^2(a-2x)}\times\frac{(a-2x)^2}{a^2-2ax+4x^2}$$

$$=\frac{(a+2x)(a-2x)}{a^2}=\frac{a^2-4x^2}{a^2}$$

10 The given expression

$$=\frac{(x+1)(x+3)}{(x+2)(x-2)}\times\frac{(x-1)(x-2)}{(x-3)(x+3)}$$

$$=\frac{(x+1)(x-1)}{(x+2)(x-3)}=\frac{x^2-1}{x^2-x-6}$$

11 The given expression

$$=\frac{(x-2)(x-5)}{(x+3)(x-5)}\times\frac{(x+3)(x-6)}{(x-2)(x-6)}=1$$

12 The given expression

$$=\frac{(x-1)(x-3)}{(x-1)(x-5)}\times\frac{(x-2)(x-5)}{(x-2)(x-3)}=1$$

13 The given expression

$$=\frac{(a^2+b^2)(a+b)(a-b)}{(a-b)^2}\times\frac{(a-b)}{a(a+b)}=\frac{a^2+b^2}{a}$$

14 The given expression

$$=\frac{(2x-1)(x-2)}{(3x+1)(x-2)}\times\frac{x(3x+1)}{2(2x-1)}=\frac{x}{2}$$

15 The given expression

$$=\frac{(x+2)(x-8)}{(x+3)(x-7)}\times\frac{(x-4)(x-7)}{(x-4)(x-8)}=\frac{x+2}{x+3}$$

16 The given expression

$$=\frac{(a+x)(a-x)}{a+b}\times\frac{(a+b)(a-b)}{x(a+x)}\times\frac{a^2}{a-x}=\frac{a^2(a-b)}{x}$$

17 The given expression

$$=\frac{x^2-ax+a^2}{a^2}\times\frac{x^2+ax+a^2}{a^2}=\frac{x^4+a^2x^2+a^4}{a^4}$$

$$=\frac{x^4}{a^4}+\frac{x^2}{a^2}+1$$

18 The given expression

$$=\frac{8ab+9x^2}{6bx}\times\frac{8ab+9x^2}{12ax}$$

$$=\frac{(8ab+9x^2)^2}{72abx^2}=\frac{64a^2b^2+144abx^2+81x^4}{72abx^2}$$

$$=\frac{8ab}{9x^2}+2+\frac{9x^2}{8ab}$$

19 The given expression

$$=\frac{a^2+b^2}{ab}\times\frac{c^2+d^2}{dc}-\frac{a^2-b^2}{ab}\times\frac{c^2-d^2}{cd}$$

$$=\frac{1}{abcd}\{a^2(c^2+d^2-c^2+d^2)+b^2(c^2+d^2+c^2-d^2)\}$$

$$=\frac{1}{abcd}(2a^2d^2+2b^2c^2)=2\left(\frac{a^2d^2}{abcd}+\frac{b^2c^2}{abcd}\right)=2\left(\frac{ad}{bc}+\frac{bc}{ad}\right)$$

20 The given expression

$$=\frac{(2x-1)(x-3)}{(2x-1)(x+4)}\times\frac{(3x-1)(x+4)}{(3x-1)(x+3)}\times\frac{(2x-5)(x+3)}{(2x-5)(x-3)}=1$$

21 The given expression

$$=\frac{b^2-(a-c)^2}{(a+c)^2-b^2}\times\frac{(b-c)^2-a^2}{(a-c)^2-b^2}$$

$$=\frac{(b+a-c)(b-a+c)}{(a+c+b)(a+c-b)}\times\frac{(b-c+a)(b-c-a)}{(a-c+b)(a-c-b)}$$

$$=\frac{(a-c+b)(a-b-c)(a+b-c)(a-b+c)}{(a+b+c)(a-b+c)(a+b-c)(a-b-c)}$$

$$=\frac{a+b-c}{a+b+c}$$

22 The given expression

$$=\frac{c^2-(a-b)^2}{b^2-(a-c)^2}\times\frac{(a-c)^2-b^2}{(a-b)^2-c^2}$$

$$=\frac{-\{(a-b)^2-c^2\}\times(a-c)^2-b^2}{-\{(a-c)^2-b^2\}\times(a-b)^2-c^2}$$

$$=\frac{-1}{-1}=1$$

Exercise 60

1 The given expression

$$=\frac{4a^2bc}{15xy^2z}\times\frac{25x^2yz}{8ab^2c}=\frac{a\times 5\times x}{3\times y\times 2\times b}=\frac{5ax}{6by}$$

2 The given expression

$$=\frac{a^2+ab}{a-b}\times\frac{a^2-b^2}{ab}=\frac{a(a+b)}{a-b}\times\frac{(a+b)(a-b)}{ab}=\frac{(a+b)^2}{b}$$

3 The given expression

$$=\frac{x^2-49}{x^2-25}\times\frac{x+5}{x+7}=\frac{(x+7)(x-7)}{(x+5)(x-5)}\times\frac{(x+5)}{(x+7)}=\frac{x-7}{x-5}$$

4 The given expression

$$=\frac{a^4-b^4}{a^2+2ab+b^2}\times\frac{a+b}{a^2+b^2}$$

$$=\frac{(a^2+b^2)(a+b)(a-b)}{(a+b)^2}\times\frac{a+b}{a^2+b^2}=(a-b)$$

5 The given expression

$$=\frac{m^2-9n^2}{m^2+5mn+6n^2}\times\frac{m^2-n^2}{m^2-2mn-3n^2}$$

$$=\frac{(m+3n)(m-3n)}{(m+3n)(m+2n)}\times\frac{(m+n)(m-n)}{(m-3n)(m+n)}=\frac{m-n}{m+2n}$$

6 The given expression

$$=\frac{m^3-n^3}{m+n}\times\frac{m^2-n^2}{m^2+mn+n^2}$$

$$=\frac{(m-n)(m^2+mn+n^2)}{m+n}\times\frac{(m+n)(m-n)}{m^2+mn+n^2}=(m-n)^2$$

7 The given expression

$$=\left(\frac{2x+y-x-y}{x+y}\right)\div\left(\frac{x+y-y}{x+y}\right)$$

$$=\left(\frac{x}{x+y}\right)\div\left(\frac{x}{x+y}\right)=1$$

8 The given expression

$$=\left\{\frac{a^2-ab+ab+b^2}{(a+b)(a-b)}\right\}-\left\{\frac{a^2+ab-ab+b^2}{(a-b)(a+b)}\right\}$$

$$=\left(\frac{a^2+b^2}{a^2-b^2}\right)-\left(\frac{a^2+b^2}{a^2-b^2}\right)=1$$

9 The given expression

$$=\frac{(x+y)^2+(x-y)^2}{x^2-y^2}-\frac{(x+y)^2-(x-y)^2}{x^2-y^2}$$

$$=\frac{2(x^2+y^2)}{x^2-y^2}-\frac{4xy}{x^2-y^2}$$

$$=\frac{2(x^2+y^2)}{x^2-y^2}\times\frac{x^2-y^2}{4xy}=\frac{x^2+y^2}{2xy}$$

10 The given expression

$$=\frac{(x+2)(x-2)}{(x-3)(x+6)}-\frac{(x+2)(x-7)}{(x-6)(x+6)}$$

$$=\frac{(x+2)(x-2)}{(x-3)(x+6)}\times\frac{(x-6)(x+6)}{(x+2)(x-7)}$$

$$=\frac{(x-2)(x-6)}{(x-3)(x-7)}=\frac{x^2-8x+12}{x^2-10x+21}$$

11 The given expression

$$=\frac{p^2+q^2-2pq}{p^2+q^2}-\frac{p^3-q^3-3pq(p-q)}{p-q}$$

$$=\frac{(p-q)^2}{p^2+q^2}-\frac{(p-q)^3}{p-q}=\frac{(p-q)^2}{p^2+q^2}\times\frac{p-q}{(p-q)^3}=\frac{1}{p^2+q^2}$$

12 The given expression

$$=\frac{(a+b)^2}{(a-b)^2}-\frac{(a+b)^3}{(a-b)^3}=\frac{(a+b)^3}{(a-b)^2}\times\frac{(a-b)^3}{(a+b)^3}$$

$$=(a+b)(a-b)=a^2-b^2$$

13 The given expression

$$=\frac{(x+y)(x^2-xy+y^2)}{(x^2+xy+y^2)}-\frac{x^2-xy+y^2}{(x-y)(x^2+xy+y^2)}\times\frac{xy}{(x+y)(x-y)}$$

$$=\frac{(x+y)(x^2-xy+y^2)}{x^2+xy+y^2}\times\frac{(x-y)(x^2+xy+y^2)}{x^2-xy+y^2}\times\frac{xy}{(x+y)(x-y)}$$

$$=xy$$

14 The given expression

$$=\frac{a\{(a-b)^2+4ab\}}{b(a+b)}\div\frac{(a+b)(a-b)}{ab}\times\frac{b\{(a+b)^2-4ab\}}{a(a-b)}$$

$$=\frac{a(a+b)^2}{b(a+b)}\times\frac{ab}{(a+b)(a-b)}\times\frac{b(a-b)^2}{a(a-b)}=ab$$

15 The given expression

$$=\frac{(x-6)(x+5)}{(x+6)(x-6)}\div\frac{(x+5)(x-2)}{(x-2)(x+4)}\div\frac{(x+4)}{2x(x+6)}$$

$$=\frac{x+5}{x+6}\times\frac{x+4}{x+5}\times\frac{2x(x+6)}{x+4}=2x$$

16 The given expression

$$=\frac{(x+12)(x-9)}{(x+8)(x-8)}\div\frac{(x-6)(x+12)}{(x+8)(x-7)}\div\frac{(x-9)(x-7)}{(x-8)(x-6)}$$

$$=\frac{(x+12)(x-9)}{(x+8)(x-8)}\times\frac{(x+8)(x-7)}{(x-6)(x+12)}\times\frac{(x-8)(x-6)}{(x-9)(x-7)}$$

$$=1$$

17 The given expression

$$=\frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)}\div\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}$$

$$=\frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)}\times\frac{x^2-y^2}{4xy}=\frac{xy}{x^2+y^2}$$

18 The given expression

$$=\frac{1}{a-b}\left\{\frac{(a+b)^2+(a^2+b^2)}{a+b}\right\}\div\frac{a-b}{a+b}\left\{\frac{a^2-ab+b^2-(a^2+ab+b^2)}{a^2-ab+b^2}\right\}$$

$$=\frac{2(a^2+ab+b^2)}{(a-b)(a+b)}\times\frac{(a+b)(a^2-ab+b^2)}{(a-b)(-2ab)}$$

$$=\frac{(a^2+ab+b^2)(a^2-ab+b^2)}{-(a-b)^2ab}=-\frac{a^4+a^2b^2+b^4}{ab(a-b)^2}$$

19 The given expression

$$=\frac{(a^2+b^2)(a+b)(a-b)}{a^3+b^3}\div\frac{(a-b)^2}{(a^2-ab+b^2)}\times\frac{a}{(a+b)^2}$$

$$=\frac{(a^2+b^2)(a-b)}{a^2-ab+b^2}\times\frac{a^2-ab+b^2}{(a-b)^2}\times\frac{a}{a^2+b^2}=\frac{a}{a-b}$$

20 The given expression

$$=\frac{(a-b)(a^2+ab+b^2)}{a^2+b^2}\div\frac{a^2+ab+b^2}{(a+b)(a^2-ab+b^2)}\times\frac{a^2+b^2}{a^2-ab+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{a^2+b^2}\times\frac{(a+b)(a^2-ab+b^2)}{a^2+ab+b^2}\times\frac{a^2+b^2}{a^2-ab+b^2}$$

$$=(a-b)(a+b)=a^2-b^2$$

21 The given expression

$$=\frac{\left(\frac{a}{b}-\frac{b}{a}\right)\left(\frac{a^2}{b^2}+1+\frac{b^2}{a^2}\right)}{\left(\frac{a}{b}-\frac{b}{a}\right)\left(\frac{a}{b}+\frac{b}{a}-1\right)}\times\frac{\left(\frac{1}{b}-\frac{1}{a}\right)\times ab}{\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{ab}\right)\times ab}$$

$$=\frac{\left(\frac{a^2}{b^2}+2+\frac{b^2}{a^2}\right)-1}{\frac{a}{b}+\frac{b}{a}-1}\times\frac{a-b}{\frac{b}{a}+\frac{a}{b}+1}$$

$$=\frac{\left(\frac{a}{b}+\frac{b}{a}\right)^2-1}{\frac{a}{b}+\frac{b}{a}-1}\times\frac{a-b}{\frac{b}{a}+\frac{a}{b}+1}$$

$$=\frac{\left(\frac{a}{b}+\frac{b}{a}+1\right)\left(\frac{a}{b}+\frac{b}{a}-1\right)\times(a-b)}{\left(\frac{a}{b}+\frac{b}{a}-1\right)\left(\frac{b}{a}+\frac{a}{b}+1\right)}=a-b$$

Exercise 61

1 $$\frac{a}{b+\frac{c}{d+\frac{c}{f}}}=\frac{a}{b+\frac{c}{\frac{df+c}{f}}}=\frac{a}{b+\frac{cf}{df+c}}=\frac{a}{\frac{bdf+bc+cf}{df+c}}=\frac{adf+ac}{bdf+bc+cf}.$$

2 $$\frac{x}{x-\frac{x-1}{1-\frac{1}{x+1}}}=\frac{x}{x-\frac{x-1}{\frac{x+1-1}{x+1}}}=\frac{x}{x-\frac{x^2-1}{x}}=\frac{x}{\frac{x^2-x^2+1}{x}}=\frac{x}{\frac{1}{x}}=x^2$$

3 $$a^2+\cfrac{b^4}{a^2-\cfrac{a^3+b^3}{a+\cfrac{b^2}{a-b}}}=a^2+\cfrac{b^4}{a^2-\cfrac{a^3+b^3}{\cfrac{a^2-ab+b^2}{a-b}}}$$

$$=a^2+\cfrac{b^4}{a^2-\cfrac{(a^3+b^3)(a-b)}{a^2-ab+b^2}}$$

$$=a^2+\frac{b^4}{a^2-(a+b)(a-b)}=a^2+\frac{b^4}{a^2-(a^2-b^2)}$$

$$=a^2+\frac{b^4}{b^2}=a^2+b^2$$

4 $$\cfrac{m}{m^2-\cfrac{m^3-1}{m+\cfrac{1}{m+1}}}=\cfrac{m}{m^2-\cfrac{(m^3-1)(m+1)}{m^2+m+1}}$$

$$=\frac{m}{m^2-(m-1)(m+1)}=\frac{m}{m^2-m^2+1}=m$$

5 The given expression

$$=\cfrac{\cfrac{(x-y)^2}{x+y}}{\cfrac{(x+y)^2-(x-y)^2}{x+y}}-1=\cfrac{(x-y)^2}{\cfrac{(x+y)^2}{4xy}-1}$$

$$=\frac{(x-y)^2\times 4xy}{(x+y)^2-4xy}=\frac{(x-y)^2\times 4xy}{(x-y)^2}=4xy$$

6 The given expression

$$=\cfrac{x^2(x+2)}{4(x+2)+\cfrac{2(x^2+4)(x^2-4)(x+2)}{(x+2)^2+(x-2)^2}}$$

$$=\cfrac{x^2(x+2)}{4(x+2)+\cfrac{2(x^2+4)(x^2-4)(x+2)}{2(x^2+4)}}$$

$$=\frac{x^2(x+2)}{(x+2)\{4+(x^2-4)\}}=\frac{x^2}{x^2}=1$$

7 $$\frac{a^3+b^3}{a-b}=\frac{(a^3-b^3)+2b^3}{a-b}=\frac{a^3-b^3}{a-b}+\frac{2b^3}{a-b}$$

$$=a^2+ab+b^2+\frac{2b^3}{a-b}$$

8 $$\frac{a^3-b^3}{a+b}=\frac{(a^3+b^3)-2b^3}{a+b}=\frac{a^3+b^3}{a+b}-\frac{2b^3}{a+b}$$
$$=a^2-ab+b^2-\frac{2b^3}{a+b}$$

9 $$\frac{a^2+2ab+3b^2}{a+b}=\frac{(a^2+2ab+b^2)+2b^2}{a+b}=\frac{(a+b)^2}{a+b}+\frac{2b^2}{a+b}$$
$$=a+b+\frac{2b^2}{a+b}$$

10 $$\frac{a^2+2ab-3b^2}{a+b}=\frac{(a^2+2ab+b^2)-4b^2}{a+b}=\frac{(a+b)^2}{a+b}-\frac{4b^2}{a+b}$$
$$=a+b-\frac{4b^2}{a+b}$$

11 $$\frac{x^6}{x^2+y^2}=\frac{(x^6+y^6)-y^6}{x^2+y^2}=\frac{x^6+y^6}{x^2+y^2}-\frac{y^6}{x^2+y^2}$$
$$=x^4-x^2y^2+y^4-\frac{y^6}{x^2+y^2}$$

12 $$\frac{x^6}{x^2-y^2}=\frac{(x^6-y^6)+y^6}{x^2-y^2}=\frac{x^6-y^6}{x^2-y^2}+\frac{y^6}{x^2-y^2}$$
$$=x^4+x^2y^2+y^4+\frac{y^6}{x^2-y^2}$$

13 $$\frac{x^2yz+xy^2z+xyz^2}{x^2y^2z^2}=\frac{x^2yz}{x^2y^2z^2}+\frac{xy^2z}{x^2y^2z^2}+\frac{xyz^2}{x^2y^2z^2}$$
$$=\frac{1}{yz}+\frac{1}{zx}+\frac{1}{xy}$$

14 $$\frac{xy^2z^2+yz^2x^2+zx^2y^2}{x^2y^2z^2}=\frac{xy^2z^2}{x^2y^2z^2}+\frac{yz^2x^2}{x^2y^2z^2}+\frac{zx^2y^2}{x^2y^2z^2}$$
$$=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

15 $$\frac{3a-6}{(a-1)(a-2)(a-3)}=\frac{(a-1)+(a-2)+(a-3)}{(a-1)(a-2)(a-3)}$$
$$=\frac{a-1}{(a-1)(a-2)(a-3)}+\frac{a-2}{(a-1)(a-2)(a-3)}$$
$$+\frac{a-3}{(a-1)(a-2)(a-3)}$$
$$=\frac{1}{(a-2)(a-3)}+\frac{1}{(a-1)(a-3)}+\frac{1}{(a-1)(a-2)}$$

16 $$\frac{3x^2-14}{(x+1)(x+2)(x+3)}=\frac{(x^2-1)+(x^2-4)+(x^2-9)}{(x+1)(x+2)(x+3)}$$

$$=\frac{x^2-1}{(x+1)(x+2)(x+3)}+\frac{x^2-4}{(x+1)(x+2)(x+3)}$$

$$+\frac{x^2-9}{(x+1)(x+2)(x+3)}$$

$$=\frac{x-1}{(x+2)(x+3)}+\frac{x-2}{(x+1)(x+3)}+\frac{x-3}{(x+1)(x+2)}$$

17 $$\begin{array}{l} 1+x\,)\ 1-x\ (\,1-2x+2x^2-2x^3 \text{ (Quotient)} \\ \qquad\ \ \underline{1+x} \\ \qquad\quad -2x \\ \qquad\quad \underline{-2x-2x^2} \\ \qquad\qquad\quad 2x^2 \\ \qquad\qquad\quad \underline{2x^2+2x^3} \\ \qquad\qquad\qquad -2x^3 \\ \qquad\qquad\qquad \underline{-2x^3-2x^4} \\ \qquad\qquad\qquad\qquad 2x^4 \text{ (Remainder)} \end{array}$$

Thus $\dfrac{1-x}{1+x}=1-2x+2x^2-2x^3+\dfrac{2x^4}{1+x}$

18 $$\begin{array}{l} x^2-a^2\,)\ a\ \left(\dfrac{a}{x^2}+\dfrac{a^3}{x^4}+\dfrac{a^5}{x^6}\right. \text{ (Quotient)} \\ \qquad\quad \underline{a-\dfrac{a^3}{x^2}} \\ \qquad\qquad \dfrac{a^3}{x^2} \\ \qquad\qquad \underline{\dfrac{a^3}{x^2}-\dfrac{a^5}{x^4}} \\ \qquad\qquad\qquad \dfrac{a^5}{x^4} \\ \qquad\qquad\qquad \underline{\dfrac{a^5}{x^4}-\dfrac{a^7}{x^6}} \\ \qquad\qquad\qquad\qquad \dfrac{a^7}{x^6} \text{ (Remainder)} \end{array}$$

Thus $\dfrac{a}{x^2-a^2}=\dfrac{a}{x^2}+\dfrac{a^3}{x^4}+\dfrac{a^5}{x^6}+\dfrac{a^7}{x^6(x^2-a^2)}$

19 $$x^3+a^3\,\big)\,a^3\,\Big(\frac{a^3}{x^3}-\frac{a^6}{x^6}+\frac{a^9}{x^9}\ \text{(Quotient)}$$

$$\begin{array}{l}
\underline{a^3+\dfrac{a^6}{x^3}} \\
\quad -\dfrac{a^6}{x^3} \\
\quad \underline{-\dfrac{a^6}{x^3}-\dfrac{a^9}{x^6}} \\
\qquad\qquad \dfrac{a^9}{x^6} \\
\qquad\qquad \underline{\dfrac{a^9}{x^6}+\dfrac{a^{12}}{x^9}} \\
\qquad\qquad\qquad -\dfrac{a^{12}}{x^9}\ \text{(Remainder)}
\end{array}$$

Thus $$\frac{a^3}{x^3+a^3}=\frac{a^3}{x^3}-\frac{a^6}{x^6}+\frac{a^9}{x^9}-\frac{a^{12}}{x^9(x^3+a^3)}$$

Again $$\frac{a^3}{x^3+a^3}=\frac{a^3}{a^3+x^3}$$

$$a^3+x^3\,\big)\,a^3\,\Big(1-\frac{x^3}{a^3}+\frac{x^6}{a^6}-\frac{x^9}{a^9}\ \text{(Quotient)}$$

$$\begin{array}{l}
\underline{a^3+x^3} \\
\quad -x^3 \\
\quad \underline{-x^3-\dfrac{x^6}{a^3}} \\
\qquad\qquad \dfrac{x^6}{a^3} \\
\qquad\qquad \underline{\dfrac{x^6}{a^3}+\dfrac{x^9}{a^6}} \\
\qquad\qquad\quad -\dfrac{x^9}{a^6} \\
\qquad\qquad\quad \underline{-\dfrac{x^9}{a^6}-\dfrac{x^{12}}{a^9}} \\
\qquad\qquad\qquad\quad \dfrac{x^{12}}{a^9}\ \text{(Remainder)}
\end{array}$$

Thus we get $$\frac{a^3}{x^3+a^3}=1-\frac{x^3}{a^3}+\frac{x^6}{a^6}-\frac{x^9}{a^9}+\frac{x^{12}}{a^9(x^3+a^3)}$$

20

$$
\begin{array}{l}
x+a\,)\,x^4-1\,(\,x^3-ax^2+a^2x-a^3 \text{ (Quotient)}\\
\quad\underline{x^4+ax^3}\\
\qquad -ax^3-1\\
\qquad \underline{-ax^3-a^2x^2}\\
\qquad\qquad a^2x^2-1\\
\qquad\qquad \underline{a^2x^2+a^3x}\\
\qquad\qquad\qquad -a^3x-1\\
\qquad\qquad\qquad \underline{-a^3x-a^4}\\
\qquad\qquad\qquad\qquad a^4-1 \text{ (Remainder)}
\end{array}
$$

Thus $\dfrac{x^4-1}{x+a}=x^3-ax^2+a^2x-a^3+\dfrac{a^4-1}{x+a}$

21 The given expression

$$=\frac{-(x+2a)(x+2b)+(x-2a)(x-2b)+4ab}{x^2-4b^2}$$

$$=\frac{-\{x^2+2x(a+b)+4ab\}+\{x^2-2x(a+b)+4ab\}+4ab}{x^2-4b^2}$$

$$=\frac{-4x(a+b)+4ab}{x^2-4b^2}\quad\left[\quad x=\frac{ab}{a+b},\qquad x(a+b)=ab\right]$$

$$=\frac{-4ab+4ab}{x^2-4b^2}=0$$

22 The left-hand expression

$$=\left(\frac{x+a}{x-a}-1\right)+\left(\frac{x+b}{x-b}-1\right)+\left(\frac{x+c}{x-c}-1\right)$$

$$=\frac{2a}{x-a}+\frac{2b}{x-b}+\frac{2c}{x-c}$$

$$=2\left\{\frac{a(x-b)(x-c)+b(x-a)(x-c)+c(x-a)(x-b)}{(x-a)(x-b)(x-c)}\right\}$$

$$=2\left[\frac{x^2(a+b+c)-x\{a(b+c)+b(a+c)+c(a+b)\}+3abc}{(x-a)(x-b)(x-c)}\right]$$

$$=2\left[\frac{x\cdot 2(ab+bc+ca)-x\cdot 2(ab+bc+ca)+3abc}{(x-a)(x-b)(x-c)}\right]$$

[since $x(a+b+c)=2(ab+bc+ca)$]

$$=\frac{2\cdot 3abc}{(x-a)(x-b)(x-c)}=\frac{6abc}{(x-a)(x-b)(x-c)}$$

23 The left-hand expression

$$=\left(\frac{a+2x}{a-2x}+1\right)+\left(\frac{b+2x}{b-2x}+1\right)+\left(\frac{c+2x}{c-2x}+1\right)$$

$$=\frac{2a}{a-2x}+\frac{2b}{b-2x}+\frac{2c}{c-2x}$$

$$=\left[\frac{a(b-2x)(c-2x)+b(a-2x)(c-2x)+c(a-2x)(b-2x)}{(a-2x)(b-2x)(c-2x)}\right]$$

$$=2\left[\frac{3abc-2x\{a(b+c)+b(c+a)+c(a+b)\}+4x^2(a+b+c)}{(a-2x)(b-2x)(c-2x)}\right]$$

$$=2\left[\frac{3abc-2x\cdot 2(ab+bc+ca)+2x\cdot 2(ab+bc+ca)}{(a-2x)(b-2x)(c-2x)}\right]$$

[since $ab+bc+ca=x(a+b+c)$]

$$=\frac{6abc}{(a-2x)(b-2x)(c-2x)}$$

24 The given expression

$$=\left(\frac{x^2-(b+c)x}{x^2-(b+c)x+bc}-1\right)+\left(\frac{x^2-(c+a)x}{x^2-(c+a)x+ac}-1\right)$$

$$+\left(\frac{x^2-(a+b)x}{x^2-(a+b)x+ab}-1\right)+3$$

$$=-\frac{bc}{(x-b)(x-c)}-\frac{ac}{(x-c)(x-a)}-\frac{ab}{(x-a)(x-b)}+3$$

$$=3-\left\{\frac{bc(x-a)+ac(x-b)+ab(x-c)}{(x-a)(x-b)(x-c)}\right\}$$

$$=3-\left\{\frac{x(ab+bc+ca)-3abc}{(x-a)(x-b)(x-c)}\right\}$$

$$=3-\left\{\frac{3abc-3abc}{(x-a)(x-b)(x-c)}\right\}$$

[since $x(ab+bc+ca)=3abc$]

$$=3$$

25 The given expression

$$=\frac{x(x+1)-y^2}{y(y+1)-x^2}=\frac{\dfrac{a-b}{a+b}\left(\dfrac{a-b}{a+b}+1\right)-\dfrac{(a+b)^2}{(a-b)^2}}{\dfrac{a+b}{a-b}\left(\dfrac{a+b}{a-b}+1\right)-\dfrac{(a-b)^2}{(a+b)^2}}$$

$$=\frac{\frac{(a-b)2a}{(a+b)^2}-\frac{(a+b)^2}{(a-b)^2}}{\frac{2a(a+b)}{(a-b)^2}-\frac{(a-b)^2}{(a+b)^2}}$$

$$=\frac{2a(a-b)^3-(a+b)^4}{2a(a+b)^3-(a-b)^4}$$

$$=\frac{2a(a^3-3a^2b+3ab^2-b^3)-(a^4+4a^3b+6a^2b^2+4ab^3+b^4)}{2a(a^3+3a^2b+3ab^2+b^3)-(a^4-4a^3b+6a^2b^2-4ab^3+b^4)}$$

$$=\frac{a^4-10a^3b-6ab^3-b^4}{a^4+10a^3b+6ab^3-b^4}$$

26 The given expression

$$=\frac{(x^2-2ab)(x^2+5ab)}{(x^2+2ab)(x^2+5ab)}\times\frac{a^2+b^2+2ab}{a^2+b^2-2ab}$$

$$=\frac{(x^2-2ab)}{(x^2+2ab)}\times\frac{a^2+b^2+2ab}{a^2+b^2-2ab}$$

$$=\frac{a^2+b^2-2ab}{a^2+b^2+2ab}\times\frac{a^2+b^2+2ab}{a^2+b^2-2ab}=1$$

27 The given expression

$$=\frac{(x^2-3ab)(y^2+6ab)}{(y^2+6ab)(y^2+3ab)}\times\frac{a^3-b^3}{a^3+b^3}$$

$$=\frac{x^2-3ab}{y^2+3ab}\times\frac{a^3-b^3}{a^3+b^3}=\frac{(a+b)^2-3ab}{(a-b)^2+3ab}\times\frac{a^3-b^3}{a^3+b^3}$$

$$=\frac{a^2+b^2-ab}{a^2+b^2+ab}\times\frac{(a-b)(a^2+b^2+ab)}{(a+b)(a^2+b^2-ab)}$$

$$=\frac{a-b}{a+b}$$

28 The given expression

$$=\frac{(x^2+2ab)(x^2-ab)}{(x^2+2ab)(y^2+ab)}-\frac{a^2+ab+b^2}{a^2-ab+b^2}$$

$$=\frac{x^2-ab}{y^2+ab}-\frac{a^2+ab+b^2}{a^2-ab+b^2}$$

$$=\frac{(a+b)^2-ab}{(a-b)^2+ab}-\frac{a^2+ab+b^2}{a^2-ab+b^2}$$

$$=\frac{a^2+ab+b^2}{a^2-ab+b^2}-\frac{a^2+ab+b^2}{a^2-ab+b^2}=0$$

29 The given expression

$$=\frac{x^{12}-x^6+x^6+x^6+x^3-1-x^3-1}{x^6-1}$$

$$=\frac{x^{12}+x^6-2}{x^6-1}=\frac{(x^6-1)(x^6+2)}{x^6-1}=x^6+2$$

30 The given expression

$$=\frac{(x+a-b)(x-a+b)}{(x+a+b)(x-a+b)}+\frac{(a+x-b)(a-x+b)}{(x+a+b)(x+a-b)}+\frac{(b+x-a)(b-x+a)}{(x+a+b)(a+b-x)}$$

$$=\frac{x+a-b}{x+a+b}+\frac{a-x+b}{x+a+b}+\frac{x-a+b}{x+a+b}$$

$$=\frac{x+a-b+a-x+b+x-a+b}{x+a+b}=\frac{x+a+b}{x+a+b}=1$$

31 The given expression

$$=\frac{(a+2b+b)(a+2b-b)}{(a+b+2b)(a+b-2b)}+\frac{(a-b+2b)(a-b-2b)}{(a-2b+b)(a-2b-b)}+\frac{(2a+3b+b)(2a+3b-b)}{(2a+b+3b)(2a+b-3b)}$$

$$=\frac{(a+3b)(a+b)}{(a+3b)(a-b)}+\frac{(a+b)(a-3b)}{(a-b)(a-3b)}+\frac{(2a+4b)(2a-2b)}{(2a+4b)(2a-2b)}$$

$$=\frac{a+b}{a-b}+\frac{a+b}{a-b}+\frac{a+b}{a-b}=\frac{3(a+b)}{(a-b)}$$

32 The given expression

$$=\frac{(x^2+x-1)(x^2-x+1)}{(x^2+x+1)(x^2-x+1)}+\frac{(x+x^2-1)(x-x^2+1)}{\{x(x+1)+1\}\{x(x+1)-1\}}+\frac{\{x(x-1)+1\}\{x(x-1)-1\}}{(x^2+x+1)(x^2-x-1)}$$

$$=\frac{x^2+x-1}{x^2+x+1}+\frac{1+x-x^2}{x^2+x+1}+\frac{x^2-x+1}{x^2+x+1}$$

$$=\frac{x^2+x-1+1+x-x^2+x^2-x+1}{x^2+x+1}=\frac{x^2+x+1}{x^2+x+1}=1$$

33 The left-hand expression

$$=\frac{2ab-a^2-b^2+c^2}{2ab}=\frac{c^2-(a-b)^2}{2ab}$$

$$=\frac{(c+a-b)(c-a+b)}{2ab}$$

$$=\frac{(a+b+c-2b)(a+b+c-2a)}{2ab}$$

$$=\frac{(2s-2b)(2s-2a)}{2ab}=\frac{2(s-a)(s-b)}{ab}$$

34 The given expression

$$=\frac{b-c}{(a+b-c)(a-b+c)}+\frac{c-a}{(b+c-a)(b-c+a)}$$

$$+\frac{a-b}{(c+a-b)(c-a+b)}$$

$$=\frac{(b-c)(b+c-a)+(c-a)(c+a-b)+(a-b)(a+b-c)}{(a+b-c)(a-b+c)(b+c-a)}$$

$$=\frac{b^2-c^2-a(b-c)+c^2-a^2-b(c-a)+(a^2-b^2)-c(a-b)}{(a+b-c)(a-b+c)(b+c-a)}$$

$$=\frac{(b^2-c^2+c^2-a^2+a^2-b^2)-\{a(b-c)+b(c-a)+c(a-b)\}}{(a+b-c)(b+c-a)+(c+a-b)}$$

$$=0$$

35 The given expression

$$=\frac{c(a+b)(a+b-c)+a(b+c)(b+c-a)+b(c+a)(c+a-b)}{2abc}$$

$$=\frac{c(a+b)^2-c^2(a+b)+a(b+c)^2-a^2(b+c)+b(c+a)^2-b^2(c+a)}{2abc}$$

$$=\frac{a^2(c-b-c+b)+b^2(c-c-a+a)+c^2(-a-b+a+b)+6abc}{2abc}$$

$$=\frac{6abc}{2abc}=3$$

36 The given expression

$$=\frac{z(x+y)(x^2+y^2-z^2)+x(y+z)(y^2+z^2-x^2)+y(z+x)(z^2+x^2-y^2)}{2xyz}$$

$$+x^2(yz+yz)+y^2(xz+xz)+z^2(xy+xy)$$

$$=\frac{x^3(z-y-z+y)+y^3(z+x-z-x)+z^3(-x-y+x+y)}{2xyz}$$

$$=\frac{2(x^2yz)+2(xy^2z)+2(xyz^2)}{2xyz}$$

$$=\frac{2xyz(x+y+z)}{2xyz}=x+y+z$$

37 The given expression

$$=\frac{c(a+b)(a^3+b^3-c^3)+a(b+c)(b^3+c^3-a^3)+b(c+a)(c^3+a^3-b^3)}{2abc}$$

$$=\frac{a^4(c-b-c+b)+b^4(c+a-c-a)+c^4(-a-b+a+b)+a^3(bc+bc)+b^3(ac+ac)+c^3(ab+ab)}{2abc}$$

$$=\frac{2a^3bc+2ab^3c+2abc^3}{2abc}=a^2+b^2+c^2$$

38 The given expression

$$=\frac{(b+c)(b^2+c^2-a^2)}{2bc}+\frac{(c+a)(a^2+c^2-b^2)}{2ac}+\frac{(a+b)(a^2+b^2-c^2)}{2ab}$$

$$=\frac{a(b+c)(b^2+c^2-a^2)+b(c+a)(a^2+c^2-b^2)+c(a+b)(a^2+b^2-c^2)}{2abc}$$

$$=\frac{a^3(-b-c+b+c)+b^3(a-c-a+c)+c^3(a+b-a-b)+a^2(bc+bc)+b^2(ac+ac)+c^2(ab+ab)}{2abc}$$

$$=\frac{2a^2bc+2ab^2c+2abc^2}{2abc}=a+b+c$$

39 The required value

$$=\frac{a-b}{x-c}+\frac{b-c}{x-a}+\frac{c-a}{x-b}+\frac{(a-b)(b-c)(c-a)}{(x-a)(x-b)(x-c)}$$

$$=\frac{(a-b)\{x^2-x(a+b)+ab\}+(b-c)\{x^2-x(b+c)+bc\}+(c-a)\{x^2-x(c+a)+ca\}}{(x-a)(x-b)(x-c)}+\frac{(a-b)(b-c)(c-a)}{(x-a)(x-b)(x-c)}$$

$$=\frac{x^2\{(a-b)+(b-c)+(c-a)\}-x\{(a^2-b^2)+(b^2-c^2)+(c^2-a^2)\}+ab(a-b)+bc(b-c)+ca(c-a)}{(x-a)(x-b)(x-c)}+\frac{(a-b)(b-c)(c-a)}{(x-a)(x-b)(x-c)}$$

$$=\frac{ab(a-b)+bc(b-c)+ca(c-a)}{(x-a)(x-b)(x-c)}+\frac{(a-b)(b-c)(c-a)}{(x-a)(x-b)(x-c)}$$

$$=\frac{-(a-b)(b-c)(c-a)}{(x-a)(x-b)(x-c)}+\frac{(a-b)(b-c)(c-a)}{(x-a)(x-b)(x-c)}=0$$

40 The left-hand expression

$$=\frac{1}{(b-c)^2}+\frac{1}{(c-a)^2}+\frac{1}{(a-b)^2}$$

$$+2\left\{\frac{1}{(b-c)(c-a)}+\frac{1}{(b-c)(a-b)}+\frac{1}{(c-a)(a-b)}\right\}$$

$$=\frac{1}{(b-c)^2}+\frac{1}{(c-a)^2}+\frac{1}{(a-b)^2}$$

$$+2\left\{\frac{(a-b)+(b-c)+(c-a)}{(a-b)(b-c)(c-a)}\right\}$$

$$=\frac{1}{(b-c)^2}+\frac{1}{(c-a)^2}+\frac{1}{(a-b)^2}+2\times 0$$

$$=\frac{1}{(b-c)^2}+\frac{1}{(c-a)^2}+\frac{1}{(a-b)^2}$$

41 The left-hand expression

$$=\frac{1}{1-x}-\left(\frac{1}{1-x}-\frac{1}{1+x}\right)+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}$$

$$=\frac{1}{1-x}-\frac{2x}{1-x^2}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}$$

$$=\frac{1}{1-x}-2x\left\{\frac{(1+x^2)-(1-x^2)}{(1-x^2)(1+x^2)}\right\}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}$$

$$=\frac{1}{1-x}-\frac{4x^3}{1-x^4}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}$$

$$=\frac{1}{1-x}-4x^3\left\{\frac{(1+x^4)-(1-x^4)}{1-x^8}\right\}+\frac{8x^7}{1+x^8}$$

$$=\frac{1}{1-x}-\frac{8x^7}{1-x^8}+\frac{8x^7}{1+x^8}$$

$$=\frac{1}{1-x}-8x^7\left\{\frac{(1+x^8)-(1-x^8)}{1-x^{16}}\right\}$$

$$=\frac{1}{1-x}-\frac{16x^{15}}{1-x^{16}}$$

42 The given expression

$$=\frac{-(b-c)-(c-a)-(a-b)}{(a-b)(b-c)(c-a)}=0$$

43 The given expression

$$=\frac{-a^2(b-c)-b^2(c-a)-c^2(a-b)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}}{(a-b)(b-c)(c-a)}$$

$$=\frac{-\{-(a-b)(b-c)(c-a)\}}{(a-b)(b-c)(c-a)}=1$$

44 The given expression

$$=\frac{-(x^2+yz)(y-z)-(y^2+zx)(z-x)-(z^2+xy)(x-y)}{(x-y)(y-z)(z-x)}$$

$$=\frac{-\{x^2(y-z)+y^2(z-x)+z^2(x-y)\} -\{yz(y-z)+zx(z-x)+xy(x-y)\}}{(x-y)(y-z)(z-x)}$$

$$=\frac{-\{-(x-y)(y-z)(z-x)\}-\{-(x-y)(y-z)(z-x)\}}{(x-y)(y-z)(z-x)}$$

$$=\frac{2(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)}=2$$

45 The given expression

$$=\frac{-bc(b-c)-ca(c-a)-ab(a-b)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-\{bc(b-c)+ca(c-a)+ab(a-b)\}}{(a-b)(b-c)(c-a)}$$

$$=\frac{-\{-(a-b)(b-c)(c-a)\}}{(a-b)(b-c)(c-a)}=1$$

46 The given expression

$$=\frac{(x^2-yz)(y+z)-(y^2+zx)(x-z)-(z^2+xy)(x-y)}{(x-y)(x-z)(y+z)}$$

$$=\frac{x^2(y+z-z-y)-y^2(z+x-z-x)-z^2(y-x+x-y)}{(x-y)(x-z)(y+z)}=0$$

47 The given expression

$$=\frac{-yz(y-z)-xz(z-x)-xy(x-y)}{xyz(x-y)(y-z)(z-x)}$$

$$=\frac{-\{yz(y-z)+xz(z-x)+xy(x-y)\}}{xyz(x-y)(y-z)(z-x)}$$

$$=\frac{(x-y)(y-z)(z-x)}{xyz(x-y)(y-z)(z-x)}=\frac{1}{xyz}$$

48 The given expression

$$=\frac{-(x-b)(x-c)(b-c)-(x-a)(x-c)(c-a)-(x-a)(x-b)(a-b)}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-\quad[\{x^2-x(b+c)+bc\}(b-c)+\{x^2-x(a+c)+ac\}(c-a)+\{x^2-x(a+b)+ab\}(a-b)]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[x^2\{(b-c)+(c-a)+(a-b)\}-x\{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)\}-\{bc(b-c)+ac(c-a)+ab(a-b)\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-\{-(a-b)(b-c)(c-a)\}}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{1}{(x-a)(x-b)(x-c)}$$

49 The given expression

$$=\frac{-a(b-c)(x-b)(x-c)-b(c-a)(x-a)(x-c)-c(a-b)(x-a)(x-b)}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[a(b-c)\{x^2-x(b+c)+bc\}+b(c-a)\{x^2-x(a+c)+ac\}+c(a-b)\{x^2-x(a+b)+ab\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[x^2\{a(b-c)+b(c-a)+c(a-b)\}-x\{a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)\}+abc\{(b-c)+(c-a)+(a-b)\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{x(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{x}{(x-a)(x-b)(x-c)}$$

50 The given expression

$$=\frac{-a^2(b-c)(x-b)(x-c)-b^2(c-a)(x-a)(x-c)-c^2(a-b)(x-a)(x-b)}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[a^2(b-c)\{x^2-x(b+c)+bc\}+b^2(c-a)\{x^2-x(a+c)+ac\}+c^2(a-b)\{x^2-x(a+b)+ab\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[x^2\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}-x\{a^2(b^2-c^2)+b^2(c^2-a^2)+c^2(a^2-b^2)\}+abc\{a(b-c)+b(c-a)+c(a-b)\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[x^2\{-(a-b)(b-c)(c-a)\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{x^2}{(x-a)(x-b)(x-c)}$$

51 The given expression

$$=\frac{-(a^2+ha+k)(b-c)(x-b)(x-c)-(b^2+hb+k)(c-a)(x-a)(x-c)-(c^2+hc+k)(a-b)(x-a)(x-b)}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[(a^2+ha+k)(b-c)\{x^2-x(b+c)+bc\}+(b^2+hb+k)(c-a)\{x^2-x(a+c)+ac\}+(c^2+hc+k)(a-b)\{x^2-x(a+b)+ab\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{\begin{aligned}-[&x^2\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}+x^2h\{a(b-c)+b(c-a)+c(a-b)\}+\\&x^2k\{-(b-c)+(c-a)+(a-b)\}-x\{a^2(b^2-c^2)+b^2(c^2-a^2)+\\&c^2(a^2-b^2)\}-hx\{a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)\}-xk\{(b^2-c^2)+\\&(c^2-a^2)+(a^2-b^2)\}+abc\{a(b-c)+b(c-a)+c(a-b)\}+abch\{(b-c)+\\&(c-a)+(a-b)\}+k\{bc(b-c)+ac(c-a)+ab(a-b)\}]\end{aligned}}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{-[x^2\{-(a-b)(b-c)(c-a)\}-hx\{(a-b)(b-c)(c-a)\}+k\{-(a-b)(b-c)(c-a)\}]}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{(a-b)(b-c)(c-a)(x^2+hx+k)}{(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)}$$

$$=\frac{x^2+hx+k}{(x-a)(x-b)(x-c)}$$

52 The left-hand expression

$$=\frac{\dfrac{a^2(c-b)}{bc}+\dfrac{b^2(a-c)}{ac}+\dfrac{c^2(b-a)}{ab}}{\dfrac{a(c-b)}{bc}+\dfrac{b(a-c)}{ac}+\dfrac{c(b-a)}{ab}}$$

$$=\frac{\dfrac{-a^3(b-c)-b^3(c-a)-c^3(a-b)}{abc}}{\dfrac{-a^2(b-c)-b^2(c-a)-c^2(a-b)}{abc}}$$

$$=\frac{a^3(b-c)+b^3(c-a)+c^3(a-b)}{a^2(b-c)+b^2(c-a)+c^2(a-b)}$$

$$=\frac{(a+b+c)(a-b)(a-c)(b-c)}{(a-b)(a-c)(b-c)}=a+b+c$$

53 The left-hand expression

$$=\frac{(ab+bc+ca)(a-b)(a-c)(b-c)}{(a-b)(a-c)(b-c)} \quad \left[\begin{array}{l}\text{See page 127}\\ \text{Example 41}\end{array}\right]$$

$$=ab+bc+ca$$

54 The left-hand expression

$$=\frac{a\{a^2+a(b+c)+bc\}}{(a-b)(a-c)}+\frac{b\{b^2+b(a+c)+ac\}}{(b-a)(b-c)}+\frac{c\{c^2+c(a+b)+ab\}}{(c-a)(c-b)}$$

$$=\frac{-a^3(b-c)-a^2(b^2-c^2)-abc(b-c)-b^3(c-a)-b^2(c^2-a^2)-abc(c-a)-c^3(a-b)-c^2(a^2-b^2)-abc(a-b)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-[\{a^3(b-c)+b^3(c-a)+c^3(a-b)\}+\{a^2(b^2-c^2)+b^2(c^2-a^2)+c^2(a^2-b^2)\}+abc\{(b-c)+(c-a)+(a-b)\}]}{(a-b)(b-c)(c-a)}$$

$$=a+b+c$$

55 The left-hand expression

$$=\frac{-b^2c^2(b^2-c^2)-a^2c^2(c^2-a^2)-a^2b^2(b^2-a^2)}{abc(a^2-b^2)(b^2-c^2)(c^2-a^2)}$$

$$=\frac{-\{b^2c^2(b^2-c^2)+a^2c^2(c^2-a^2)+a^2b^2(b^2-a^2)\}}{abc(a^2-b^2)(b^2-c^2)(c^2-a^2)}$$

$$=\frac{(a^2-b^2)(b^2-c^2)(c^2-a^2)}{abc(a^2-b^2)(b^2-c^2)(c^2-a^2)}$$

$$=\frac{1}{abc}$$

56 The given expression

$$=\frac{-bc(x^2-2ax+a^2)(b-c)-ac(x^2-2bx+b^2)(c-a)-ab(x^2-2cx+c^2)(a+b)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-x^2\{bc(b-c)+ac(c-a)+ab(a-b)\}+2abcx\{(b-c)+(c-a)+(a-b)\}-abc\{a(b-c)+b(c-a)+c(a-b)\}}{(a-b)(b-c)(c-a)}$$

$$=\frac{-x^2\{-(a-b)(b-c)(c-a)\}}{(a-b)(b-c)(c-a)}$$

$$=x^2$$

Exercise 62

1
$$
\begin{aligned}
& 4x+3=2x+5 \\
\text{or}\quad & 4x-2x=5-3 \\
\text{or}\quad & 2x=2\,, \\
& x=1
\end{aligned}
$$

2
$$
\begin{aligned}
& 3x+2=x+6 \\
\text{or}\quad & 3x-x=6-2 \\
\text{or}\quad & 2x=4\,, \\
& x=2
\end{aligned}
$$

3
$$
\begin{aligned}
& 5x-6=2x+3 \\
\text{or}\quad & 5x-2x=3+6 \\
\text{or}\quad & 3x=9\,, \\
& x=3
\end{aligned}
$$

4
$$
\begin{aligned}
& 15x-9=11x-25 \\
\text{or}\quad & 15x-11x=-25+9 \\
\text{or}\quad & 4x=-16\,, \\
& x=-4
\end{aligned}
$$

5
$$
\begin{aligned}
& 4(x-3)=2(x-6) \\
\text{or}\quad & 4x-12=2x-12 \\
\text{or}\quad & 4x-2x=-12+12 \\
\text{or}\quad & 2x=0\,, \\
& x=0
\end{aligned}
$$

6
$$
\begin{aligned}
& 2(x-15)=5(x-11)+4 \\
\text{or}\quad & 2x-30=5x-55+4=5x-51 \\
\text{or}\quad & 2x-5x=-51+30 \\
\text{or}\quad & -3x=-21\,; \\
& x=7
\end{aligned}
$$

7
$$
\begin{aligned}
& 19-3x=5x+35 \\
\text{or}\quad & -3x-5x=35-19 \\
\text{or}\quad & -8x=16\,; \\
& x=-2
\end{aligned}
$$

8
$$
\begin{aligned}
& 3(x-2)+7(2x-3)=5(1-2x)-59 \\
\text{or}\quad & 3x-6+14x-21=5-10x-59 \\
\text{or}\quad & 17x-27=-10x-54 \\
\text{or}\quad & 17x+10x=-54+27 \\
\text{or}\quad & 27x=-27\,, \qquad x=-1
\end{aligned}
$$

9
$$
\begin{aligned}
& 13x-4(5x-8)+17=0 \\
\text{or}\quad & 13x-20x+32+17=0 \\
\text{or}\quad & -7x+49=0 \\
\text{or}\quad & -7x=-49\,, \qquad x=7
\end{aligned}
$$

10
$$
\begin{aligned}
& 14(x-4)+3(x+5)=6(7-2x)+4 \\
\text{or}\quad & 14x-56+3x+15=42-12x+4 \\
\text{or}\quad & 17x-41=46-12x \\
\text{or}\quad & 17x+12x=46+41 \\
\text{or}\quad & 29x=87\,, \qquad x=3
\end{aligned}
$$

11 $8(2x-7)-9(3x-14)=15$

or $16x-56-27x+126=15$

or $-11x+70=15$

or $-11x=15-70=-55$, $x=5$.

12 $3x-13(2x-13)=4x-20$

or $3x-26x+169=4x-20$

or $-23x+169=4x-20$

or $-23x-4x=-20-169$

or $-27x=-189$, $x=7$

13 $49+13(5x+27)=8(5+x)-3x$

or $49+65x+351=40+8x-3x$

or $65x+400=40+5x$

or $65x-5x=40-400$

or $60x=-360$, $x=-6$

14 $16-5(7x-2)=13(x-2)+4(13-x)$

or $16-35x+10=13x-26+52-4x$

or $26-35x=9x+26$

or $-35x-9x=26-26$

or $-44x=0$, $x=0$

15 $8x+5(x+7)+9(2x+23)-3(x+6)=0$

or $8x+5x+35+18x+207-3x-18=0$

or $8x+5x+18x-3x=-35-207+18$

or $28x=-224$, $x=-8$

16 $(x-7)(4x-29)=(2x-5)(2x-17)+1$

or $4x^2-57x+203=4x^2-44x+85+1$

or $-57x+203=-44x+86$

or $-57x+44x=86-203$

or $-13x=-117$, $x=9$.

17 $(3x+2)(2x-6)=(4-3x)(1-2x)-10$

or $6x^2-14x-12=4-11x+6x^2-10$

or $-14x-12=-11x-6$

or $-14x+11x=-6+12$

or $-3x=6$, $x=-2$

18 $(3x+5)(6x-7)=(3x+2)(9x-13)-(3x+1)(3x-1)$

or $18x^2+9x-35=27x^2-21x-26-9x^2+1$

or $18x^2+9x-35=18x^2-21x-25$

or $9x+21x=-25+35$

or $30x=10\,,$ $\qquad x=\frac{1}{3}$

19 $(x+2)(2x+5)=2(x+1)^2+13$

or $2x^2+9x+10=2x^2+4x+2+13$

or $9x+10=4x+15$

or $9x-4x=15-10$

or $5x=5\,,$ $\qquad x=1$

20 $(x+1)(4x-7)-(x-1)(x+5)=3(x+2)^2+5$

or $4x^2-3x-7-(x^2+4x-5)=3x^2+12x+12+5$

or $3x^2-3x-4x-7+5=3x^2+12x+17$

or $-3x-4x-12x=17+7-5$

or $-19x=19\,,$ $\qquad x=-1$

21 $3(x-4)^2+5(x-3)^2=(2x-5)(4x-1)+24$

or $3(x^2-8x+16)+5(x^2-6x+9)=8x^2-22x+5+24$

or $3x^2-24x+48+5x^2-30x+45=8x^2-22x+29$

or $-24x-30x+22x=29-48-45$

or $-32x=-64$ $\qquad x=2$

22 $(6x+9)^2+(8x-7)^2=(10x+3)^2-71$

or $36x^2+108x+81+64x^2-112x+49$

$=100x^2+60x+9-71$

or $108x-112x-60x=9-71-81-49$

or $-64x=-192\,,$ $\qquad x=3$

23 $5(x+1)^2+7(x+3)^2=12(x+2)^2$

or $5(x^2+2x+1)+7(x^2+6x+9)=12(x^2+4x+4)$

or $5x^2+10x+5+7x^2+42x+63=12x^2+48x+48$

or $10x+42x-48x=48-5-63$

or $4x=-20\,,$ $\qquad x=-5$

24 $(3x-14)^2+(4x-19)^2-(5x-23)^2=22$

or $9x^2-84x+196+16x^2-152x+361-(25x^2-230x+529)$
$=22$

or $-84x-152x+230x=22-196-361+529$

or $-6x=-6$, $x=1$

25 $(5x-8)^2+(12x-7)^2=(13x-10)^2+37$

or $25x^2-80x+64+144x^2-168x+49$
$=169x^2-260x+100+37$

or $-80x-168x+260x=100+37-64-49$

or $12x=24$, $x=2$

26 $(x-1)^3+(x+1)^3=2x(x^2-1)+4$

or $2(x^3+3x)=2(x^3-x+2)$

or $x^3+3x=x^3-x+2$

or $3x+x=2$

or $4x=2$, $x=\frac{1}{2}$

27. $(x-2)^3+2x^3+(x+2)^3=4x^2(x+2)$

or $2(x^3+12x)+2x^3=4x^2(x+2)$

or $2x^3+24x+2x^3=4x^3+8x^2$

or $24x=8x^2$

or $24=8x$, $x=3$

28 $(x+2)(x+3)(x+4)+96=x^2(x+9)+5(3x+13)$

or $x^3+(2+3+4)x^2+(6+8+12)x+24+96$
$=x^3+9x^2+15x+65$

or $9x^2+26x+120=9x^2+15x+65$

or $26x-15x=65-120$

or $11x=-55$, $x=-5$

29 $3(x^2-14)=(x+1)^2+(x-2)^2+(x-5)^2$

or $3(x^2-14)=x^2+2x+1+x^2-4x+4+x^2-10x+25$

or $3(x^2-14)=3(x^2-4x+10)$

or $x^2-14=x^2-4x+10$

or $4x=10+14=24$, $x=6$

30. $$a(x-a)=b(x-b)$$
or $ax-a^2=bx-b^2$
or $ax-bx=a^2-b^2$
or $x(a-b)=(a+b)(a-b)$, $\qquad x=a+b$

31 $$3(x-a)+5(2x-3a)=8a$$
or $3x-3a+10x-15a=8a$
or $13x=8a+3a+15a=26a$, $\qquad x=2a$

32 $$(x+a)(x+b)-(a+b)^2=(x-a)(x-b)$$
or $x^2+x(a+b)+ab-(a+b)^2=x^2-x(a+b)+ab$
or $2x(a+b)=(a+b)^2$, $\qquad x=\frac{1}{2}(a+b)$

33 $$a^2(x-a)+b^2(x-b)=abx$$
or $x(a^2+b^2)-(a^3+b^3)=abx$
or $x(a^2+b^2-ab)=a^3+b^3$
$=(a+b)(a^2-ab+b^2)$, $\qquad x=a+b$

34 $$m^2(x-m)+n^2(x+n)+mnx=0$$
or $x(m^2+n^2+mn)-(m^3-n^3)=0$
or $x(m^2+mn+n^2)=(m^3-n^3)$
$=(m-n)(m^2+mn+n^2)$, $\qquad x=m-n$

35 $$b(x-2a)+a(x-2b)=(a-b)^2$$
or $x(a+b)-4ab=(a-b)^2$
or $x(a+b)=(a-b)^2-4ab$
$=(a+b)^2$; $\qquad x=a+b$

36 $$a(4x-a)\times b(4x-b)-2ab=0$$
or $4x(a+b)-(a^2+b^2+2ab)=0$
or $4x(a+b)=(a+b)^2$, $\qquad x=\frac{1}{4}(a+b)$

37 $$x(x-a)+x(x-b)-2(x-a)(x-b)=0$$
or $2x^2-x(a+b)-2\{x^2-x(a+b)+ab\}=0$
or $x(a+b)-2ab=0$
or $x(a+b)=2ab$; $\qquad x=\dfrac{2ab}{a+b}$

38. $(x+3a)(x-3b)+3(x-3a)(x+3b)=4(x-3a)(x-3b)$

or $x^2+3x(a-b)-9ab+3\{x^2-3x(a-b)-9ab\}$
$=4\{x^2-3x(a+b)+9ab\}$

or $3x\{(a-b)-3(a-b)+4(a+b)\}=9ab(4+3+1)$

or $3x(2a+6b)=9ab\ 8$

or $x(2a+6b)=3ab\ 8$

or $2x(\ a+3b)=3ab\ 8$

or $x(a+3b)=12ab$, $\quad x=\dfrac{12ab}{a+3b}$

39 $(2b+2c-x)^2+(2b-2c+x)^2=(2b+2d-x)^2$
$+(2b-2d+x)^2$

or $(2b+2c)^2-2x(2b+2c)+x^2+(2b-2c)^2+2x(2b-2c)+x^2$
$=(2b+2d)^2-2x(2b+2d)$
$+x^2+(2b-2d)^2+2x(2b-2d)+x^2$

or $2(4b^2+4c^2)-2x(4c)=2(4b^2+4d^2)-2x(4d)$

or $-8x(c-d)=-8(c^2-d^2)$

or $x(c-d)=(c+d)(c-d)$, $\quad x=c+d$

40 $(x-a)^3+(x-b)^3+(x-c)^3=3(x-a)(x-b)(x-c)$

or $x^3-3ax^2+3a^2x-a^3+x^3-3bx^2+3b^2x-b^3$
$+x^3-3cx^2+3c^2x-c^3$
$=3\{x^3-x^2(a+b+c)+x(ab+ac+bc)-abc\}$

or $3x^2\{(a+b+c)-a-b-c\}+3x(a^2+b^2+c^2-ab-ac-bc)$
$=a^3+b^3+c^3-3abc$

or $3x(a^2+b^2+c^2-ab-ac-bc)=(a+b+c)(a^2+b^2+c^2$
$-ab-ac-bc)$, $\quad x=\frac{1}{3}(a+b+c)$

41. $(x+a)^3+(x+b)^3+(x+c)^3=3(x+a)(x+b)(x+c)$

or $x^3+3ax^2+3a^2x+a^3+x^3+3bx^2+3b^2x+b^3+x^3+3cx^2$
$+3c^2x+c^3=3\{x^3+x^2(a+b+c)+x(ab+ac+bc)+abc\}$

or $3x^2\{a+b+c-(a+b+c)\}+3x(a^2+b^2+c^2-ab-ac-bc)$
$=3abc-(a^3+b^3+c^3)$

or $3x(a^2+b^2+c^2-ab-ac-bc)=-(a^3+b^3+c^3-3abc)$
$=-(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$x=-\frac{1}{3}(a+b+c)$

42 $$\frac{x}{2}+5=\frac{x}{3}+7,$$

or $$\left(\frac{x}{2}-\frac{x}{3}\right)=7-5$$

or $$\frac{x}{6}=2, \qquad x=12$$

43 $$\frac{x}{a}+a=\frac{x}{b}+b,$$

or $$\frac{x}{a}-\frac{x}{b}=b-a$$

or $$x\left(\frac{b-a}{ab}\right)=b-a,$$

$$x=ab,$$

44 $$\frac{x}{6}-\frac{x}{5}=\frac{x}{15}-\frac{x}{3}+7$$

or $$-\frac{x}{30}=-x\left(\frac{4}{15}\right)+7$$

or $$x\left(\frac{4}{15}-\frac{1}{30}\right)=7$$

or $$x\left(\frac{8-1}{30}\right)=7$$

or $$x\left(\frac{7}{30}\right)=7, \qquad x=30$$

45 $$\frac{x}{2}-\frac{x}{3}+\frac{x}{4}=2-\frac{x}{6}+\frac{5x}{12}$$

or $$x(\tfrac{1}{2}-\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{6}-\tfrac{5}{12})=2$$

or $$x\left(\frac{6-4+3+2-5}{12}\right)=2$$

or $$x(\tfrac{2}{12})=2,$$

46 $$\frac{a}{bx}-\frac{b}{ax}=a^2-b^2$$

or $$\frac{a^2-b^2}{abx}=a^2-b^2$$

or $$abx=1, \qquad x=\frac{1}{ab}$$

47 $$\tfrac{1}{2}(x+1)+\tfrac{1}{3}(x+2)+\tfrac{1}{4}(x+3)=16$$

or $$\tfrac{1}{12}\{(6x+6)+(4x+8)+(3x+9)\}=16$$

or $$13x+23=192$$

or $$13x=192-23=169, \qquad x=13$$

48 $$\frac{x-6}{5}+\frac{x-4}{3}=8-\frac{x-2}{7}$$

or $\frac{1}{15}\{(3x-18)+(5x-20)\}=\frac{1}{7}(56-x+2)$

or $7(8x-38)=15(58-x)$

or $56x+15x=870+266$

or $71x=1136$, $x=16$

49 $$\frac{x}{10}+\frac{2x-13}{9}=8-\frac{4x-35}{15}$$

or $\frac{1}{90}(9x+20x-130)=\frac{1}{15}(120-4x+35)$

or $29x-130=6(155-4x)$

or $29x+24x=930+130$

or $53x=1060$, $x=20$

50 $$\frac{x+7}{2}+\frac{x+13}{5}+\frac{x+17}{7}=\frac{x+27}{4}$$

or $\frac{35(x+7)+14(x+13)+10(x+17)}{70}=\frac{x+27}{4}$

or $70(x+7)+28(x+13)+20(x+17)=35(x+27)$

or $70x+490+28x+364+20x+340=35x+945$

or $70x+28x+20x-35x=945-490-364-340$

or $83x=-249$, $x=-3$

51 $$6\tfrac{1}{3}-\frac{x-7}{3}=\frac{4x-2}{5}$$

or $\frac{19-x+7}{3}=\frac{4x-2}{5}$

or $5(26-x)=3(4x-2)$

or $130-5x=12x-6$

or $-5x-12x=-6-130$

or $-17x=-136$, $x=8$

52 $$\frac{x-1}{3}-\frac{x-9}{2}+\frac{3x-2(x-2)}{7}=4\tfrac{1}{2}$$

or $\frac{x-1}{3}-\frac{x-9}{2}+\frac{3x-2x+4}{7}=\frac{9}{2}$

or $14(x-1)-21(x-9)+6(x+4)=189$

or $14x-14-21x+189+6x+24=189$

or $14x-21x+6x=14-24$, $x=10$

53 $$\frac{2x-9}{27}+\frac{x}{18}-\frac{x-3}{4}=8\tfrac{1}{3}-x$$

or $4(2x-9)+6x-27(x-3)=36\times 25-108x$

or $8x-36+6x-27x+81=900-108x$

or $8x+6x-27x+108x=900+36-81$

or $95x=855$, $x=9$

54 $$\frac{9x+7}{2}-\left(x-\frac{x-2}{7}\right)=36$$

or $7(9x+7)-2(7x-x+2)=14\times 36$

or $63x+49-12x-4=504$

or $63x-12x=504-49+4$

or $51x=459$, $x=9$

55 $$\frac{7x+9}{4}-\left(x-\frac{2x-1}{9}\right)=7$$

or $9(7x+9)-4(9x-2x+1)=36\times 7$

or $63x+81-28x-4=252$

or $63x-28x=252-81+4$

or $35x=175$, $x=5$

56 $$\frac{x+7}{3}-5\tfrac{3}{4}=\frac{2x+5}{7}+\frac{10-5x}{8}$$

or $56(x+7)-23\times 42=24(2x+5)+21(10-5x)$

or $56x+392-966=48x+120+210-105x$

or $56x-48x+105x=120+210-392+966$

or $113x=904$, $x=8$

57 $$x-\left(3x-\frac{2x-5}{10}\right)=\tfrac{1}{6}(2x-57)-\tfrac{5}{3}$$

or $30x-3(30x-2x+5)=5(2x-57)-50$

or $30x-84x-15=10x-285-50$

or $30x-84x-10x=-285-50+15$

or $-64x=-320$, $x=5$

58 $$\frac{4x-21}{7}+7\tfrac{5}{8}+\frac{7x-28}{3}=x+3\tfrac{3}{4}-\frac{9-7x}{8}+\tfrac{1}{12}$$

or $x(\tfrac{4}{7}+\tfrac{7}{3})+7\tfrac{5}{8}-3-9\tfrac{1}{3}=x(1+\tfrac{7}{8})+3\tfrac{3}{4}-\tfrac{9}{8}+\tfrac{1}{12}$

or $x\left(\frac{12+49}{21}\right)+7\frac{5}{6}-12\frac{1}{3}=\frac{15}{8}x+3\frac{5}{6}-1\frac{1}{8}$

or $x\left(\frac{61}{21}-\frac{15}{8}\right)=3\frac{5}{6}-1\frac{1}{8}-7\frac{5}{6}+12\frac{1}{3}$

or $x\left(\frac{488-315}{168}\right)=-4+11+\frac{1}{3}-\frac{1}{8}$

or $x\,\frac{173}{168}=7\frac{5}{24}=\frac{173}{24}$, $x=\frac{168}{24}=7$

59 $\frac{1}{2}\left(x-\frac{a}{3}\right)-\frac{1}{3}\left(x-\frac{a}{4}\right)+\frac{1}{4}\left(x-\frac{a}{5}\right)=0$

or $6\left(x-\frac{a}{3}\right)-4\left(x-\frac{a}{4}\right)+3\left(x-\frac{a}{5}\right)=0$

or $6x-2a-4x+a+3x-\frac{3a}{5}=0$

or $6x-4x+3x=2a-a+\frac{3a}{5}$

or $5x=8\frac{a}{5}$, $x=\frac{8a}{25}$

60 $\frac{x-3}{7}-\frac{\frac{1}{2}x-3}{3}=\frac{\frac{1}{6}x+2}{2}-\frac{x-6}{3}+\frac{x}{8}$

or $x(\frac{1}{7}-\frac{1}{6}-\frac{1}{12}+\frac{1}{3}-\frac{1}{8})=1+2+\frac{3}{7}-1$

or $x\left(\frac{24-28-14+56-21}{24\times7}\right)=\frac{17}{7}$

or $x\,\frac{17}{24\times7}=\frac{17}{7}$, $x=24$.

61. $\frac{1}{8}(x-2)-\frac{1}{7}(x-4)=\frac{1}{12}(2x-3)-2\frac{3}{4}$

or $x(\frac{1}{8}-\frac{1}{7}-\frac{1}{6})=\frac{1}{4}-\frac{11}{4}-\frac{1}{4}-\frac{4}{7}$

or $x\left(\frac{21-24-28}{8\times7\times3}\right)=\frac{7-77-7-16}{7\times4}$

or $x\,\frac{-31}{8\times7\times3}=\frac{-93}{7\times4}$

or $\frac{x}{6}=3$, $x=18$

62 $$\frac{a-x}{a}+\frac{2a-x}{2a}=\frac{3a-x}{3a}$$

or $$1-\frac{x}{a}+1-\frac{x}{2a}=1-\frac{x}{3a}$$

or $$x\left(-\frac{1}{a}-\frac{1}{2a}+\frac{1}{3a}\right)=-1$$

or $$x\left(\frac{-6-3+2}{6a}\right)=-1$$

or $$x\,\frac{-7}{6a}=-1, \qquad x=\frac{6a}{7}$$

63 $$\frac{2x-13}{9}-\frac{x-1}{11}=\frac{x}{8}+\frac{x}{7}-9$$

or $$x\left(\tfrac{2}{9}-\tfrac{1}{11}-\tfrac{1}{8}-\tfrac{1}{7}\right)=-9-\tfrac{1}{11}+\tfrac{13}{9}$$

or $$x\left(\frac{22-9}{99}-\frac{7+8}{56}\right)=\frac{-891-9+143}{99}$$

or $$x\left(\frac{13}{99}-\frac{15}{56}\right)=\frac{-757}{99}$$

or $$x\left(\frac{728-1485}{99\times 56}\right)=\frac{-757}{99}$$

or $$x\,\frac{-757}{99\times 56}=\frac{-757}{99},$$

64 $$\frac{2x-3}{6}+\frac{3x-8}{11}=\frac{4x+15}{33}+\frac{1}{2}$$

or $$x\left(\tfrac{1}{3}+\tfrac{3}{11}-\tfrac{4}{33}\right)=\tfrac{1}{2}+\tfrac{15}{33}+\tfrac{1}{2}+\tfrac{8}{11}$$

or $$x\left(\frac{11+9-4}{33}\right)=1+\frac{15+24}{33}$$

or $$x\,\tfrac{16}{33}=1+\tfrac{13}{11}$$

or $$x\,\tfrac{16}{33}=\tfrac{24}{11}, \qquad x=\tfrac{24}{11}\times\tfrac{33}{16}=\tfrac{9}{2}=4\tfrac{1}{2}$$

65 $$\frac{4x+3}{9}+\frac{13x}{108}=\frac{8x+19}{18}$$

or $$12(4x+3)+13x=6(8x+19)$$

or $$48x+36+13x=48x+114$$

or $$13x=114-36=78, \qquad x=6$$

66 $$\frac{x^2-2\frac12}{4}-\frac{x-3\frac12}{5}=\frac{2x^2-3}{8}-\frac{x-5\frac12}{3}$$

or $$\frac{x^2}{4}-\frac58-\frac{x}{5}+\frac{7}{10}=\frac{x^2}{4}-\frac38-\frac{x}{3}+\frac{11}{6}$$

or $$x\left(-\tfrac15+\tfrac13\right)=-\tfrac38+\tfrac{11}{6}+\tfrac58-\tfrac{7}{10}$$

or $$x\left(\frac{-3+5}{15}\right)=\frac{-45+220+75-84}{120}$$

or $$x\tfrac{2}{15}=\tfrac{166}{120}\ ,\ \therefore\quad x=\tfrac{166}{120}\times\tfrac{15}{2}=\tfrac{83}{8}=10\tfrac38.$$

67 $$\frac{a-x^2}{bx}-\frac{b-x}{c}=\frac{c-x}{b}-\frac{b-x^2}{cx}$$

or $$c(a-x^2)-bx(b-x)=cx(c-x)-b(b-x^2)$$

or $$ac-cx^2-b^2x+bx^2=c^2x-cx^2-b^2+bx^2$$

or $$ac-b^2x=c^2x-b^2$$

or $$-b^2x-c^2x=-b^2-ac$$

or $$x(b^2+c^2)=b^2+ac\ ,\qquad x=\frac{ac+b^2}{b^2+c^2}$$

68 $$\frac{x+2\frac12}{15}+\frac{x+3\frac13}{25}=\frac{x+4\frac16}{55}$$

or $$\frac{x+2\frac12}{3}+\frac{x+3\frac13}{5}=\frac{x+4\frac16}{11}$$

or $$x\left(\tfrac13+\tfrac15-\tfrac{1}{11}\right)=\tfrac{25}{66}-\tfrac56-\tfrac23$$

or $$x\left(\frac{55+33-15}{15\times 11}\right)=\frac{25-55-44}{66}$$

or $$x\tfrac{73}{165}=-\tfrac{74}{66}=-\tfrac{37}{33}$$

$$x=-\tfrac{37}{11}\times\tfrac{55}{73}=-\tfrac{185}{73}=-2\tfrac{39}{73}$$

69 $$\frac{11x-13}{25}+\frac{19x+3}{7}-\frac{5x-25\frac13}{4}=28\tfrac17-\frac{17x+4}{21}$$

or $$84(11x-13)+300(19x+3)-525(5x-\tfrac{76}{3})$$
$$=300\times 197-100(17x+4)$$

or $$x(924+5700-2625+1700)=59100-400$$
$$-13300-900+1092$$

or $$5699x=45592\ ,\qquad \therefore\ x=8$$

70 $$\frac{x-\frac{5}{39}}{2}-\frac{2-6x}{13}=x-\frac{5x-\frac{1}{3}(10-3x)}{39}$$

or $39(x-\frac{5}{39})-6(2-6x)=78x-2(5x-\frac{10}{3}+\frac{2}{3}x)$

or $x(39+36-78+10+\frac{2}{3})=5+12+\frac{152}{3}$

or $x\frac{17}{3}=\frac{187}{3}, \qquad x=11$

71. $$\frac{3x-\frac{2}{3}(1+x)}{4}+\frac{1-\frac{1}{5}x}{5\frac{1}{2}}=\frac{2\frac{2}{5}+\frac{1}{25}(x-1)}{2\frac{1}{5}}$$

or $\frac{9x-2-2x}{12}+\frac{2(5-x)}{55}=\frac{60+x-1}{55}$

or $55(7x-2)+24(5-x)=12(59+x)$

or $385x-110+120-24x=708+12x$

or $385x-24x-12x=708+110-120$

or $349x=698, \qquad x=2$

72 $$\frac{1}{3}(x-a)-\frac{1}{5}(2x-3b)-\frac{1}{2}(a-x)=30(10a+11b)$$

or $10(x-a)-6(2x-3b)+15(x-a)=30(10a+11b)$

or $10x-12x+15x=300a+330b+10a-18b+15a$

or $13x=325a+312b=13(25a+24b)$

$x=25a+24b.$

73 $$\frac{2x+a}{b}-\frac{x-b}{a}=\frac{3ax+(a-b)^2}{ab}$$

or $a(2x+a)-b(x-b)=3ax+(a-b)^2$

or $x(2a-b-3a)=(a-b)^2-a^2-b^2$

or $-x(a+b)=-2ab, \qquad x=\frac{2ab}{a+b}$

74 $$\frac{2x+1}{29}-\frac{402-3x}{12}=9-\frac{471-6x}{2}$$

or $12(2x+1)-29(402-3x)=29\times108-174(471-6x)$

or $24x+87x-1044x=3132-81954-12+11658$

or $-933x=-67176, \qquad x=72$

75 $$\frac{x+5}{4x-9}=\frac{x+10\frac{1}{3}}{4x-7}$$

or $(4x-7)(x+5)=\frac{1}{3}(4x-9)(3x+31)$

or $3(4x^2+13x-35)=12x^2+97x-297$

or $39x-97x=-279+105$

or $-58x=-174$, $\quad x=3$

76 $$\frac{7x+2}{17x+14}=\frac{7x+6}{17x+26}$$

or $$\frac{7x+2}{17x+14}=\frac{7x+2}{17x+26}+\frac{4}{17x+26}$$

or $$(7x+2)\left(\frac{1}{17x+14}-\frac{1}{17x+26}\right)=\frac{4}{17x+26}$$

or $$(7x+2)\left\{\frac{17x+26-17x-14}{(17x+14)(17x+26)}\right\}=\frac{4}{17x+26}$$

or $$\frac{12(7x+2)}{17x+14}=4$$

or $3(7x+2)=17x+14$

or $21x-17x=14-6$

or $4x=8$, $\quad x=2$

77 $$\frac{2x-7}{3x-8}=\frac{3(2x-1)}{9x-2}$$

or $(2x-7)(9x-2)=(3x-8)(6x-3)$

or $18x^2-67x+14=18x^2-57x+24$

or $-67+57x=24-14$

or $-10x=10$; $\quad x=-1$

78 $$\frac{13-6x}{3+4x}=\frac{7-9x}{7+6x}$$

or $(13-6x)(7+6x)=(7-9x)(3+4x)$

or $91+36x-36x^2=21+x-36x^2$

or $36x-x=21-91$

or $35x=-70$, $\quad x=-2$

79 $$\frac{10(7+4x)}{7+15x}=\frac{27+8x}{4+3x}$$

or $10(7+4x)(4+3x)=(7+15x)(27+8x)$

or $280+370x+120x^2=189+461x+120x^2$

or $370x-461x=189-280$

or $-91x=-91$, $\quad x=1$

80 $$\frac{15-\frac{2}{3}x}{5}-\frac{2x+5}{2\frac{1}{2}}=\frac{17-\frac{14}{3}x}{3}$$

or $3-\frac{2}{15}x-\frac{4}{5}x-2=\frac{17}{3}-\frac{14}{9}x$

or $-\frac{2}{15}x-\frac{4}{5}x+\frac{14}{9}x=\frac{17}{3}-3+2$

or $\frac{-6x-36x+70x}{45}=\frac{14}{3}$

or $\frac{28x}{45}=\frac{14}{3}$ or $\frac{2x}{15}=1$, $x=\frac{15}{2}=7\frac{1}{2}$

Exercise 63

1 $5x-2x=3x-1\ 5$

or $5x-x(2+3)=-1\ 5$

or $5x-5x=-1\ 5$

or $-05x=-1\ 5$, $x=\frac{1\ 5}{05}=\frac{15}{5}=\frac{15+9}{5}=27$

2 $3\ 75x+5=2\ 25x+8$

or $x(3\ 75-2\ 25)=8-5$

or $1\ 5x=7\ 5$, $x=\frac{75}{15}=5$

3 $1\ 2x-\frac{18x-05}{5}=4x+8\ 9$

or $6x-18x-05=2x+4\ 45$

or $6x-18x-2x=4\ 45-05$

or $22x=4\ 4$, $x=\frac{440}{22}=20$

4 $\frac{x+75}{125}-\frac{x-25}{25}=15$

or $2x+1\ 5-x+25=3\ 75$ (multiplying both sides by 25)

$x=3\ 75-1\ 5-25=2$

5 $\frac{x}{5}-\frac{1}{05}+\frac{x}{005}-\frac{1}{0005}=0$

or $x-10+100x-1000=0$

or $101x=1010$, $x=10$

6 $$\cdot 5x+\frac{\cdot 45x-\cdot 75}{\cdot 6}=\frac{1\cdot 2}{\cdot 2}-\frac{\cdot 3x-\cdot 6}{\cdot 9}$$

or $\cdot 5x+\frac{1\cdot 5x-2\cdot 5}{2}=6-\frac{x-2}{3}$

or $3x+4\cdot 5x-7\cdot 5=36-2x+4$

or $3x+4\cdot 5x+2x=36+4+7\cdot 5$

or $9\cdot 5x=47\cdot 5$; $\therefore\ x=5$

7. $$\frac{x+2}{x-2}+6\cdot\dot{8}=5\cdot 3\dot{8}$$

or $\frac{x+2}{x-2}=5\cdot 3\dot{8}-6\cdot\dot{8}$

or $\frac{x+2}{x-2}=-1\cdot 5$

or $x+2=-1\cdot 5x+3$

or $2\cdot 5x=1$; $\therefore\ x=\frac{10}{25}=\frac{2}{5}$

8 $$\cdot 7x-\cdot 4=\cdot 6\dot{7}x-\cdot 5$$

or $\cdot 7x-\cdot 6\dot{7}x=\cdot 5-\cdot 4$

or $\cdot 0\dot{2}x=\cdot 1$ $\therefore\ x=\frac{1}{9}\times\frac{90}{2}=5$

9 $$\frac{2x-3}{3x-4}=\frac{\cdot\dot{4}x-\cdot\dot{6}}{\cdot 0\dot{6}x-\cdot 0\dot{7}}$$

or $\frac{2x-3}{3x-4}=\frac{4x-6}{6x-7}=\frac{4x-6}{6x-7}$

or $(2x-3)(6x-7)=(4x-6)(3x-4)$

or $12x^2-32x+21=12x^2-34x+24$

or $-32x+34x=24-21=3$

or $2x=3$, $\therefore\ x=1\frac{1}{2}$

10 $$\cdot 15x+\frac{\cdot 135x-\cdot 225}{\cdot 6}=\frac{\cdot 36}{\cdot 2}-\frac{\cdot 09x-\cdot 18}{\cdot 9}$$

or $\cdot 15x+\frac{\cdot 45x-\cdot 75}{2}=1\cdot 8-\cdot 1x+\cdot 2$

or $\cdot 3x+\cdot 45x-\cdot 75=3\cdot 6-\cdot 2x+\cdot 4$

or $30x+45x-75=360-20x+40$

or $30x+45x+20x=360+40+75$

or $95x=475$. $\therefore\ x=5$

11 $$5x+\frac{02x+07}{03}-\frac{x+2}{9}=9\ 5$$

or $4\ 5x+6x+21-x-2=85\ 5$

or $4\ 5x+6x-x=85\ 5-21+2$

or $9\ 5x=66{\cdot}5$, $x=7$

12 $$\frac{4\ 05}{9x}-\frac{3}{8-2x}=\frac{1\ 8}{x}-\frac{3\ 6}{2\ 4-6x}$$

or $\frac{13\ 5}{9x}-\frac{1}{2(4-x)}=\frac{6}{x}-\frac{12}{6(4-x)}$

or $\frac{1\ 5}{x}-\frac{1}{2(4-x)}=\frac{6}{x}-\frac{2}{(4-x)}$

or $-\frac{1}{2(4-x)}+\frac{2}{(4-x)}=\frac{6}{x}-\frac{1\ 5}{x}$

or $\frac{3}{2}\left(\frac{1}{4-x}\right)=\frac{4\ 5}{x}$

or $\frac{1}{2(4-x)}=\frac{1\ 5}{x}$

or $x=1\ 2-3x$

or $4x=1\ 2$, $x=3$

13 $$011x+\frac{001x-125}{6}=\frac{5-x}{03}-145$$

or $11x+\frac{x-125}{6}=\frac{5000-1000x}{03}-145$

or $33x+5x-625=500000-100000x-435$

01 $38x+100000x=500000-435+625$

or $100038x=500190$, $x=5$

Exercise (64)

1 $$\frac{5x+6}{4}+\frac{64x-35}{15}=\frac{20x+23}{16}+\frac{13x-7}{3}$$

or $\frac{5x+6}{4}-\frac{20x+23}{16}=\frac{13x-7}{3}-\frac{64x-35}{15}$

or $\frac{20x+24-20x-23}{16}=\frac{65x-35-64x+35}{15}$

or $\frac{1}{16}=\frac{x}{15}$, $x=\frac{15}{16}$

2 $$\frac{17x-13}{9}+\frac{108x-75}{32}=\frac{27x+19}{8}+\frac{(50\frac{7}{16})x-39}{27}$$

or $$\frac{17x-13}{9}-\frac{(50\frac{7}{16})x-39}{27}=\frac{27x+19}{8}-\frac{108x+75}{32}$$

or $$\frac{51x-39-50\frac{7}{16}x+39}{27}=\frac{108x+76-108x-75}{32}$$

or $\frac{\frac{9}{16}x}{27}=\frac{1}{32}$ or $\frac{9}{16}x\times 32=27$, $x=\frac{27}{18}=$

3 $$\frac{29x-18}{8}+\frac{189x-93}{49}=\frac{(86\frac{13}{21})x-54}{24}+\frac{27x-13}{7}$$

or $$\frac{29x-18}{8}-\frac{86\frac{13}{21}x-54}{24}=\frac{27x-13}{7}-\frac{189x-93}{49}$$

or $$\frac{87x-54-86\frac{13}{21}x+54}{24}=\frac{189x-91-189x+93}{49}$$

or $\frac{\frac{8}{21}x}{24}=\frac{2}{49}$ or $\frac{8x}{21\times 24}=\frac{2}{49}$,

$$x=\frac{63\times 2}{49}=\frac{18}{7}=2\frac{4}{7}$$

4 $$\frac{16x-17}{9}-\frac{23x-15}{16}=\frac{142\frac{7}{16}x-153}{81}-\frac{92x-65}{64}$$

or $$\frac{16x-17}{9}-\frac{142\frac{7}{16}x-153}{81}=-\frac{92x-65}{64}+\frac{23x-15}{16}$$

or $$\frac{144x-153-142\frac{7}{16}x+153}{81}=\frac{-92x+65+92x-60}{64}$$

or $\frac{1\frac{9}{16}x}{81}=\frac{5}{64}$, $x=\frac{81\times 5\times 16}{64\times 25}=\frac{81}{20}=4.05$

5 $$\frac{18x-19}{7}+\frac{135x+62\frac{1}{2}}{65}=\frac{27x+14}{13}+\frac{106\frac{5}{13}x-114}{42}$$

or $$\frac{18x-19}{7}-\frac{106\frac{5}{13}x-114}{42}=\frac{27x+14}{13}-\frac{135x+62\frac{1}{2}}{65}$$

or $$\frac{108x-114-106\frac{5}{13}x+114}{42}=\frac{135x+70-135x-62\frac{1}{2}}{65}$$

or $\frac{1\frac{8}{13}x}{42}=\frac{7\frac{1}{2}}{65}$ or $\frac{21x}{13\times 42}=\frac{15}{2\times 65}$

or $\frac{x}{2\times 13}=\frac{3}{2\times 13}$, $x=3$

6 $$\frac{33-19x}{15}-\frac{41+27x}{28}+\frac{164+(107\frac{11}{15})x}{112}-\frac{164\frac{13}{28}-95x}{75}=0$$

or $$\frac{33-19x}{15}-\frac{164\frac{13}{28}-95x}{75}=\frac{41+27x}{28}-\frac{164+(107\frac{11}{15})x}{112}$$

or $$\frac{165-95x-164\frac{13}{28}+95x}{75}=\frac{164+108x-164-107\frac{11}{15}x}{112}$$

or $$\frac{\frac{15}{28}}{75}=\frac{\frac{4}{15}x}{112} \qquad \text{or} \qquad \frac{1}{5\times 28}=\frac{x}{15\cdot 28}, \qquad x=3$$

7 $$\frac{18-41x}{9}-\frac{17-16x}{8}+\frac{9\frac{16}{21}-10x}{5}-\frac{14-32x}{7}=0$$

or $$\frac{18-41x}{9}-\frac{14-32x}{7}=\frac{17-16x}{8}-\frac{9\frac{16}{21}-10x}{5}$$

or $$\frac{126-287x-126+288x}{63}=\frac{85-80x-78\frac{2}{21}+80x}{40}$$

or $$\frac{x}{63}=\frac{6\frac{19}{21}}{40}, \qquad x=\frac{145\times 63}{21\times 40}=\frac{87}{8}=10\frac{7}{8}$$

8 $$\frac{98x-73}{21}=\frac{14x-9}{3}-\frac{13x-16}{15x-9}$$

or $$\frac{98x-73}{7}=14x-9-\frac{13x-16}{5x-3}$$

or $$\frac{98x-73-98x+63}{7}=-\frac{13x-16}{5x-3}$$

or $$\frac{10}{7}=\frac{13x-16}{5x-3}$$

or $$50x-30=91x-112$$

or $$50x-91x=-112+30$$

or $$41x=82 \qquad x=2$$

9 $$\frac{95x-159}{35}=\frac{19x-29}{7}-\frac{17x-47}{23x-59}$$

or $$\frac{95x-159}{35}-\frac{19x-29}{7}=-\frac{17x-47}{23x-59}$$

or $$\frac{95x-159-95x+145}{35}=-\frac{17x-47}{23x-59}$$

or $$-\frac{14}{35}=-\frac{17x-47}{23x-59}$$

or $46x - 118 = 85x - 235$

or $39x = 117$, $x = 3$

10 $$\frac{91x-21}{56} + \frac{24x-93}{35x-138} = \frac{13x+9}{8}$$

or $$\frac{91x-21}{56} - \frac{13x+9}{8} = -\frac{24x-93}{35x-138}$$

or $$\frac{91x-21-91x-63}{56} = -\frac{24x-93}{35x-138}$$

or $$\frac{-84}{56} = -\frac{24x-93}{35x-138}$$

or $$\frac{3}{2} = \frac{24x-93}{35x-138}$$

or $105x - 414 = 48x - 186$

or $105x - 48x = -186 + 414$

or $57x = 228$; $x = 4$

11 $$\frac{117x-26}{135} + \frac{16x-77}{23x-110} = \frac{13x+4}{15} + \frac{3\frac{4}{5}}{27}$$

or $$\frac{16x-77}{23x-110} = \frac{13x+4}{15} + \frac{\frac{19}{5}}{27} - \frac{117x-26}{135}$$

or $$\frac{16x-77}{23x-110} = \frac{117x+36+19-117x+26}{135}$$

or $$\frac{16x-77}{23x-110} = \frac{81}{135} = \frac{3}{5}$$

or $80x - 385 = 69x - 330$

or $80x - 69x = -330 + 385 = 55$

or $11x = 55$, $x = 5$

12 $$\frac{6x-7\frac{1}{3}}{13-2x} + 2x + \frac{1+16x}{24} = 4\frac{5}{12} - \frac{12\frac{5}{8}-8x}{3}$$

or $$\frac{6x-7\frac{1}{3}}{13-2x} = \frac{53}{12} - \frac{\frac{101}{8}-8x}{3} - 2x - \frac{1+16x}{24}$$

or $$\frac{6x-7\frac{1}{3}}{13-2x} = \frac{106-101+64x-48x-1-16x}{24} = \frac{4}{24} = \frac{1}{6}$$

or $36x - 44 = 13 - 2x$

or $38x = 57$, $x = \frac{57}{38} = 1\frac{1}{2}$

13
$$\frac{2x+8\frac{1}{2}}{9}-\frac{13x-2}{17x-32}+\frac{x}{3}=\frac{7x}{12}-\frac{x+16}{36}$$

or
$$\frac{13x-2}{17x-32}=\frac{2x+\frac{17}{2}}{9}+\frac{x}{3}-\frac{7x}{12}+\frac{x+16}{36}$$
$$=\frac{8x+34+12x-21x+x+16}{36}=\frac{50}{36}=\frac{25}{18}$$

or $234x-36=425x-800$

or $234x-425x=-800+36$

or $-191x=-764$, $\qquad x=4$

14
$$\frac{41-35x}{105}-\frac{7-2x^2}{14(x-1)}=\frac{1+3x}{21}-\frac{2x-2\frac{1}{5}}{6}$$

or
$$\frac{7-2x^2}{14(x-1)}=\frac{41-35x}{105}-\frac{1+3x}{21}+\frac{2x-\frac{11}{5}}{6}$$
$$=\frac{82-70x-10-30x+70x-77}{210}$$
$$=\frac{-5-30x}{210}=-\frac{1+6x}{14\times 3}$$

or
$$\frac{7-2x^2}{x-1}=-\frac{1+6x}{3}$$

or $21-6x^2=-x+1-6x^2+6x$

or $5x=20$, $\qquad x=4$

15
$$\frac{1}{x-1}-\frac{2}{x+7}=\frac{1}{7(x-1)}$$

or
$$\frac{1}{x-1}-\frac{1}{7(x-1)}=\frac{2}{x+7}$$

or
$$\frac{6}{7(x-1)}=\frac{2}{x+7} \text{ or } \frac{3}{7(x-1)}=\frac{1}{x+7}$$

or $3x+21=7x-7$

or $3x-7x=-7-21=-28$

or $-4x=-28$; $\qquad x=7$

16
$$\frac{2}{5(3x+4)}+\frac{4}{2x+3}=\frac{6}{3x+4}$$

or
$$\frac{2}{5(3x+4)}-\frac{6}{3x+4}=-\frac{4}{2x+3}$$

or $$\frac{2-30}{5(3x+4)}=-\frac{4}{2x+3}$$

or $$\frac{28}{5(3x+4)}=\frac{4}{2x+3} \text{ or } \frac{7}{5(3x+4)}=\frac{1}{2x+3}$$

or $$14x+21=15x+20\,, \qquad x=1$$

17 $$\frac{3}{3x-5}-\frac{6}{7(4x-7)}=\frac{7}{9(3x-5)}+\frac{2}{4x-7}$$

or $$\frac{3}{3x-5}-\frac{7}{9(3x-5)}=\frac{2}{4x-7}+\frac{6}{7(4x-7)}$$

or $$\frac{27-7}{9(3x-5)}=\frac{14+6}{7(4x-7)}$$

or $$\frac{20}{9(3x-5)}=\frac{20}{7(4x-7)}$$

or $$27x-45=28x-49\,, \qquad x=4$$

18 $$\frac{11}{12(14x-19)}+\frac{7}{9(13x-14)}=\frac{3}{14x-19}-\frac{2}{13x-14}$$

or $$\frac{11}{12(14x-19)}-\frac{3}{14x-19}=-\frac{2}{13x-14}-\frac{7}{9(13x-14)}$$

or $$\frac{11-36}{12(14x-19)}=\frac{-18-7}{9(13x-14)}$$

or $$\frac{25}{4(14x-19)}=\frac{25}{3(13x-14)}$$

or $$56x-76=39x-42$$

or $$56x-39x=-42+76$$

or $$17x=34\,, \qquad x=2$$

19 $$\frac{50}{3x-1}+\frac{37-\frac{1}{3}x}{12x-1}=\frac{35}{12x-1}+\frac{49-\frac{1}{12}x}{3x-1}$$

or $$\frac{50-49+\frac{1}{12}x}{3x-1}=\frac{35-37+\frac{1}{3}x}{12x-1}$$

or $$\frac{\frac{1}{12}x+1}{3x-1}=\frac{\frac{1}{3}x-2}{12x-1}$$

or $$x^2+11\tfrac{11}{12}x-1=x^2-6\tfrac{1}{3}x+2$$

or $$18\tfrac{1}{4}x=3\,, \qquad x=\frac{3\times 4}{73}=\frac{12}{73}$$

20
$$\frac{(1\frac{3}{7})x+19\frac{11}{12}}{2x+5}-\frac{\frac{8}{11}x+8}{x+8}=\frac{20\frac{11}{12}-(1\frac{4}{7})x}{2x+5}+\frac{(1\frac{6}{11})x-9}{2(x+8)}$$

or $$\frac{(1\frac{3}{7})x+19\frac{11}{12}-20\frac{11}{12}+(1\frac{4}{7})x}{(2x+5)}=\frac{(1\frac{6}{11})x-9+(1\frac{5}{11})x+16}{2(x+8)}$$

or $$\frac{3x-1}{2x+5}=\frac{3x+7}{2(x+8)}$$

or $$6x^2+46x-16=6x^2+29x+35$$

or $$17x=51\,, \qquad x=3$$

21
$$\frac{(9\frac{12}{17})x-32}{4x+7}+\frac{65x+4\frac{14}{15}}{8x+29}=\frac{75x+5\frac{14}{15}}{8x+29}+\frac{(4\frac{12}{17})x-29}{4x+7}$$

or $$\frac{(9\frac{12}{17})x-32-(4\frac{12}{17})x+29}{4x+7}=\frac{75x+5\frac{14}{15}-65x-4\frac{14}{15}}{8x+29}$$

or $$\frac{5x-3}{4x+7}=\frac{10x+1}{8x+29}$$

or $$40x^2+121x-87=40x^2+74x+7$$

or $$121x-74x=7+87$$

or $$47x=94\,, \qquad x=2$$

22
$$\frac{1}{x-1}+\frac{2}{x-2}=\frac{3}{x-3}$$

or $$\frac{1}{x-1}-\frac{1}{x-3}=\frac{2}{x-3}-\frac{2}{x-2}$$

or $$\frac{x-3-x+1}{(x-1)(x-3)}=2\left\{\frac{x-2-x+3}{(x-3)(x-2)}\right\}$$

or $$\frac{-2}{x-1}=\frac{2}{x-2}$$

or $$-x+2=x-1 \quad \text{or} \quad 2x=3\,,$$

$$x=\tfrac{3}{2}$$

23
$$\frac{3}{4x+1}+\frac{4}{4x+5}=\frac{7}{4x+3}$$

or $$\frac{3}{4x+1}-\frac{3}{4x+3}=\frac{4}{4x+3}-\frac{4}{4x+5}$$

or $$3\left\{\frac{4x+3-4x-1}{(4x+1)(4x+3)}\right\}=4\left\{\frac{4x+5-4x-3}{(4x+3)(4x+5)}\right\}$$

or $$\frac{6}{4x+1}=\frac{8}{4x+5} \text{ or } \frac{3}{4x+1}=\frac{4}{4x+5}$$

or $$12x+15=16x+4$$

or $$4x=11\,, \qquad x=2\tfrac{3}{4}$$

24 $$\frac{15}{3x+11}-\frac{8}{3x+17}=\frac{7}{3x+5}$$

or $$\frac{8}{3x+11}-\frac{8}{3x+17}=\frac{7}{3x+5}-\frac{7}{3x+11}$$

or $$8\left\{\frac{3x+17-3x-11}{(3x+11)(3x+17)}\right\}=7\left\{\frac{3x+11-3x-5}{(3x+5)(3x+11)}\right\}$$

or $$\frac{8\times 6}{3x+17}=\frac{7\times 6}{3x+5}$$

or $$24x+40=21x+119$$

or $$3x=79\,, \qquad x=\tfrac{79}{3}=26\tfrac{1}{3}$$

25 $$\frac{6}{5x+7}-\frac{4}{5x+13}=\frac{9}{5x+13}-\frac{7}{5x+19}$$

or $$\frac{6}{5x+7}-\frac{6}{5x+13}=\frac{7}{5x+13}-\frac{7}{5x+19}$$

or $$6\left\{\frac{5x+13-5x-7}{(5x+7)(5x+13)}\right\}=7\left\{\frac{5x+19-5x-13}{(5x+13)(5x+19)}\right\}$$

or $$\frac{6\times 6}{5x+7}=\frac{7\times 6}{5x+19}$$

or $$30x+114=35x+49$$

or $$5x=65\,, \qquad x=13$$

26 $$\frac{8}{2x+17}-\frac{12}{2x+25}=\frac{5}{2x+25}-\frac{9}{2x+33}$$

or $$\frac{8}{2x+17}-\frac{8}{2x+25}=\frac{9}{2x+25}-\frac{9}{2x+33}$$

or $$8\left\{\frac{2x+25-2x-17}{(2x+17)(2x+25)}\right\}=9\left\{\frac{2x+33-2x-25}{(2x+25)(2x+33)}\right\}$$

or $$\frac{8\times 8}{2x+17}=\frac{9\times 8}{2x+33}$$

or $$16x+264=18x+153$$

or $$2x=111\,, \qquad x=\tfrac{111}{2}=55\tfrac{1}{2}$$

27. $$\frac{5}{3-4x}+\frac{9}{4x+13}-\frac{4}{4x+5}=0$$

or $$\frac{5}{3-4x}+\frac{5}{4x+13}=\frac{4}{4x+5}-\frac{4}{4x+13}$$

or $$5\left\{\frac{4x+13+3-4x}{(3-4x)(4x+13)}\right\}=4\left\{\frac{4x+13-4x-5}{(4x+5)(4x+13)}\right\}$$

or $$\frac{5\times16}{3-4x}=\frac{4\times8}{4x+5}$$

or $$20x+25=6-8x$$

or $$28x=-19\,, \qquad x=-\tfrac{19}{28}$$

28 $$\frac{6}{5-6x}+\frac{13}{6x+19}=\frac{7}{6x+7}$$

or $$\frac{6}{5-6x}+\frac{6}{6x+19}=\frac{7}{6x+7}-\frac{7}{6x+19}$$

or $$6\left\{\frac{6x+19+5-6x}{(5-6x)(6x+19)}\right\}=7\left\{\frac{6x+19-6x-7}{(6x+7)(6x+19)}\right\}$$

or $$\frac{6\times24}{5-6x}=\frac{7\times12}{6x+7}$$

or $$72x+84=35-42x$$

or $$114x=-49\,, \qquad x=-\tfrac{49}{114}$$

29 $$\frac{7}{4-5x}-\frac{8}{17-5x}=\frac{1}{5x+9}$$

or $$\frac{7}{4-5x}-\frac{7}{17-5x}=\frac{1}{5x+9}+\frac{1}{17-5x}$$

or $$7\left\{\frac{17-5x-4+5x}{(4-5x)(17-5x)}\right\}=\frac{17-5x+5x+9}{(5x+9)(17-5x)}$$

or $$\frac{7\times13}{4-5x}=\frac{26}{5x+9}$$

or $$35x+63=8-10x$$

or $$45x=-55\,; \qquad x=-\tfrac{11}{9}$$

30 $$\frac{9}{3-7x}+\frac{1}{7x+15}=\frac{8}{12-7x}$$

or $$\frac{1}{3-7x}+\frac{1}{7x+15}=\frac{8}{12-7x}-\frac{8}{3-7x}$$

or $$\frac{7x+15+3-7x}{(3-7x)(7x+15)}=8\left\{\frac{3-7x-12+7x}{(12-7x)(3-7x)}\right\}$$

or $$\frac{18}{7x+15}=\frac{8\times(-9)}{12-7x}$$

or $$\frac{1}{7x+15}=\frac{-4}{12-7x}$$

or $$12-7x=-28x-60$$

or $$21x=-72\,, \qquad x=-\tfrac{24}{7}.$$

31 $$\frac{2}{2x-5}+\frac{1}{x-3}=\frac{6}{3x-1}$$

or $$\frac{2}{2x-5}-\frac{3}{3x-1}=\frac{3}{3x-1}-\frac{1}{x-3}$$

or $$\frac{6x-2-6x+15}{(2x-5)(3x-1)}=\frac{3x-9-3x+1}{(3x-1)(x-3)}$$

or $$\frac{13}{2x-5}=\frac{-8}{x-3}$$

or $$13x-39=-16x+40$$

or $$29x=79\,, \qquad x=\tfrac{79}{29}=2\tfrac{21}{29}$$

32 $$\frac{10}{2x-5}+\frac{1}{x+5}=\frac{18}{3x-5}$$

or $$\frac{10}{2x-5}-\frac{15}{3x-5}=\frac{3}{3x-5}-\frac{1}{x+5}$$

or $$\frac{30x-50-30x+75}{(2x-5)(3x-5)}=\frac{3x+15-3x+5}{(3x-5)(x+5)}$$

or $$\frac{25}{2x-5}=\frac{20}{x+5}$$

or $$5x+25=8x-20$$

or $$3x=45\,, \qquad x=15$$

33 $$\frac{9}{3x-4}+\frac{20}{4x+1}=\frac{8}{x+7}$$

or $$\frac{9}{3x-4}-\frac{3}{x+7}=\frac{5}{x+7}-\frac{20}{4x+1}$$

or $$\frac{9x+63-9x+12}{(3x-4)(x+7)}=\frac{20x+5-20x-140}{(x+7)(4x+1)}$$

or $$\frac{75}{3x-4}=\frac{-135}{4x+1}$$

or $$5(4x+1)=-9(3x-4)$$

or $$20x+5=-27x+36$$

or $$47x=31\,, \qquad x=\tfrac{31}{47}$$

34 $$\frac{10}{5x-9}+\frac{14}{2x+9}=\frac{9}{x+8}$$

or $$\frac{10}{5x-9}-\frac{2}{x+8}=\frac{7}{x+8}-\frac{14}{2x+9}$$

or $$\frac{10x+80-10x+18}{(5x-9)(x+8)}=\frac{14x+63-14x-112}{(x+8)(2x+9)}$$

or $$\frac{98}{5x-9}=\frac{-49}{2x+9}$$

or $$4x+18=-5x+9$$

or $$9x=-9\,, \qquad x=-1$$

35 $$\frac{12}{3x-8}=\frac{20}{4x-13}-\frac{1}{x+9}$$

or $$\frac{12}{3x-8}-\frac{16}{4x-13}=\frac{4}{4x-13}-\frac{1}{x+9}$$

or $$\frac{48x-156-48x+128}{(3x-8)(4x-13)}=\frac{4x+36-4x+13}{(4x-13)(x+9)}$$

or $$\frac{-28}{3x-8}=\frac{49}{x+9}$$

or $$\frac{-4}{3x-8}=\frac{7}{x+9}$$

or $$-4x-36=21x-56$$

or $$25x=20\,, \qquad x=\tfrac{4}{5}$$

36 $$\frac{a+b}{x-c}=\frac{a}{x-a}+\frac{b}{x-b}$$

or $$\frac{a}{x-c}-\frac{a}{x-a}=\frac{b}{x-b}-\frac{b}{x-c}$$

or $$a\left\{\frac{x-a-x+c}{(x-c)(x-a)}\right\}=b\left\{\frac{x-c-x+b}{(x-b)(x-c)}\right\}$$

or $$\frac{a(c-a)}{x-a}=\frac{b(b-c)}{(x-b)}$$

or $$a(c-a)(x-b)=b(b-c)(x-a)$$

or $$(ac-a^2)x-b(ac-a^2)=(b^2-bc)x-a(b^2-bc)$$

or $$x(ac-a^2-b^2+bc)=-ab^2+abc+abc-a^2b$$
$$=ab(2c-a-b)$$
$$x=\frac{ab(2c-a-b)}{ac-a^2-b^2+bc}=\frac{ab(a+b-2c)}{a^2+b^2-ac-bc}$$

37 $$\frac{a^2}{ax-b}+\frac{b^2}{bx-a}=\frac{a+b}{x+c}$$

or $$\frac{a^2}{ax-b}-\frac{a}{x+c}=\frac{b}{x+c}-\frac{b^2}{bx-a}$$

or $$\frac{a^2x+a^2c-a^2x+ab}{(ax-b)(x+c)}=\frac{b^2x-ab-b^2x-b^2c}{(x+c)(bx-a)}$$

or $$\frac{a(ac+b)}{ax-b}=\frac{-b(a+bc)}{bx-a}$$

or $$abx(ac+b)-a^2(ac+b)=-abx(a+bc)+b^2(a+bc)$$

or $$abx(ac+b+a+bc)=ab^2+b^3c+a^3c+a^2b$$

or $$abx\{c(a+b)+(a+b)\}=ab(a+b)+c(a+b)(a^2-ab+b^2)$$

or $$abx(c+1)=ab+c(a^2-ab+b^2),$$
$$x=\frac{c(a^2-ab+b^2)+ab}{ab(c+1)}$$

38 $$\frac{m(x+a)}{x+b}+\frac{n(x+b)}{x+a}=m+n$$

or $$\frac{m(x+a)}{x+b}-m=n-\frac{n(x+b)}{x+a}$$

or $$m\left(\frac{x+a-x-b}{x+b}\right)=n\left(\frac{x+a-x-b}{x+a}\right)$$

or $$\frac{m(a-b)}{x+b}=\frac{n(a-b)}{x+a}$$

or $$m(x+a)=n(x+b)$$

or $$x(m-n)=nb-am, \qquad x=\frac{nb-am}{m-n}$$

39 $$\frac{b-c}{x+a}+\frac{a-b}{x+b}=\frac{a-c}{x+c}$$

or $$\frac{b-c}{x+a}+\frac{a-b}{x+b}=\frac{a-b+b-c}{x+c}$$

or $$b-c\left\{\frac{1}{x+a}-\frac{1}{x+c}\right\}=a-b\left\{\frac{1}{x+c}-\frac{1}{x+b}\right\}$$

or $$b-c\left\{\frac{x+c-x-a}{(x+a)(x+c)}\right\}=a-b\left\{\frac{x+b-x-c}{(x+c)(x+b)}\right\}$$

or $$\frac{(b-c)(c-a)}{x+a}=\frac{(a-b)(b-c)}{x+b}$$

or $$(c-a)(x+b)=(a-b)(x+a)$$

or $$x\{(c-a)-(a-b)\}=a(a-b)-b(c-a)$$
$$=a^2-ab-bc+ab$$

or $x(b+c-2a)=a^2-bc$, $$x=\frac{a^2-bc}{b+c-2a}$$

40 $$\frac{2a-3b}{x-a+b}-\frac{2b-3a}{x+a-b}=\frac{5(a-b)}{x+a+b}$$

or $$\frac{2a-3b}{x-a+b}-\frac{2b-3a}{x+a-b}=\frac{2a-3b+3a-2b}{x+a+b}$$

or $$(2a-3b)\left\{\frac{1}{(x-a+b)}-\frac{1}{(x+a+b)}\right\}$$
$$=(3a-2b)\left\{\frac{1}{(x+a+b)}-\frac{1}{(x+a-b)}\right\}$$

or $$(2a-3b)\left\{\frac{x+a+b-x+a-b}{(x-a+b)(x+a+b)}\right\}$$
$$=(3a-2b)\left\{\frac{x+a-b-x-a-b}{(x+a+b)(x+a-b)}\right\}$$

or $$\frac{2a(2a-3b)}{x-a+b}=\frac{-2b(3a-2b)}{x+a-b}$$

or $$a(2b-3b)(x+a-b)=b(2b-3a)(x-a+b)$$

or $$x(2a^2-3ab-2b^2+3ab)$$
$$=-b(2b-3a)(a-b)-a(a-b)(2a-3b)$$
$$=(a-b)(-2b^2+3ab-2a^2+3ab)$$

or $2x(a+b)(a-b)=2(a-b)(3ab-a^2-b^2)$,
$$x=\frac{3ab-a^2-b^2}{a+b}$$

41 $$\frac{1}{x-6a}+\frac{2}{x+3a}+\frac{3}{x-2a}=\frac{6}{x-a}$$

or $$\frac{1}{x-6a}-\frac{1}{x-a}+\frac{2}{x+3a}-\frac{2}{x-a}+\frac{3}{x-2a}-\frac{3}{x-a}=0$$

or $$\frac{x-a-x+6a}{(x-6a)(x-a)}+\frac{2x-2a-2x-6a}{(x-a)(x+3a)}+\frac{3x-3a-3x+6a}{(x-2a)(x-a)}=0$$

or $$\frac{5a}{x-6a}+\frac{3a}{x-2a}=\frac{8a}{x+3a}=\frac{5a+3a}{x+3a}$$

or $$5a\left\{\frac{1}{x-6a}-\frac{1}{x+3a}\right\}=3a\left\{\frac{1}{x+3a}-\frac{1}{x-2a}\right\}$$

or $$\frac{5a\,9a}{x-6a}=\frac{3a\times(-5a)}{x-2a}$$

or $$\frac{3}{x-6a}=-\frac{1}{x-2a}$$

or $$3x-6a=-x+6a$$

or $$4x=12a\,, \qquad x=3a$$

42 $$\frac{1}{x+1}+\frac{4}{2x-1}+\frac{9}{3x-1}=\frac{36}{6x-1}$$

or $$\frac{1}{x+1}-\frac{6}{6x-1}+\frac{4}{2x-1}-\frac{12}{6x-1}+\frac{9}{3x-1}-\frac{18}{6x-1}=0$$

or $$\frac{6x-1-6x-6}{(x+1)(6x-1)}+\frac{24x-4-24x+12}{(2x-1)(6x-1)}+\frac{54x-9-54x+18}{(3x-1)(6x-1)}=0$$

or $$\frac{-7}{x+1}+\frac{8}{2x-1}+\frac{9}{3x-1}=0$$

or $$\frac{8}{2x-1}-\frac{4}{x+1}+\frac{9}{3x-1}-\frac{3}{x+1}=0$$

or $$\frac{8x+8-8x+4}{(2x-1)(x+1)}+\frac{9x+9-9x+3}{(3x-1)(x+1)}=0$$

or $$\frac{12}{2x-1}+\frac{12}{3x-1}=0$$

or $$3x-1+2x-1=0$$

or $$5x=2\,, \qquad x=\frac{2}{5}$$

Exercise (65)

1
$$\frac{2x-1}{x-1}+\frac{3x-4}{x-2}=\frac{5x-12}{x-3}$$

or $$2+\frac{1}{x-1}+3+\frac{2}{x-2}=5+\frac{3}{x-3}$$

or $$\frac{1}{x-1}-\frac{1}{x-3}=\frac{2}{x-3}-\frac{2}{x-2}$$

or $$\frac{x-3-x+1}{(x-1)(x-3)}=\frac{2x-4-2x+6}{(x-3)(x-2)}$$

or $$\frac{-2}{x-1}=\frac{2}{x-2}$$

or $$-x+2=x-1$$

or $$2x=3, \qquad x=1\tfrac{1}{2}$$

2
$$\frac{2x+7}{x+2}+\frac{4x+29}{x+6}-\frac{6x-10}{x-3}=0$$

or $$2+\frac{3}{x+2}+4+\frac{5}{x+6}-6-\frac{8}{x-3}=0$$

or $$\frac{3}{x+2}-\frac{3}{x-3}+\frac{5}{x+6}-\frac{5}{x-3}=0$$

or $$\frac{3x-9-3x-6}{(x+2)(x-3)}+\frac{5x-15-5x-30}{(x+6)(x-3)}=0$$

or $$\frac{15}{x+2}+\frac{45}{x+6}=0$$

or $$\frac{1}{x+2}+\frac{3}{x+6}=0$$

or $$x+6+3x+6=0$$

or $$4x=-12, \qquad x=-3$$

3
$$\frac{25x-40}{5x-6}-\frac{7x+9}{x+2}+\frac{6x-1}{3x+4}=0$$

or $$5-\frac{10}{5x-6}-7+\frac{5}{x+2}+2-\frac{9}{3x+4}=0$$

or $$-\frac{10}{5x-6}+\frac{2}{x+2}+\frac{3}{x+2}-\frac{9}{3x+4}=0$$

or $$\frac{-10x-20+10x-12}{(5x-6)(x+2)}+\frac{9x+12-9x-18}{(x+2)(3x+4)}=0$$

or $\quad -\dfrac{32}{5x-6}-\dfrac{6}{3x+4}=0$

or $\quad \dfrac{16}{5x-6}+\dfrac{3}{3x+4}=0$

or $\quad 48x+64+15x-18=0$

or $\quad 63x=-46\,, \qquad x=-\dfrac{46}{63}$

4 $\quad \dfrac{12x+5}{4x+3}+\dfrac{15x+11}{3x+4}=\dfrac{8x+44}{x+6}$

or $\quad 3-\dfrac{4}{4x+3}+5-\dfrac{9}{3x+4}=8-\dfrac{4}{x+6}$

or $\quad \dfrac{4}{4x+3}-\dfrac{1}{x+6}=\dfrac{3}{x+6}-\dfrac{9}{3x+4}$

or $\quad \dfrac{4x+24-4x-3}{(4x+3)(x+6)}=\dfrac{9x+12-9x-54}{(x+6)(3x+4)}$

or $\quad \dfrac{21}{4x+3}=\dfrac{-42}{3x+4}$

or $\quad \dfrac{1}{4x+3}=\dfrac{-2}{3x+4}$

or $\quad 3x+4=-8x-6$

or $\quad 11x=-10\,, \qquad x=-\frac{10}{11}$

5 $\quad \dfrac{15x-13}{3x-5}+\dfrac{12x+59}{3x+5}=\dfrac{9x+80}{x+7}$

or $\quad 5+\dfrac{12}{3x-5}+4+\dfrac{39}{3x+5}=9+\dfrac{17}{x+7}$

or $\quad \dfrac{12}{3x-5}-\dfrac{4}{x+7}=\dfrac{13}{x+7}-\dfrac{39}{3x+5}$

or $\quad \dfrac{12x+84-12x+20}{(3x-5)(x+7)}=\dfrac{39x+65-39x-273}{(x+7)(3x+5)}$

or $\quad \dfrac{104}{3x-5}=\dfrac{-208}{3x+5}$

or $\quad \dfrac{1}{3x-5}=\dfrac{-2}{3x+5}$

or $\quad 3x+5=-6x+10$

or $\quad 9x=5\,, \qquad x=\frac{5}{9}$

6 $$\frac{42x-37}{6x-1}+\frac{20x+13}{5x+12}=\frac{11x+76}{x+8}$$

or $$7-\frac{30}{6x-1}+4-\frac{35}{5x+12}=11-\frac{12}{x+8}$$

or $$\frac{30}{6x-1}+\frac{35}{5x+12}=\frac{12}{x+8}$$

or $$\frac{30}{6x-1}-\frac{5}{x+8}=\frac{7}{x+8}-\frac{35}{5x+12}$$

or $$\frac{30x+240-30x+5}{(6x-1)(x+8)}=\frac{35x+84-35x-280}{(x+8)(5x+12)}$$

or $$\frac{245}{6x-1}=\frac{-196}{5x+12}$$

or $$\frac{5}{6x-1}=\frac{-4}{5x+12}$$

or $$25x+60=-24x+4$$

or $$49x=-56\,, \qquad x=-\frac{56}{49}=-\frac{8}{7}$$

7 $$\frac{15x-7}{5x-4}+\frac{4x+3}{4x-3}=\frac{8x+1}{2x-1}$$

or $$3+\frac{5}{5x-4}+1+\frac{6}{4x-3}=4+\frac{5}{2x-1}$$

or $$\frac{5}{5x-4}-\frac{2}{2x-1}=\frac{3}{2x-1}-\frac{6}{4x-3}$$

or $$\frac{10x-5-10x+8}{(5x-4)(2x-1)}=\frac{12x-9-12x+6}{(2x-1)(4x-3)}$$

or $$\frac{3}{5x-4}=\frac{-3}{4x-3}$$

or $$5x-4=-4x+3$$

or $$9x=7\,, \qquad x=\frac{7}{9}$$

8 $$\frac{4x-7}{4x+5}+\frac{15x+11}{5x+7}=\frac{12x+1}{3x+4}$$

or $$1-\frac{12}{4x+5}+3-\frac{10}{5x+7}=4-\frac{15}{3x+4}$$

or $$\frac{12}{4x+5}+\frac{10}{5x+7}=\frac{15}{3x+4}$$

or $$\frac{12}{4x+5}-\frac{9}{3x+4}=\frac{6}{3x+4}-\frac{10}{5x+7}$$

or $$\frac{36x+48-36x-45}{(4x+5)(3x+4)}=\frac{30x+42-30x-40}{(3x+4)(5x+7)}$$

or $$\frac{3}{4x+5}=\frac{2}{5x+7}$$

or $$15x+21=8x+10$$

or $$7x=-11\,, \qquad x=-\tfrac{11}{7}$$

9 $$\frac{4x^3+4x^2+8x+1}{2x^2+2x+3}=\frac{2x^2+2x+1}{x+1}$$

or $$\frac{2x(2x^2+2x+3)+2x+1}{2x^2+2x+3}=\frac{2x(x+1)+1}{x+1}$$

or $$2x+\frac{2x+1}{2x^2+2x+3}=2x+\frac{1}{x+1}$$

or $$(2x+1)(x+1)=2x^2+2x+3$$

or $$2x^2+3x+1=2x^2+2x+3\,, \qquad x=2$$

10 $$\frac{12x^3+16x^2+29x-1}{3x^2+4x+8}=\frac{4x^2+20x-1}{x+5}$$

or $$\frac{4x(3x^2+4x+8)-(3x+1)}{3x^2+4x+8}=\frac{4x(x+5)-1}{x+5}$$

or $$4x-\frac{3x+1}{3x^2+4x+8}=4x-\frac{1}{x+5}$$

or $$3x^2+4x+8=(3x+1)(x+5)=3x^2+16x+5$$

or $$-12x=-3\,, \qquad x=\tfrac{1}{4}$$

11 $$\frac{x^2-x+1}{x-1}+\frac{x^2-2x+1}{x-2}=2x+\frac{2}{x-3}$$

or $$x+\frac{1}{x-1}+x+\frac{1}{x-2}=2x+\frac{2}{x-3}$$

or $$\frac{1}{x-1}-\frac{1}{x-3}=\frac{1}{x-3}-\frac{1}{x-2}$$

or $$\frac{x-3-x+1}{(x-1)(x-3)}=\frac{x-2-x+3}{(x-3)(x-2)}$$

or $$\frac{-2}{x-1}=\frac{1}{x-2}$$

or $$-2x+4=x-1$$

or $$-3x=-5\,, \qquad x=\tfrac{5}{3}$$

12
$$\frac{x^2+3}{x-1}+\frac{x^2-x+1}{x-2}=\frac{2x^2-4x+1}{x-3}$$

or $$\frac{(x^2-1)+4}{x-1}+\frac{(x-2)(x+1)+3}{x-2}=\frac{(2x+2)(x-3)+7}{x-3}$$

or $$(x+1)+\frac{4}{x-1}+(x+1)+\frac{3}{x-2}=2x+2+\frac{7}{x-3}$$

or $$\frac{4}{x-1}-\frac{4}{x-3}=\frac{3}{x-3}-\frac{3}{x-2}$$

or $$\frac{4(x-3-x+1)}{(x-1)(x-3)}=\frac{3(x-2-x+3)}{(x-3)(x-2)}$$

or $$\frac{-8}{x-1}=\frac{3}{x-2}$$

or $$-8x+16=3x-3$$

or $$-11x=-19 \qquad x=\tfrac{19}{11}=1\tfrac{8}{11}$$

13
$$\frac{2x^2-3x+7}{2x-1}+\frac{6x^2+2x+21}{3x+1}=\frac{3x^2+8x+7}{x+3}$$

or $$x-\frac{2x-7}{2x-1}+2x+\frac{21}{3x+1}=3x-\frac{x-7}{x+3}$$

or $$-1+\frac{6}{2x-1}+\frac{21}{3x+1}=-1+\frac{10}{x+3}$$

or $$\frac{6}{2x-1}-\frac{3}{x+3}=\frac{7}{x+3}-\frac{21}{3x+1}$$

or $$\frac{6x+18-6x+3}{(2x-1)(x+3)}=\frac{21x+7-21x-63}{(x+3)(3x+1)}$$

or $$\frac{21}{2x-1}=\frac{-56}{3x+1}$$

or $$\frac{3}{2x-1}=\frac{-8}{3x+1}$$

or $$9x+3=-16x+8$$

or $$25x=5, \qquad x=\tfrac{1}{5}$$

14
$$\frac{3+2x}{1+2x}-\frac{5+2x}{7+2x}=1-\frac{4x^2-2}{7+16x+4x^2}$$

or $$1+\frac{2}{1+2x}-1+\frac{2}{7+2x}=1-1+\frac{16x+9}{7+16x+4x^2}$$

or $$\frac{2(7+2x+1+2x)}{(1+2x)(7+2x)}=\frac{16x+9}{(7+2x)(1+2x)}$$

or $$16+8x=16x+9$$

or $$-8x=-7\,, \qquad x=\tfrac{7}{8}$$

15 $$\frac{2x-3}{x-2}+\frac{3x-20}{x-7}=\frac{x-3}{x-4}+\frac{4x-19}{x-5}$$

or $$2+\frac{1}{x-2}+3+\frac{1}{x-7}=1+\frac{1}{x-4}+4+\frac{1}{x-5}$$

or $$\frac{1}{x-2}-\frac{1}{x-4}=\frac{1}{x-5}-\frac{1}{x-7}$$

or $$\frac{x-4-x+2}{(x-2)(x-4)}=\frac{x-7-x+5}{(x-5)(x-7)}$$

or $$\frac{-2}{(x-2)(x-4)}=\frac{-2}{(x-5)(x-7)}$$

$$(x-2)(x-4)=(x-5)(x-7)$$

or $$x^2-6x+8=x^2-12x+35$$

or $$6x=27\,, \qquad x=\tfrac{27}{6}=4\tfrac{1}{2}$$

16 $$\frac{3x-8}{x-3}+\frac{4x-35}{x-9}=\frac{2x-9}{x-5}+\frac{5x-34}{x-7}$$

or $$3+\frac{1}{x-3}+4+\frac{1}{x-9}=2+\frac{1}{x-5}+5+\frac{1}{x-7}$$

or $$\frac{1}{x-3}-\frac{1}{x-5}=\frac{1}{x-7}-\frac{1}{x-9}$$

or $$\frac{x-5-x+3}{(x-3)(x-5)}=\frac{x-9-x+7}{(x-7)(x-9)}$$

or $$\frac{-2}{(x-3)(x-5)}=\frac{-2}{(x-7)(x-9)}$$

or $$(x-3)(x-5)=(x-7)(x-9)$$

or $$x^2-8x+15=x^2-16x+63$$

or $$8x=48\,, \qquad x=6$$

17 $$\frac{3x-13}{x-4}+\frac{4x-41}{x-10}=\frac{2x-13}{x-6}+\frac{5x-41}{x-8}$$

or $$3-\frac{1}{x-4}+4-\frac{1}{x-10}=2-\frac{1}{x-6}+5-\frac{1}{x-8}$$

or $$\frac{1}{x-4}-\frac{1}{x-6}=\frac{1}{x-8}-\frac{1}{x-10}$$

or $$\frac{x-6-x+4}{(x-4)(x-6)}=\frac{x-10-x+8}{(x-8)(x-10)}$$

or $$\frac{-2}{(x-4)(x-6)}=\frac{-2}{(x-8)(x-10)}$$

or $$(x-4)(x-6)=(x-8)(x-10)$$

or $$x^2-10x+24=x^2-18x+80$$

or $$8x=56\,, \qquad x=7$$

18 $$\frac{4x+21}{x+5}+\frac{5x-69}{x-14}=\frac{3x-5}{x-2}+\frac{6x-41}{x-7}$$

or $$4+\frac{1}{x-5}+5+\frac{1}{x-14}=3+\frac{1}{x-2}+6+\frac{1}{x-7}$$

or $$\frac{1}{x+5}+\frac{1}{x-14}=\frac{1}{x-2}+\frac{1}{x-7}$$

or $$\frac{1}{x+5}-\frac{1}{x-7}=\frac{1}{x-2}-\frac{1}{x-14}$$

or $$\frac{x-7-x-5}{(x+5)(x-7)}=\frac{x-14-x+2}{(x-2)(x-14)}$$

or $$\frac{-12}{(x+5)(x-7)}=\frac{-12}{(x-2)(x-14)}$$

or $$(x+5)(x-7)=(x-2)(x-14)$$

or $$x^2-2x-35=x^2-16x+28$$

or $$-2x+16x=28+35$$

or $$14x=63\,, \qquad x=4\tfrac{1}{2}$$

19 $$\frac{5-6x}{3x-1}-\frac{2x+7}{x+3}=\frac{31-12x}{3x-7}+\frac{4x+21}{x+5}$$

or $$-2+\frac{3}{3x-1}+2+\frac{1}{x+3}=-4+\frac{3}{3x-7}+4+\frac{1}{x+5}$$

or $$\frac{3}{3x-1}-\frac{1}{x+5}=\frac{3}{3x-7}-\frac{1}{x+3}$$

or $$\frac{3x+15-3x+1}{(3x-1)(x+5)}=\frac{3x+9-3x+7}{(3x-7)(x+3)}$$

or $$\frac{16}{(3x-1)(x+5)}=\frac{16}{(3x-7)(x+3)}$$

or $$(3x-1)(x+5)=(3x-7)(x+3)$$

or $$3x^2+14x-5=3x^2+2x-21$$

or $$12x=-16\,, \qquad x=-\tfrac{4}{3}$$

20 $$\frac{x^2+3x+3}{x+2}+\frac{x^2-15}{x-4}=\frac{x^2+7x+11}{x+5}+\frac{x^2-4x-20}{x-7}$$

or $\dfrac{(x+2)(x+1)+1}{x+2}+\dfrac{(x^2-16)+1}{x-4}=\dfrac{(x+5)(x+2)+1}{x+5}$
$+\dfrac{(x-7)(x+3)+1}{x-7}$

or $(x+1)+\dfrac{1}{x+2}+(x+4)+\dfrac{1}{x-4}=(x+2)+\dfrac{1}{x+5}+(x+3)$
$+\dfrac{1}{x-7}$

or $\dfrac{1}{x+2}-\dfrac{1}{x-7}=\dfrac{1}{x+5}-\dfrac{1}{x-4}$

or $\dfrac{x-7-x-2}{(x+2)(x-7)}=\dfrac{x-4-x-5}{(x-5)(x-4)}$

or $\dfrac{-9}{(x+2)(x-7)}=\dfrac{-9}{(x+5)(x-4)}$

or $x^2-5x-14=x^2+x-20$

or $6x=6\,,\qquad x=1$

21 $\dfrac{2x+11}{x+5}-\dfrac{9x-9}{3x-4}=\dfrac{4x+13}{x+3}-\dfrac{15x-47}{3x-10}$.

or $2+\dfrac{1}{x+5}-3-\dfrac{3}{3x-4}=4+\dfrac{1}{x+3}-5-\dfrac{3}{3x-10}$

or $\dfrac{3x-4-3x-15}{(x+5)(3x-4)}=\dfrac{3x-10-3x-9}{(x+3)(3x-10)}$

or $\dfrac{-19}{(x+5)(3x-4)}=\dfrac{-19}{(x+3)(3x-10)}$

or $(x+5)(3x-4)=(x+3)(3x-10)$

or $3x^2+11x-20=3x^2-x-30$

or $12x=-10\,,\qquad x=-\frac{5}{6}$

22 $\dfrac{x-2}{x-3}+\dfrac{x-3}{x-4}=\dfrac{x-1}{x-2}+\dfrac{x-4}{x-5}$

or $1+\dfrac{1}{x-3}+1+\dfrac{1}{x-4}=1+\dfrac{1}{x-2}+1+\dfrac{1}{x-5}$

or $\dfrac{1}{x-3}-\dfrac{1}{x-2}=\dfrac{1}{x-5}-\dfrac{1}{x-4}$

or $\dfrac{x-2-x+3}{(x-3)(x-2)}=\dfrac{x-4-x+5}{(x-5)(x-4)}$

or $\dfrac{1}{(x-3)(x-2)}=\dfrac{1}{(x-5)(x-4)}$

or $$(x-3)(x-2)=(x-5)(x-4)$$
or $$x^2-5x+6=x^2-9x+20$$
or $$4x=14, \qquad x=3\tfrac{1}{2}$$

Exercise 66.

1 $$\frac{2x}{x-4}+\frac{7x-3}{x+1}=9$$

or $$2+\frac{8}{x-4}+7-\frac{10}{x+1}=9$$

or $$\frac{4}{x-4}-\frac{5}{x+1}=0$$

or $$\frac{4}{x-4}=\frac{5}{x+1}$$

or $$4x+4=5x-20, \qquad x=24$$

2 $$\frac{x+4a+b}{x+a+b}+\frac{4x+a+2b}{x+a-b}=5$$

or $$1+\frac{3a}{x+a+b}+4-\frac{3a-6b}{x+a-b}=5$$

or $$\frac{a}{x+a+b}-\frac{a-2b}{x+a-b}=0$$

or $$a(x+a-b)=a(x+a+b)-2b(x+a+b)$$
or $$-ab=ab-2bx-2ab-2b^2$$
or $$bx=-b^2, \qquad x=-b$$

3 $$\frac{3x+5}{x+1}=\frac{4x+8}{3x+3}+\frac{10x+1}{6x+3}$$

or $$\frac{3x+5}{x+1}-\frac{4x+8}{3(x+1)}=\frac{10x+1}{3(2x+1)}$$

or $$\frac{9x+15-4x-8}{3(x+1)}=\frac{10x+1}{3(2x+1)}$$

or $$\frac{5x+7}{x+1}=\frac{10x+1}{2x+1}$$

or $$5+\frac{2}{x+1}=5-\frac{4}{2x+1}$$

or $$4x+2=-4x-4$$
or $$8x=-6, \qquad x=-\tfrac{3}{4}$$

4 $$\frac{6x+8}{2x+1}-\frac{2x+38}{x+12}=1$$

or $$3+\frac{5}{2x+1}-2-\frac{14}{x+12}=1$$

or $$\frac{5}{2x+1}=\frac{14}{x+12}$$

or $$5x+60=28x+14$$

or $$-23x=-46, \qquad x=2$$

5 $$\frac{x+18}{x-2}-\frac{27-3x}{3x-19}=2$$

or $$1+\frac{20}{x-2}-1-\frac{8}{3x-19}=2$$

or $$\frac{5}{x-2}=\frac{2}{3x-19}$$

or $$15x-95=2x-4$$

or $$13x=91 \qquad x=7$$

6 $$\frac{x-b}{x-a}-\frac{x-a}{(x-b)}=\frac{2(a-b)}{x-(a+b)}$$

or $$1+\frac{a-b}{x-a}-1+\frac{a-b}{x-b}=\frac{2(a-b)}{x-(a+b)}$$

or $$\frac{1}{x-a}-\frac{1}{x-b}=\frac{2}{x-(a+b)}$$

or $$\frac{1}{x-a}-\frac{1}{x-(a+b)}=\frac{1}{x-(a+b)}-\frac{1}{x-b}$$

or $$\frac{x-a-b-x+a}{(x-a)\{x-(a+b)\}}=\frac{x-b-x+a+b}{\{x-(a+b)\}(x-b)}$$

or $$\frac{-b}{x-a}=\frac{a}{x-b}$$

or $$-bx+b^2=ax-a^2$$

or $$x(a+b)=a^2+b^2, \qquad x=\frac{a^2+b^2}{a+b}$$

7 $$\frac{x+2a}{2b-x}+\frac{x-2a}{2b+x}-\frac{4ab}{x^2-4b^2}=0$$

or $$-1+\frac{2(a+b)}{2b-x}+1-\frac{2(a+b)}{2b+x}-\frac{4ab}{4b^2-x^2}=0$$

or $$\frac{2(a+b)(2b+x-2b+x)}{4b^2-x^2}=\frac{4ab}{4b^2-x^2}$$

or $$4x(a+b)=4ab\,, \qquad x=\frac{ab}{a+b}$$

8 $$\frac{(x-a)(x-b)}{x-a-b}=\frac{(x-c)(x-d)}{x-c-d}$$

or $$\frac{x^2-(a+b)x+ab}{x-(a+b)}=\frac{x^2-(c+d)x+cd}{x-(c+d)}$$

or $$x+\frac{ab}{x-(a+b)}=x+\frac{cd}{x-(c+d)}$$

or $$xab-ab(c+d)=xcd-cd(a+b)$$

or $$x(ab+cd)=ab(c+d)-cd(a+b)\,,$$

$$x=\frac{ab(c+d)-cd(a+b)}{ab-cd}$$

9 $$\frac{1}{x^2+3x+2}+\frac{2x}{x^2+4x+3}+\frac{1}{x^2+5x+6}=14-\frac{60+4x}{x+3}$$

or $$\frac{1}{(x+1)(x+2)}+\frac{2x}{(x+1)(x+3)}-\frac{2}{x+3}+\frac{1}{(x+2)(x+3)}=14-\frac{62+4x}{x+3}$$

or $$\frac{1}{(x+1)(x+2)}+\frac{2x-2x-2}{(x+1)(x+3)}+\frac{1}{(x+2)(x+3)}=\frac{14x+42-62-4x}{x+3}$$

or $$\frac{1}{(x+1)(x+2)}-\frac{2}{(x+1)(x+3)}+\frac{1}{(x+2)(x+3)}=\frac{10x-20}{x+3}$$

or $$\frac{x+3-2x-4+x+1}{(x+1)(x+2)(x+3)}=\frac{10x-20}{x+3}$$

or $$10x-20=0\,, \qquad x=2$$

10 $$\frac{a+x}{a^2+ax+x^2}+\frac{a-x}{a^2-ax+x^2}=\frac{3a}{x(a^4+a^2x^2+x^4)}$$

or $$\frac{a^3+x^3+a^3-x^3}{a^4+a^2x^2+x^4}=\frac{3a}{x(a^4+a^2x^2+x^4)}$$

or $$2a^3=\frac{3a}{x}\,, \qquad x=\frac{3a}{2a^3}=\frac{3}{2a^2}$$

11
$$\frac{x}{x-2}+\frac{x-9}{x-7}=\frac{x+1}{x-1}+\frac{x-8}{x-6}$$

or
$$1+\frac{2}{x-2}+1-\frac{2}{x-7}=1+\frac{2}{x-1}+1-\frac{2}{x-6}$$

or
$$\frac{1}{x-2}-\frac{1}{x-7}=\frac{1}{x-1}-\frac{1}{x-6}$$

or
$$\frac{x-7-x+2}{(x-2)(x-7)}=\frac{x-6-x+1}{(x-1)(x-6)}$$

or
$$\frac{-5}{(x-2)(x-7)}=\frac{-5}{(x-1)(x-6)}$$

or
$$(x-2)(x-7)=(x-1)(x-6)$$

or
$$x^2-9x+14=x^2-7x+6$$

or
$$2x=8\,, \qquad x=4$$

12
$$\frac{1}{(x+a)^2-b^2}+\frac{1}{(x+b)^2-a^2}=\frac{1}{x^2-(a+b)^2}+\frac{1}{x^2-(a-b)^2}$$

or
$$\frac{1}{(x+a+b)(x+a-b)}+\frac{1}{(x+a+b)(x-a+b)}$$
$$=\frac{1}{(x+a+b)(x-a-b)}+\frac{1}{(x-a+b)(x+a-b)}$$

or
$$\frac{1}{(x+a+b)(x+a-b)}-\frac{1}{(x+a+b)(x-a-b)}$$
$$=\frac{1}{(x-a+b)(x+a-b)}-\frac{1}{(x+a+b)(x-a+b)}$$

or
$$\frac{x-a-b-x-a+b}{(x+a+b)(x+a-b)(x-a-b)}$$
$$=\frac{x+a+b-x-a+b}{(x-a+b)(x+a-b)(x+a+b)}$$

or
$$\frac{-a}{x-a-b}=\frac{b}{x-a+b}$$

or
$$-ax+a^2-ab=bx-ab-b^2$$

or
$$x(a+b)=a^2+b^2 \qquad x=\frac{a^2+b^2}{a+b}$$

13
$$\frac{3x^2+5x+8}{5x^2+6x+12}=\frac{3x+5}{5x+6}$$

or
$$\frac{x(3x+5)+8}{x(5x+6)+12}=\frac{3x+5}{5x+6}$$

Now putting m for $3x+5$ and n for $5x+6$, we get

$$\frac{xm+8}{xn+12}=\frac{m}{n}$$

or $$xmn+8n=xmn+12m$$

or $$8n=12m$$

or $$2n=3m$$

or (substituting) $$2(5x+6)=3(3x+5)$$

or $$10x+12=9x+15\ ; \qquad x=3$$

14 $$\frac{58x^2+87x+7}{87x^2+145x+11}=\frac{2x+3}{3x+5}$$

or $$\frac{29x(2x+3)+7}{29x(3x+5)+11}=\frac{2x+3}{3x+5}$$

Putting a for $2x+3$ and b for $3x+5$, we get

$$\frac{29xa+7}{29xb+11}=\frac{a}{b}$$

or $$29xab+7b=29\,abx+11a$$

or $$7b=11a$$

or (substituting) $$7(3x+5)=11(2x+3)$$

or $$21x+35=22x+33\ , \qquad x=2$$

15 $$\frac{a^2(a-2b)}{b(a-b)^2}x+\frac{2abc}{a-b}-\frac{ax}{b}=2cx-\frac{a^2b^2}{(a-b)^3}$$

or $$x\left\{\frac{a^2(a-2b)}{b(a-b)^2}-\frac{a}{b}-2c\right\}=-\frac{ab}{a-b}\left\{\frac{ab}{(a-b)^2}+2c\right\}$$

or $$x\left\{\frac{a^3-2a^2b-a^3+2a^2b-ab^2}{b(a-b)^2}-2c\right\}$$

$$=-\frac{ab}{a-b}\left\{\frac{ab}{(a-b)^2}+2c\right\}$$

or $$x\left\{\frac{-ab^2}{b(a-b)^2}-2c\right\}=-\frac{ab}{a-b}\left\{\frac{ab}{(a-b)^2}+2c\right\}$$

or $$x\left\{-\frac{ab}{(a-b)^2}-2c\right\}=\frac{ab}{a-b}\left\{-\frac{ab}{(a-b)^2}-2c\right\}$$

$$x=\frac{ab}{a-b}$$

16 $$(x-23)^3+(x-27)^3=2(x-25)^3$$

Putting a for $x-25$, we get

$$(a+2)^3+(a-2)^3=2a^3$$

or $\quad a^3+6a^2+12a+8+a^3-6a^2+12a-8=2a^3$

or $\quad 2a^3+24a=2a^3$

or $\quad 24a=0$

or $\quad a=0$

or $\quad x-25=0\,, \qquad x=25$

17 $\quad \dfrac{4x-17}{9}-\dfrac{3\frac{2}{3}-22x}{33}=x-\dfrac{6}{x}\left(1-\dfrac{x^2}{54}\right)$

or $\quad \dfrac{4x-17}{9}-\dfrac{11-66x}{99}=x-\dfrac{6}{x}\left(\dfrac{54-x^2}{54}\right)$

or $\quad \dfrac{4x-17}{9}-\dfrac{1-6x}{9}=x-\dfrac{54-x^2}{9x}$

or $\quad \dfrac{4x-17-1+6x}{9}=\dfrac{9x^2-54+x^2}{9x}$

or $\quad x(10x-18)=10x^2-54$

or $\quad -18x=-54\,, \qquad x=3$

18 $\quad \left(\dfrac{x-2a}{x+2b}\right)^2=\dfrac{x-2a-2b}{x+2a+2b}$

or $\quad \left\{1-\dfrac{2(a+b)}{x+2b}\right\}^2=1-\dfrac{4(a+b)}{x+2a+2b}$

or $\quad 1-\dfrac{4(a+b)}{x+2b}+\dfrac{4(a+b)^2}{(x+2b)^2}=1-\dfrac{4(a+b)}{x+2a+2b}$

or $\quad \dfrac{1}{x+2b}-\dfrac{1}{x+2a+2b}=\dfrac{a+b}{(x+2b)^2}$

or $\quad \dfrac{x+2a+2b-x-2b}{(x+2b)(x+2a+2b)}=\dfrac{a+b}{(x+2b)^2}$

or $\quad \dfrac{2a}{x+2a+2b}=\dfrac{a+b}{x+2b}$

or $\quad 2ax+4ab=ax+bx+2(a+b)^2$

or $\quad x(2a-a-b)=2\{(a+b)^2-2ab\}$

or $\quad x(a-b)=2(a^2+b^2)\,, \qquad x=\dfrac{2(a^2+b^2)}{a-b}$

19 $\quad \dfrac{x+19}{x+10}=\left(\dfrac{2x+33}{2x+24}\right)^2$

or $\quad 1+\dfrac{9}{x+10}=\left(1+\dfrac{9}{2x+24}\right)^2$

$\quad =1+\dfrac{9}{x+12}+\dfrac{81}{4(x+12)^2}$

or $$\frac{1}{x+10}-\frac{1}{x+12}=\frac{9}{4(x+12)^2}$$

or $$\frac{x+12-x-10}{(x+10)(x+12)}=\frac{9}{4(x+12)^2}$$

or $$\frac{2}{x+10}=\frac{9}{4(x+12)}$$

or $$8(x+12)=9(x+10)$$

or $$8x+96=9x+90\,, \qquad x=6$$

20 $$\left(\frac{x-a}{x+b}\right)^3=\frac{x-2a-b}{x+a+2b}$$

or $$\left(1-\frac{a+b}{x+b}\right)^3=1-\frac{3(a+b)}{x+a+2b}$$

or $$1-\frac{3(a+b)}{(x+b)}+\frac{3(a+b)^2}{(x+b)^2}-\frac{(a+b)^3}{(x+b)^3}$$
$$=1-\frac{3(a+b)}{x+a+2b}$$

or $$\frac{3(a+b)}{(x+b)^2}-\frac{(a+b)^2}{(x+b)^3}=3\left(\frac{1}{x+b}-\frac{1}{x+a+2b}\right)$$
$$=3\left\{\frac{x+a+2b-x-b}{(x+b)(x+a+2b)}\right\}$$
$$=\frac{3(a+b)}{(x+b)(x+a+2b)}$$

or $$\frac{3}{x+b}-\frac{3}{(x+a+2b)}=\frac{a+b}{(x+b)^2}$$

or $$3\left\{\frac{x+a+2b-x-b}{(x+b)(x+a+2b)}\right\}=\frac{a+b}{(x+b)^2}$$

or $$\frac{3(a+b)}{(x+a+2b)}=\frac{a+b}{(x+b)}$$

or $$3x+3b=x+a+2b$$

or $$2x=a-b\,; \qquad x=\tfrac{1}{2}(a-b)$$

21 $$\frac{3x}{2}-\frac{81x^2-9}{(3x-1)(x+3)}=3x-\frac{3}{2}\,\frac{2x^2-1}{x+3}-\frac{57-3x}{2}$$

or $$-\frac{9(3x+1)}{x+3}=\frac{3x}{2}-\frac{3}{2}\,\frac{2x^2-1}{x+3}-\frac{57-3x}{2}$$

or $$-\frac{3(3x+1)}{x+3}=\frac{x}{2}-\frac{1}{2}\,\frac{2x^2-1}{x+3}-\frac{19-x}{2}$$

or $$\frac{2x^2-1}{2(x+3)}-\frac{3(3x+1)}{x+3}=\frac{x}{2}-\frac{19-x}{2}=\frac{2x-19}{2}$$

or $$\frac{2x^2-1-18x-6}{2(x+3)}=\frac{2x-19}{2}$$

or $$2x^2-18x-7=2x^2-13x-57$$

or $$-5x=-50\,, \qquad x=10$$

Exercise (67)

1 Let the other number be y, then we have $x+y=15$,
$$y=15-x$$

2 Let y be the other number, then we have $x-y=20$,
$$y=x-20$$

3 Let y be the other number then we have $y-x=25$,
$$y=x+25$$

4 Let x be the required excess, then $y-x=25$,
$$y=25+x$$

5 Let z be the required defect, then we have $y-z=2x$
$$z=y-2x$$

6 Let y be the other factor, then we have $xy=21$,
$$y=\frac{21}{x}$$

7 Let y be the required number, then we have $y+3x=100$,
$$y=100-3x$$

8 Let z be the required number, then we have $4x-z=3y$,
$$z=4x-3y$$

9 Since in one hour y mile is travelled,
in x hours $(x\times y)$ or (xy) mile is travelled

10 Since y mile is travelled in 1 hour,

1 mile $\quad \frac{1}{y}$th of an hour,

x mile $\quad \frac{x}{y}$ hour

11 Let y years be the age 20 years later,
and z years 3 years ago,
then $y-20=x$, $\quad y=x+20$,
and $x-z=3$, $\quad z=x-3$

12 Let z miles be the required rate,
$$xz=60\,, \qquad z=\frac{60}{x}$$

13 Since in x hours 30 miles is travelled

or in $(x \times 60 \times 60)$ seconds $(30 \times 1760 \times 3)$ feet is travelled,

in 1 second $\frac{30 \times 1760 \times 3}{60 \times 60 \times x}$ feet or $\frac{44}{x}$ feet is travelled

14 In one week x annas is spent

or $\frac{x}{16}$ Rupees is spent,

in one year $\left(\frac{x}{16} \times 52\right)$ Rupees is spent

or $\frac{13}{4}x$ Rupees is spent

Now therefore the saving $= \left(5x - \frac{13}{4}x\right)$Rs $= \frac{7x}{4}$ Rs

15 As the preceding number is evidently greater than the succeeding one, the two numbers just preceding x are $(x-1)$ and $(x-2)$ and those just succeeding x are $(x+1)$ and $(x+2)$ Thus the required numbers are $x-2, x-1, x, x+1$ and $x+2$

16 The three numbers are $x-1$, x and $x+1$,

the required sum $=(x-1)+x+(x+1)=3x$

17 The next number after $2m+1$ is

$\{(2m+1)+1\}$, i e, $2(m+1)$ which is even,

the required odd number is $2(m+1)+1$, i e, $2m+3$

18 The even number next before $2x$ is evidently $2x-2$

19 x men take 10 days to do the work

1 man takes $10x$ days

y men will take $\frac{10x}{y}$ days to do the work

20 a yds $= 3a$ ft,

the required measure $=(3a \times b)$sq ft $= 3ab$ sq ft

21 The length and breadth evidently become

$\frac{3a}{4} + \frac{b}{4}$ respectively,

the number of sq units required is $\frac{3ab}{16}$

22 In y hours he walks x miles

in 1 hour $\frac{x}{y}$ miles

in 1 minute $\frac{x}{60y}$ miles

in 20 minutes $\frac{x}{3y}$ miles

23 He walks x miles in a hours

1 miles in $\frac{a}{x}$ hours

16 miles in $\frac{16a}{x}$ hours

24 His present age $=\{(x-5)+20\}$ years $=x+15$ years

and his age 30 years hence $=\{(x+15)+30\}$ years

$=(x+45)$ years

25 The number is evidently equal to $10y+x$

26 The number is evidently equal to $100x+10y+z$

27 The number in this case evidently becomes $100z+10y+x$

Exercise (68)

1 Let x be the greater number

Then the other number $=50-x$,

also that number $=x-30$,

$50-x=x-30$

or $2x=80$, $x=40$,

the other number $=50-40=10$

2 Let x be the number

Then $x=5(96-x)$

or $6x=5\times 96$,

$x=5\times 16=80$

3 Let x be the number

Then $8x=\frac{1}{2}x+90$

or $16x=x+180$

or $15x=180$, $x=12$

4 Let x be the required number

Then $x-40=\frac{1}{3}x$

or $3x-120=x$

or $2x=120$

$x=60$

5 Let x be the number

Then $x-35=(67-x)-22$

or $2x=67-22+35=80$

$x=40$

6 Let x be the number

Then $4(x-16)=416-x$

or $5x=416+64=480$,

$x=96$

7 Let x be the middle number

Then $3x=129$,

$x=43$,

the numbers are 42, 43 and 44

8 Let x be the number

Then $7x-132=132-x$

or $8x=264$,

$x=33$

9 Let x be one part

Then the other part $=90-x$

$3x+4(90-x)=335$

or $x=335-360$

$=-25$,

$x=25$,

and the other part is $(90-25)=65$

10 Let x be one of the numbers

the other number $=39-x$;

$\frac{x}{5}=\frac{39-x}{8}$

or $8x=195-5x$

or $13x=195$,

$$x=15,$$

the other number is $(39-15)=24$

11 Let x be the number

$$\tfrac{1}{4}x-\tfrac{1}{9}x=5$$

or $9x-4x=180$

or $5x=180,$

$$x=36$$

12 Let x be the number

Then $\tfrac{1}{6}x-\tfrac{1}{8}x=3$

or $8x-6x=144$

or $2x=144,$

$$x=72$$

13 Let x be one of the parts

Then the other part $=21-x,$

$$10x=9(21-x)+1$$

or $19x=189+1=190,$

$$x=10,$$

the other part $=21-10=11$

14 Let x be the price of the house

Then the price of the garden $=\dfrac{5x}{12},$

$$x+\frac{5x}{12}=£850$$

or $12x+5x=£10200$

or $17x=£10200$

$$x=£600,$$

the price of the garden $=\frac{5}{12}\times£600=£250$

15 Let one of the persons get x shillings or $£\dfrac{x}{20}$ and the other gets x half-crown or $£\dfrac{x}{8}$

We have $\dfrac{x}{20}+\dfrac{x}{8}=420$

or $2x+5x=420\times40,$

or $7x=420\times40,$

$$x = 60 \times 40 = 2400,$$

they will receive 2400 shillings and 2400 half-crowns respectively i e, £120 and £300

16 Let x be the value of a sheep,

$$72x + £35 = 92x - £35$$

or $$20x = £70$$

$$x = £3 \ \ 10s$$

17 Let x be the age of the younger person,

the age of the elder man $= (x + 10)$ years,

$$(x + 10) - 15 = 2(x - 15)$$

or $$x - 5 = 2x - 30,$$

$$x = 25,$$

the age of the elder man $= 25 + 10 = 35$

18 Let x be the breadth of the smaller field,

$2x$ is the length of the smaller field,

$$2x^2 + 6800 = (x + 10)(2x + 50)$$

$$= 2x^2 + 70x + 500,$$

$$70x = 6300,$$

$$x = 90$$

The size of the smaller field is 90 yds by 180 yds, and the size of the larger field is 100 yds by 230 yds

19 Let the speed of the luggage train be x miles per hour

Now the luggage train travels $(15 + 10)$ mts before it is overtaken by the train

$$\frac{35 \times 10}{60} = \frac{25x}{60}$$

$$25x = 35 \times 10$$

or $$x = \frac{35 \times 10}{25} = 14$$

The required speed is 14 miles per hour

20 Let x miles be the required distance Then he must take $\frac{x}{8}$ hrs to ride on and $\frac{x}{2}$ hours to come back

$$\frac{x}{8} + \frac{x}{2} = 5$$

or $x + 4x = 40$, $x = 8$ miles

21 Let x years be the son's age Then the father's age is $3x$ years Ten years afterwards the father's age will be $(3x+10)$ and the son's age $(x+10)$

We get $3x+10=2(x+10)$

or $x=10$,

the father's age is 30 years,

and the son's age is 10 years

22 Let x ft be the breadth of the room Then the length is $(x+3)$ ft Also the increased length becomes $(x+6)$ ft and the diminished breadth becomes $(x-2)$ ft

We get $x(x+3)=(x+6)(x-2)$

or $x^2+3x=x^2+4x-12$

or $x=12$,

the breadth is 12 ft

and the length is 15 ft

23 Let x pounds be the required sum After losing £$\frac{5}{11}$ B had £$\frac{6x}{11}$ with him

A gained £$\left\{\frac{1}{2}\left(\frac{6x}{11}\right)+6\right\}$

or £$\left(\frac{3x}{11}+6\right)$,

we get $\frac{5x}{11}=\frac{3x}{11}+6$

or $\frac{2x}{11}=6$, or $x=33$

They had at first £33

24 Let x years be the son's age, then the father's age is $(80-x)$ years

We get $2x=(80-x)+10$

or $3x=90$, $x=30$, and

$80-x=50$

The father's age is 50 years,

and the son's age is 30 years

25 Let there were x men, therefore there were $(36-x)$ women

$$\text{We get } \frac{3}{20}x+\frac{5}{2\times 20}(36-x)=5$$

or $$6x+5(36-x)=200$$

or $$x=200-180=20\ ,$$

$$\text{and } 36-x=16\ ,$$

there were 20 men and 16 women

26 Let them have travelled for x hours, then A travelled $\frac{3x}{2}$ miles and B travelled $\frac{5x}{4}$ miles

$$\text{We get } \frac{3x}{2}+\frac{5x}{4}=154$$

or $$6x+5x=616\ ,$$

$$11x=616\ ,$$

$$x=56,$$

and $\frac{3x}{2}=84$ and $\frac{5x}{4}=70$

A travelled 84 miles and B 70 miles in 56 hours

27 Let him have worked for x days, then he was absent for $(36-x)$ days

$$\text{We get } x(2\tfrac{1}{2})-(36-x)(1\tfrac{1}{2})=58$$

or $$5x-(36-x)3=116$$

or $$8x=116+108=224\ ,$$

$$x=28\ ,$$

he worked for 28 days

28 Let x shillings be the cost of the picture or that of the frame,

$$\text{we have } x+15=2(x-20)$$

or $$x=55\ ,$$

the cost of the picture $=55s=£2\ \ 15s$

29 Let x feet be the required length

$$\text{We get } \frac{x}{4}+\frac{x}{3}+10=x$$

or $$3x+4x+120=12x$$

or $$5x=120$$

or $$x=24\ ,$$

the required length $=24$ ft

30. Let x be the number of working days, $\therefore$ he remained idle for $(30-x)$ days For x days he received $x\,(2\frac{1}{2})s$ and $(30-x)$ days he lost $(30-x)s$

$$\text{We get } x(2\tfrac{1}{2})s-(30-x)s=£2\ 7s$$

$$\text{or}\qquad \frac{5x}{2}-30+x=47$$

$$\text{or}\qquad 5x-60+2x=94$$

$$\text{or}\qquad 7x=154$$

$$\text{or}\qquad x=22,$$

$\therefore$ he worked for 22 days

31 Let x be the required number of days Then in one day they can together do $\frac{1}{x}$ of the work

A can do $\frac{1}{9}$ of the work in one day, B can do $\frac{1}{18}$ and C can do $(\frac{1}{9}\times\frac{3}{4})$ or $\frac{1}{12}$ of the work in the same time

$$\text{We get } \frac{1}{9}+\frac{1}{18}+\frac{1}{12}=\frac{1}{x}$$

$$\text{or}\qquad \frac{4+2+3}{36}=\frac{1}{x}$$

$$\text{or}\qquad \frac{9}{36}=\frac{1}{x},\ \text{or,}\ \frac{1}{4}=\frac{1}{x},\ \text{or,}\ x=4$$

$\therefore$ They would require 4 days to do the work

32 Let x pounds and x shillings be the required sums

$$\text{We get } x+\frac{x}{20}=54\frac{3}{5}=\frac{273}{5}$$

$$\text{or}\qquad 20x+x=1092$$

$$\text{or}\qquad 21x=1092$$

$$\text{or}\qquad x=52,$$

$\therefore$ the required sums are £52 and 52s

33 Let x pounds be the sum to be divided

A is to get $\left(\frac{x}{2}-30\right)$ pounds,

B is to get $\left(\frac{x}{3}-10\right)$ pounds,

and C is to get $\left(\frac{x}{4}+8\right)$ pounds,

we get $\frac{x}{2}-30+\frac{x}{3}-10+\frac{x}{4}+8=x$

or $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}-x=32$

or $\frac{6x+4x+3x-12x}{12}=32$

or $\frac{x}{12}=32$, $x=£384$

A receives $£\left(\frac{384}{2}-30\right)$ or £162,

B " $£\left(\frac{384}{3}-10\right)$ or £118,

and *C* " $£\left(\frac{384}{4}+8\right)$ or £104

34 Let x be the number of sheep

Then we get $x(£2\ 2s)-(£1\ 8s)=x(£2)+£2$

or $x(2s)=£3\ 8s=68s$

or $x=34$, there were 34 sheep,

and he had with him £(34 2+2), or £70

35 Let them meet x hours after starting

Then we get

$9\frac{1}{2}x+9\frac{1}{4}x=200$

or $18\frac{3}{4}x=200$

or $75x=800$

or $x=10\frac{2}{3}$

They met at a distance of $(9\frac{1}{4}\times 10\frac{2}{3})$,

or $98\frac{2}{3}$ miles from London, also the required time is $10\frac{2}{3}$ hours

36 Let the number of apples at 3 a penny be x, the number of the kind at 4 a penny is therefore $\frac{5x}{6}$

The sum spent in buying them is $\left(\frac{x}{3}+\frac{5x}{6\times 4}\right)d$ and the sum obtained by selling them is $\left\{\left(x+\frac{5x}{6}\right)\frac{6}{16}\right\}d$ As the selling price is greater than the buying price by $3\frac{1}{2}d$,

we get $\frac{x}{3}+\frac{5}{24}x+\frac{7}{2}=\frac{11}{16}x$

or $16x+10x+168=33x$

or $33x-26x=168$

or $7x=168$, or $x=24$,

the total number of apples bought

$=24+\frac{5\cdot 24}{6}=24+20=44$

37 Let the number representing the left digit be x. The number representing the other digit is $(5-x)$, the required number is $10x+(5-x)$, or $9x+5$

Also we get $x+1=\frac{1}{8}(9x+5)$

or $8x+8=9x+5$

or $x=3$

The required number $=32$

38 Let the digit in the unit's place be x

Then the ten's place is $(x+5)$,

the number is $10(x+5)+x$ or $11x+50$

Also we get $(11x+50)-5\{x+(x+5)\}=10x+(x+5)$

or $11x+50-10x-25=11x+5$

or $10x=20$, $x=2$

The required number $=72$

39 Let the digit in the ten's place be x,

the other digit is $(5-x)$,

the number is $10x+(5-x)$ or $9x+5$

Also $10x+4(5-x)=10(5-x)+x$

or $6x+20=50-9x$

or $15x=30$, $x=2$

The required number is 23

40 Let the common result be x

The 1*st* part is $x-1$,

" 2*nd* $x+2$,

" 3*rd* $\frac{x}{3}$

" 4*th* $4x$

We get $(x-1)+(x+2)+\frac{x}{3}+4x=39$

or $6x+\frac{x}{3}=38$

or $18x+x=114$

or $19x=114$

or $x=6$

The parts are $(6-1)$, $(6+2)$, $(\frac{6}{3})$ and (6×4)

or 5, 8, 2 and 24

41 Let the common result be x

The 1st part $=x+3$,

" 2nd " $=x-11$,

" 3rd " $=\frac{x}{4}$,

" 4th " $=2x$

We get $(x+3)+(x-11)+\frac{x}{4}+2x=60$

or $4x+\frac{x}{4}=60+11-3=68$

or $16x+x=68\ 4$

or $17x=68\ 4$

or $x=16$

The parts are $(16+3)$, $(16-11)$, $(\frac{16}{4})$ and (16×2)

or 19, 5, 4 and 32

42 Let the common result be x

The 1st part $=x-5$,

" 2nd " $=x+4$,

" 3rd " $=\frac{x}{3}$,

" 4th " $=2x$

We get $(x-5)+(x+4)+\frac{x}{3}+2x=116$

or $4x+\frac{x}{3}=117$

or $12x+x=117\ 3$

or $13x=117\ 3$, $x=27$

The parts are $(27-5)$, $(27-4)$, $(\frac{27}{3})$ and (27×2)

or 22, 31, 9, 54.

Exercise 69

1. Let x minutes past 3 o'clock be the required time. Now the minute hand travels 12 times faster than the hour-hand so that $x=12(x-15)$

or $11x=180$ or $x=\frac{180}{11}=16\frac{4}{11}$

$\therefore$ the required time is $16\frac{4}{11}$ minutes past 3

2. Let x mts past 5 be the required time.

We get $x=12(x-25)$

or $11x=300$

$x=\frac{300}{11}=27\frac{3}{11}$

$\therefore$ The required time is $27\frac{3}{11}$ minutes past 5

3. At 7 the hands are 35 minute-divisions apart.

(i) They will be opposite to each other when they will be 30 minute-divisions apart.

Let x minutes past 7 be the required time.

We get $x=12\{x-(35-30)\}=12(x-5)$

or $11x=60$ $\qquad x=\frac{60}{11}=5\frac{5}{11}$

The hands will be opposite to each other at $5\frac{5}{11}$ minutes past 7

(ii) The hands will be at right angles when they will be 15 minute-divisions apart.

Let x minutes past 7 be the required time

We get $x=12\{x-(35\pm 15)\}$

$=12(x-50)$

or $12(x-20)$

or $11x=600$ or 240

or $x=54\frac{6}{11}$ or $21\frac{9}{11}$

The hands will be at right angles at $54\frac{6}{11}$ minutes and $21\frac{9}{11}$ minutes past 7

(iii) Let the required time be x minutes past 7.

We get $x = 12(x-35)$

or $\qquad 11x = 420$,

$$x = \frac{420}{11} = 38\tfrac{2}{11}$$

The required time is $38\frac{2}{11}$ minutes past 7.

4 At 6 the hands are 30 minute-divisions apart.

(i) they will be again opposite when the minute hand will pass 60 minute-divisions more than the hour-hand.

Let x minutes past 6 be the required time.

We get $x = 12(x-60)$

or $\qquad 11x = 720$

or $\qquad x = \frac{720}{11} = 65\tfrac{5}{11}$

The required time is $65\frac{5}{11}$ minutes past 6 i.e. $5\frac{5}{11}$ minutes past 7.

(ii) The hands will be at right angles to each other when they will be 15 minute divisions apart.

Let x be the required time.

$$\text{We get } x = 12\{x-(30-15)\}$$
$$= 12(x-15)$$

or $\qquad 11x = 180, \qquad x = 16\tfrac{4}{11}$

The required time is $16\frac{4}{11}$ minutes past 6.

5 Let x miles be the required distance. Then the man from A walked x miles in $\frac{x}{p}$ hours and the man from B walked $(a-x)$ miles in $\frac{a-x}{q}$ hours.

$$\text{We get } \frac{x}{p} = \frac{a-x}{q}$$

or $\qquad xq = ap - xp$

or $\qquad x(p+q) = ap, \qquad x = \frac{ap}{p+q}$

The required distance is $\frac{ap}{p+q}$ miles.

6 Let the second meeting place be x miles from the place whence the man walking 6 miles an hour started. Then he

walked $\{22+(22-x)\}$ miles or $(44-x)$ miles in $\frac{44-x}{6}$ hours and the other walked $(22+x)$ miles in $\frac{22+x}{5}$ hours

$$\frac{44-x}{6}=\frac{22+x}{5}$$

or $$220-5x=132+6x$$

or $$11x=88$$

$$x=8$$

Thus the required place is 8 miles distant from the starting place of the faster walker and the required time $=\frac{44-x}{6}$ hours

$$=\frac{36}{6} \text{ hours } =6 \text{ hours}$$

7 Let x miles be the distance from A to B Then the man travelled $\frac{x}{3}$ miles in $\frac{x}{3a}$ hours and $\frac{2x}{3}$ miles in $\frac{2x}{3\ 2b}$ hours or $\frac{x}{3b}$ hours and at the rate of $3c$ miles per hour he would have ridden $2x$ miles in $\frac{2x}{3c}$ hours

We get $$\frac{2x}{3c}=\frac{x}{3a}+\frac{x}{3b}$$

or $$\frac{2}{3c}=\frac{1}{a}+\frac{1}{b}$$

8 Let x minutes be the time for which B ran

In 5 minutes B ran $(900-75)$ yds $=825$ yds , in x minutes he ran $\frac{825}{5}x$ yds or $165x$ yds

Again A ran before he fell $\frac{900}{5}$ yds or 180 yds per minute and after the accident he ran 160 yds per minute , in the remaining $(x-5+\frac{1}{2})$ minutes he ran $(x-\frac{9}{2})\times 160$ yds or $(160x-720)$ yds

We get $$165x=160x-720+900$$

or $$5x=180$$

or $$x=36$$

The time required is 36 minutes

9 Let x miles per hour be his rate when he first started

Then $\frac{1}{x}$($\frac{5}{6}$ of the journey in miles) is the fixed time

Also $\left\{\frac{1}{x}\right.$($\frac{1}{4}$ of the journey in miles) $+\frac{1}{x+1}$($\frac{3}{4}$ of the journey in miles$\left.\right\}$ is the fixed time

We get $\frac{1}{x}\cdot\frac{5}{6}=\frac{1}{x}\cdot\frac{1}{4}+\frac{1}{x+1}\cdot\frac{3}{4}$

or $\frac{1}{x}\left(\frac{5}{6}-\frac{1}{4}\right)=\frac{1}{x+1}\cdot\frac{3}{4}$

or $\frac{1}{x}\left(\frac{10-3}{12}\right)=\frac{1}{x+1}\cdot\frac{9}{12}$

or $\frac{7}{x}=\frac{9}{x+1}$, or $7x+7=9x$, or $x=\frac{7}{2}=3\frac{1}{2}$

The required rate is $3\frac{1}{2}$ miles and $4\frac{1}{2}$ miles per hour

10 Let x be the required number of bushels

Then (£1 15s.) ÷ {£80+(£1 5s.)x} is the number of acres in his farm

Also (£2) ÷ {£80+(£1 10s.)x}

We get $1\frac{3}{4}\div(80+1\frac{1}{4}x)=2\div(80+1\frac{1}{2}x)$

or $\frac{7}{4(80+\frac{5}{4}x)}=\frac{2}{80+\frac{3}{2}x}$

or $560+\frac{21}{2}x=640+10x$

or $\frac{1}{2}x=80$, or $x=160$

The required number of bushels $=160$

11 Let x pounds be the value of the livery ∴ $(8+x)$ pounds is his yearly pay ∴ $\frac{7}{12}(8+x)$ pounds is his 7 months' pay

Again (£2 3s. 4d. + £x) is his 7 months' pay

We get $\frac{7}{12}(8+x)=2\frac{1}{6}+x$

or $56+7x=26+12x$

or $5x=30$

∴ $x=6$,

∴ the required value $=$ £6

12 Let x be the required number of leaps of the greyhound

But in that time the hare took $\frac{4x}{3}$ leaps Also x leaps of the hound are as much as $\frac{3x}{2}$ leaps of the hare

$$\text{We get } 50+\frac{4x}{3}=\frac{3x}{2}$$

or $$300+8x=9x$$

or $$x=300$$

The required number of leaps of the hound $=300$

13 Let x be the number of leaps made by the hound Then the number of leaps made by the hare is $\frac{5x}{4}$ Again x leaps of the hound are as much as $\frac{4x}{3}$ leaps of the hare

$$\text{We get } \frac{4x}{3}=\frac{5x}{4}+\frac{4}{3}(60)$$

or $$16x=15x+960$$

or $$x=960$$

and $$\tfrac{5}{4}x=\tfrac{5}{4}(960)=1200$$

The number of leaps made by the hound $=960$ and that made by the hare $=1200$.

14 Let x be the required number of strokes pulled by the Caius The number of strokes pulled by the Johnious is $\frac{4x}{3}$ But the x strokes of the Caius are equivalent to $\frac{3x}{2}$ of the Johnious

$$\text{We get } \frac{3x}{2}=30+\frac{4x}{3}$$

or $$9x=180+8x$$

or $$x=180$$

The required number of strokes $=180$

15 Let x be the number of shillings in the purse Then A took $\{2+\frac{1}{6}(x-2)\}$ shillings or $\left(\frac{x}{6}+\frac{5}{3}\right)$ shillings and B took

$$\left\{3+\frac{1}{6}\left(x-\frac{x}{6}-\frac{5}{3}-3\right)\right\} \text{ shillings or } \left(\frac{5x}{36}+\frac{20}{9}\right)\text{shillings.}$$

We get $\frac{x}{6}+\frac{5}{3}=\frac{5x}{36}+\frac{20}{9}$

or $\qquad 6x+60=5x+80$

or $\qquad x=20,$

and $\quad \frac{x}{6}+\frac{5}{3}=\frac{20}{6}+\frac{5}{3}=\frac{30}{6}=5$

There were 20 shillings in the purse and 5 shillings were taken by each of A and B

16 Let x be the required number of crew Then there were $60x$ pounds of biscuit Evidently after 20 days there remained $40x$ pounds of biscuits Again for the remaining $(40+24)$ days or 64 days the remaining $(x-5)$ men were supplied with $\{\frac{5}{7}(x-5)64\}$ pounds of biscuit

We get $40x=\frac{5\times 64(x-5)}{7}$

or $\qquad 280x=320x-1600$

or $\qquad 40x=1600$

or $\qquad x=40$

The required number of crew is 40

17 Let x lbs be the weight of gold in 106 lbs of the mass, then the weight of silver there is $(106-x)$ lbs Again x lbs of gold will weigh $\frac{18}{19}x$ lbs in water and $(106-x)$ lbs of silver will weigh $\frac{9(106-x)}{10}$ lbs in water

We get $\frac{18x}{19}+\frac{9(106-x)}{10}=99$

or $\qquad 180x+171(106-x)=99\times 190$

or $\qquad 180x+18126-171x=18810$

or $\qquad 9x=684, \quad$ or $\quad x=76,$

and $\qquad 106-x=106-76=30$

There were 76 lbs of gold and 30 lbs of silver

18 Let x hours be the time of his going with the current, then $(10-x)$ hours is the time of his returning. The rates of his going with and against the current are $\frac{20}{x}$ miles and $\frac{20}{10-x}$ miles per hour respectively

We get $\frac{20}{x}=\frac{3}{2}\cdot\frac{20}{10-x}$

or $\frac{1}{x}=\frac{3}{2(10-x)}$

or $20-2x=3x$

or $5x=20$, or $x=4$, and $10-x=6$

The time of going with the current is 4 hours and the time of returning is 6 hours

19 Let x years be the required age of the son Then the father's age is $2x$ years Also the father's age is

$$\frac{2x}{6}+\frac{2x}{12}+\frac{2x}{7}+5+x+4$$

We get $\frac{2x}{6}+\frac{2x}{12}+\frac{2x}{7}+x+9=2x$

or $14x+7x+12x+42x+378=84x$

or $9x=378$, $x=42$

The required age is 42 years

20 Let x oz. be taken from the first bar and then $(20-x)$ oz is taken from the second bar Now x oz of the first bar contains $(x\times\frac{14}{20})$ oz of silver and $(20-x)$ oz of the second bar contains $\{(20-x)\times\frac{8}{20}\}$ oz of silver

We get $\frac{14}{20}x+\frac{8}{20}(20-x)=10$

or $\frac{7}{10}x+\frac{2}{5}(20-x)=10$

or $7x+80-4x=100$

or $3x=20$

or $x=\frac{20}{3}=6\frac{2}{3}$ and $20-x=13\frac{1}{3}$

$6\frac{2}{3}$ oz. from the first bar and $13\frac{1}{3}$ oz. from the second bar should be taken

21 Let the greater sum be £x, then the smaller is (£607 1*s* 8*d* − £x)

The interest on the greater sum for 2 years at $3\frac{1}{2}$ p c is $3\frac{1}{2}\times2\times\frac{£x}{100}$ or £$\frac{7x}{100}$ and that on the smaller for $2\frac{1}{2}$ years at $3\frac{1}{4}$ p c. is $\frac{5}{2}\times\frac{13}{4}\left(\frac{£607\ 1s\ 8d-£x}{100}\right)$ or $\frac{65}{8}\left(\frac{£607\ 1s\ 8d-£x}{100}\right)$

We get $\frac{7x}{100}=\frac{65}{800}\left(607\frac{20}{240}-x\right)+18\frac{16}{20}$

or $\frac{7x}{100}=\frac{65}{800}\left(607\frac{1}{12}-x\right)-18\frac{4}{5}$

or $\frac{7x}{100}=\frac{65}{800}\left(\frac{7285-12x}{12}\right)+\frac{94}{5}$

or $56\times 12x=65\times 7285-65\times 12x+94\times 12\times 8\times 20$

or $12x(56+65)=5\times 47(13\times 155+2\times 96\times 4)$

or $121\times 12x=235(2015+768)$

$=235\times 2783$

or $12x=235\times 23=5405$,

$x=£450\frac{5}{12}=£450\ 8s\ 4d$

The greater sum $=£450\ \ 8s\ \ 4d$,

and the other sum $=£156\ \ 13s\ \ 4d$

22 Let there be x pice in each group

Each man of the 3rd group obtained $\frac{x+5}{4}$ pice and each man of the 4th group obtained $\frac{x+9}{5}$ pice ,

we have $\frac{x+5}{4}=\frac{x+9}{5}$

or $5x+25=4x+36$

or $x=11$,

there were 11 pice in each group

Each man of the 1st set obtained

$\left(\frac{11+1}{2}=\right)$ 6 pice,

each of the 2nd set $\left(\frac{11+4}{3}=\right)$ 5 pice,

and each of the 3rd and 4th sets $\left(\frac{11+5}{4}=\right)$ 4 pice

23 Let there be x guineas in the parcel

The weight of the parcel $\frac{4}{15}(x-9)$ oz ,

The weight of each guinea $=\frac{4(x-9)}{15x}$ oz

Again the weight of $\left(\frac{x}{2}-10\frac{1}{2}\right)$ guineas

was $\left\{\frac{4}{15}\times\left(\frac{x-21}{2}\right)-\frac{1}{3}\right\}$ oz. or $\frac{1}{3}\left(\frac{x-21}{10}-1\right)$ oz.

∴ The weight of each guinea

$=\left\{\frac{4(x+11)}{30}\div\frac{x-21}{2}\right\}$ oz.

$=\frac{4(x-11)}{15(x-21)}$ oz.

∴ We get $\frac{4(x+11)}{15(x-21)}=\frac{4(x-9)}{15x}$

or $\frac{x-11}{x-21}=\frac{x-9}{x}$

or $x^2-11x=x^2-12x-189$

or $x=189$

There were 189 guineas

24. Let x oz. be the weight of the 1st. wrought plate sold. Then price of x oz. of unwrought plate $=£10-£3.\ 15s$

$=£6\ \ 5s=£\frac{25}{4}$

∴ the price of 8 oz. of that plate

$=£\frac{25}{4}\times\frac{8}{x}=£\frac{50}{x}$

Again the price of 12 oz. of wrought plate $=£\frac{12\times10}{x}$

∴ The price of 8 oz. of unwrought plate

$=£\left(\frac{120}{x}-2\frac{4}{5}\right)$

∴ We get $\frac{50}{x}=\frac{120}{x}-\frac{14}{5}$

or $-\frac{70}{x}=-\frac{14}{5}$

or $14x=350$

or $=25$

∴ The weight of the first plate sold is 25 oz. and the price of wrought plate per oz. $=£\frac{10}{25}=8s$

25 Let xs be the charge for the luggage allowed free Then if all the luggage belonged to one man, he would have been charged $(xs + 2s\ 10d + 7s\ 6d)$

$$\text{or}\quad (xs + 10s\ 4d) \quad \text{or} \quad \left(x+\frac{31}{3}\right)s\ ,$$

$$\text{we get } \left(x+\frac{31}{3}\right)s = 14s\ 6d = \frac{29}{2}s$$

$$\text{or}\quad x+\frac{31}{3}=\frac{29}{2}$$

$$\text{or}\quad x=\frac{29}{2}-\frac{31}{3}=\frac{87-62}{6}=\frac{25}{6}=4\frac{1}{6}$$

The charge for the luggage allowed free $=4\frac{1}{6}s = 4s\ 2d$

If none had been allowed free the passengers would have been charged $\{(2s\ 10d)+(4s\ 2d)\}$ or $7s$, and $\{(7s\ 6d)+(4s\ 2d)\}$ or $11s\ 8d$ respectively

26 Let x be the required number of bundles Then the buying price of the two kinds of hay $= \left(\frac{5x}{1000}+\frac{5600\times 6}{1000}\right)$Rs

$$=\left(\frac{x}{200}+\frac{168}{5}\right)\text{ Rs}$$

The selling price must be

$$\frac{120}{100}\left(\frac{x}{200}+\frac{168}{5}\right)\text{ Rs to have a profit of } 20\ p\ c$$

$$\text{We get } \frac{11}{16}\times\frac{x+5600}{100}=\frac{120}{100}\left(\frac{x}{200}+\frac{168}{5}\right)$$

$$\text{or}\quad \frac{11x+61600}{1600}=\frac{120}{100}\times\frac{x+6720}{200}$$

$$\text{or}\quad \frac{11x+61600}{8}=\frac{6(x+6720)}{5}$$

$$\text{or}\quad 55x+308000=48x+322560$$

$$\text{or}\quad 7x = 14560\ ,$$

$$x = 2080\ ,$$

the required number of bundles $= 2080$

27 Let xd be the required selling price of each orange The buying price of 3 of the 1st kind and one of the 2nd kind

is $(2+\frac{1}{2}=)\frac{5}{2}d$ the selling price of those four must be $\frac{120\times 5}{100\times 2}d$ or $3d$

We get $4x=3$, $x=\frac{3}{4}$,

the required price is $\frac{3}{4}d$ each

Let x be the number bought,

the selling price of x oranges is

$$\frac{120}{20}(64)d \quad \text{or} \quad (6\times 64)d$$

$$x=\frac{6\times 64}{\frac{3}{4}}=6\times 64\times\frac{4}{3}=512,$$

the required number is 512

28 Let x be the required number of coins The new coins were equivalent to $(1-\frac{1}{5}=)\frac{4}{5}$ of the original coins With x original coins he made $\frac{5}{4}x$ light coins If he could pass all these coins his gain would have been equivalent to $\left(\frac{5x}{4}-x=\right)\frac{x}{4}$ original coins or $\frac{1}{4}\left(\frac{5x}{4}\right)$ light coins $\left\{\frac{1}{3}\left(\frac{5x}{4}\right)-1\right\}$ light coins were seized, and his original gain was equivalent to $\frac{2}{3}\left\{\frac{1}{4}\left(\frac{5x}{4}\right)\right\}$ light coins, and by passing the one light coin with which he decamped his gain will be $\frac{1}{4}$ of a light coin more

$$\text{We get } \left\{\frac{1}{3}\left(\frac{5x}{4}\right)-1\right\}-\frac{2}{3}\left\{\frac{1}{4}\left(\frac{5x}{4}\right)\right\}-\frac{1}{4}$$
$$=\frac{1}{3}\left\{\frac{1}{4}\left(\frac{5x}{4}\right)\right\}$$

or $$\frac{5x}{12}-1-\frac{5x}{24}-\frac{1}{4}=\frac{5x}{48}$$

or $$20x-48-10x-12=5x$$

or $$5x=60$$

$$x=12,$$

there were at first 12 coins

29 Let x be the digit in the unit's place Then the required number $=100(x+2)+10(x+1)+x$ and the number formed by inverting the digits $=100x+10(x+1)+(x+2)$, we get

$$\{100(x+2)+10(x+1)+x\}-\frac{3}{4}\{100x+10(x+1)+(x+2)\}$$
$$=36\{(x+2)+(x+1)+x\}$$

or $100x+200+10x+10+x-\frac{1}{4}(100x+10x+10+x+2)$
$=108(x+1)$

or $400x+800+40x+40+4x-100x-10x-10-x-2$
$=432x+432$

or $444x-111x-432x=432-840+12$

or $-99x=-396$, $x=4$

The required number $=654$

30 Let x be the number of men in front of the square, the required number of men $=x^2+60$

Again the number of men in the new complete column will be $(x+5)(x-3)$, we get

$$x^2+60=(x+5)(x-3)-1$$
$$=x^2+2x-15-1$$

or $60=2x-16$

or $2x=76$ $x=38$,

and $x^2+60=1444+60=1504$

The required number of men $=1504$

31 Let x be the number of men in front of the hollow square

Then the total number of men in the regiment

$$=x^2-\{x-2(10)\}^2=40x-400,$$

we get $40x-400=2800$

or $40x=3200$, $x=80$,

the number of men in front of the hollow square $=80$

32 Let x be the number of men in front of the 1st square

Then the number of men in front of the 2nd square$=(x-19)$

From the 1st square we get the number of men$=x^2-(x-8)^2$ and from the 2nd square it becomes$=(x-19)^2-(x-19-16)^2$,

we get $x^2-(x-8)^2=(x-19)^2-(x-35)^2$

or $(2x-8)8=16(2x-54)$

or $x-4=2x-54$

or $-x=-50$, $x=50$,

the required number of men

$$=(50)^2-(42)^2=92\times 8=736$$

33 Let x be the number of men in front of the 1st column

The number of men in the depth of the column $=(x+5)$,

the total number of men $=x(x+5)$

Again from the 2nd column we get the number to be equal to $5(x+845)$,

$$x(x+5)=5(x+845)$$

or $$x^2+5x=5x+845\times 5$$

or $$x^2=845\times 5$$

or $$x=65$$

The required number of men $65(65+5)=65\times 70=4550$

Exercise 70.

1 From the 1st equation $x=14-4y$, and substituting this value of x in the 2nd equation we get $7(14-4y)-3y=5$

or $$98-28y-3y=5$$

or $$31y=93, \quad y=3,$$

and $$x=14-4y=14-12=2,$$

$$\left.\begin{matrix}x=2\\y=3\end{matrix}\right\}$$

2 From the 1st equation $x=\frac{9\times 8y}{5}$, and substituting this value of x in the 2nd equation we get $13\left(\frac{9+8y}{5}\right)+7y=79$

or $$117+104y+35y=395$$

or $$139y=278, \quad y=2,$$

and $$x=\frac{9+8y}{5}=\frac{9+16}{5}=5,$$

$$\left.\begin{matrix}x=5\\y=2\end{matrix}\right\}$$

3 From the 1st equation $x=\frac{32-3y}{2}$, and substituting this value of x in the 2nd equation we get $11y-9\left(\frac{32-3y}{2}\right)=3$

or $$22y-288+27y=6$$

or $$49y=294, \quad y=6,$$

and $$x=\frac{32-3y}{2}=\frac{32-18}{2}=7,$$

$$\left.\begin{matrix}x=7\\y=6\end{matrix}\right\}$$

4 From the 1st equation $x=\frac{8+4y}{9}$, and substituting this value of x in the 2nd equation we get $13\left(\frac{8+4y}{9}\right)+7y=101$

or $$104+52y+63y=909$$

or $$115y=805\,, \qquad y=7\,,$$

$$\text{and } x=\frac{8+4y}{9}=\frac{8+28}{9}=4\,,$$

$$\left.\begin{matrix}x=4\\y=7\end{matrix}\right\}$$

5 From the 1st equation $x=b-ay$, and substituting this value of x in the 2nd equation we get

$$a(b-ay)-by=c$$

or $$ab-a^2y-by=c$$

or $$y(a^2+b)=ab-c\,, \qquad y=\frac{ab-c}{a^2+b}\,,$$

$$\text{and } x=b-a\left(\frac{ab-c}{a^2+b}\right)=\frac{a^2b+b^2-a^2b+ac}{a^2+b}=\frac{b^2+ac}{a^2+b}\,,$$

$$x=\frac{b^2+ac}{a^2+b} \text{ and } y=\frac{ab-c}{a^2+b}$$

6 The equations become

$$\left.\begin{matrix}10x-(y-3)=20\\9y+(x-2)=27\end{matrix}\right\} \text{ From the 1st equation}$$

$$x=\frac{20+y-3}{10}=\frac{17+y}{10},$$

and substituting this value of x in the 2nd we get

$$9y+\left(\frac{17+y}{10}-2\right)=27$$

or $$90y+17+y-20=270$$

or $$91y=270+20-17=273\,, \qquad y=3,$$

$$\text{and } x=\frac{17+y}{10}=\frac{20}{10}=2\,,$$

$$\left.\begin{matrix}x=2\\y=3\end{matrix}\right\}$$

7 The equations become

$$\left.\begin{matrix}3(x+y)=2(2x+4)\\2(x-y)=3(x-24)\end{matrix}\right\}$$

or $\left.\begin{array}{r}3x+3y-4x-8=0\\2x-2y-3x+72=0\end{array}\right\}$

or $\left.\begin{array}{r}3y-x-8=0\\x+2y-72=0\end{array}\right\}$ From the 1st equation

$x=3y-8$, and substituting this value of x in the 2nd equation we get $3y-8+2y-72=0$

or $5y=80$, $y=16$,

and $x=3y-8=48-8=40$,

$\left.\begin{array}{r}x=40\\y=16\end{array}\right\}$

8 The equations become

$\left.\begin{array}{r}4(x-y)=3(y-1)\\4x-5y=7(x-7)\end{array}\right\}$

or $\left.\begin{array}{r}4x-4y-3y+3=0\\4x-5y-7x+49=0\end{array}\right\}$

or $\left.\begin{array}{r}4x-7y+3=0\\3x+5y-49=0\end{array}\right\}$ From the 1st equation

$x=\dfrac{7y-3}{4}$, and substituting this value of x in the 2nd equation

we get $3\left(\dfrac{7y-3}{4}\right)+5y-49=0$

or $21y-9+20y-196=0$

or $41y=205$, $y=5$,

and $x=\dfrac{7y-3}{4}=\dfrac{35-3}{4}=8$, $\left.\begin{array}{r}x=8\\y=5\end{array}\right\}$

9 The equations become

$\left.\begin{array}{r}2(3x-2y)-12=(2x-y)\\3(5x-4y)-18=2(4x-3y)\end{array}\right\}$

or $\left.\begin{array}{r}6x-4y-12-2x+y=0\\15x-12y-18-8x+6y=0\end{array}\right\}$

or $\left.\begin{array}{r}4x-3y-12=0\\7x-6y-18=0\end{array}\right\}$ From the 1st equation

$x=\dfrac{3y+12}{4}$, and substituting this value of x in the 2nd equation

we get $7\left(\dfrac{3y+12}{4}\right)-6y-18=0$

or $21xy+84-24y-72=0$

or $-3y=-12$, $y=4$,

$$\text{and } x=\frac{3y+12}{4}=\frac{12+12}{4}=6,$$

$$\left.\begin{array}{l}x=6\\y=4\end{array}\right\}$$

10 The equations become

$$\left.\begin{array}{r}2x+3y+2x=48\\7y-3x-2y=22\end{array}\right\}$$

or
$$\left.\begin{array}{r}4x+3y=48\\-3x+5y=22\end{array}\right\}$$ From the 1st equation

$x=\frac{48-3y}{4}$, and substituting this value of x in the 2nd equation we get $-3\left(\frac{48-3y}{4}\right)+5y=22$

or $$-144+9y+20y=88$$

or $$29y=232, \qquad y=8,$$

$$\text{and } x=\frac{48-3y}{4}=\frac{48-24}{4}=6,$$

$$\left.\begin{array}{l}x=6\\y=8\end{array}\right\}$$

Exercise 71

1 From the 1st equation, we have $5x=9+3y$

$$x=\frac{9+3y}{5} \qquad (1)$$

Again from the 2nd equation, we get $2x=16-5y$,

$$x=\frac{16-5y}{2} \qquad (2)$$

Hence, from (1) and (2) we have

$$\frac{9+3y}{5}=\frac{16-5y}{2}$$

or $$18+6y=80-25y$$

or $$31y=62$$

$$y=2$$

$$\text{Hence, } x=\frac{9+3y}{5}=\frac{9+6}{5}=3$$

$$\left.\begin{array}{l}x=3\\y=2\end{array}\right\}$$

2 From the 1st equation, we have $3y=1+4x$,

$$y=\frac{1+4x}{3} \qquad (1)$$

From the 2nd equation, we have $4y=18-3x$,

$$y=\frac{18-3x}{4} \qquad (2)$$

Hence, from (1) and (2), we have

$$\frac{1+4x}{3}=\frac{18-3x}{4}$$

or $\quad 4+16x=54-9x$

or $\quad 25x=50, \qquad x=2$

Hence, $\quad y=\frac{1+4x}{3}=\frac{1+8}{3}=3$

$$\left.\begin{array}{l}x=2\\y=3\end{array}\right\}$$

3 From the 1st equation, we have $3x=7+7y$,

$$x=\frac{7+7y}{3} \qquad (1)$$

From the 2nd equation, we have $11x=87-5y$,

$$x=\frac{87-5y}{11} \qquad (2)$$

Hence, from (1) and (2), we have

$$\frac{7+7y}{3}=\frac{87-5y}{11}$$

or $\quad 77+77y=261-15y$

or $\quad 92y=184, \qquad y=2$

Hence, $\quad x=\frac{7+7y}{3}=\frac{7+14}{3}=7$

$$\left.\begin{array}{l}x=7\\y=2\end{array}\right\}$$

4 From the 1st equation, we have $3y+xy=7x+xy$,

$$y=\frac{7x}{3} \qquad (1)$$

From the 2nd equation, we have $5y=4x+9+14$,

$$y=\frac{4x+23}{5} \qquad (2)$$

Hence, from (1) and (2), we have

$$\frac{7x}{3}=\frac{4x+23}{5}$$

or $$35x=12x+69$$

or $$23x=69$$

$$x=3$$

Hence, $$y=\frac{7x}{3}=\frac{21}{3}=7$$

$$\left.\begin{array}{l}x=3\\y=7\end{array}\right\}$$

5 From the 1st equation, we have $32x=28+25y$,

$$x=\frac{28+25y}{32} \qquad (1)$$

From the 2nd equation, we have $14x=116-15y$,

$$x=\frac{116-15y}{14} \qquad (2)$$

Hence, from (1) and (2), we have

$$\frac{28+25y}{32}=\frac{116-15y}{14}$$

or $$196+175y=1856-240y$$

or $$415y=1660,$$

$$y=4$$

Hence, $$x=\frac{28+25y}{32}=\frac{28+100}{32}=4.$$

$$\left.\begin{array}{l}x=4\\y=4\end{array}\right\}$$

6 From the 1st equation, we have

$15x+5y=14x+7y+7$ [Multiplying both sides by 35]

or $$x=2y+7 \qquad (1)$$

From the 2nd equation, we have

$$40-(x-y)=30$$

or $$x-y=10$$

or $$x=y+10 \qquad (2)$$

Hence, from (1) and (2), we have

$$2y+7=y+10$$

$$y=3,$$

from (1) we get $x=6+7=13$

$$\left.\begin{array}{l}x=13\\y=3\end{array}\right\}$$

7 Multiplying both sides of the 1st equation by 12,

$$55x-66y+39x=52y-26$$

or $$94x=118y-26$$

or $$47x=59y-13,$$

$$x=\frac{59y-13}{47} \qquad (1)$$

Multiplying both sides of the 2nd equation by 12,

$$10x+12y-9x+6y=24y-24$$

or $$x+18y=24y-24,$$

$$x=6y-24 \qquad (2)$$

Hence, from (1) and (2) we have

$$\frac{59y-13}{47}=6y-24$$

or $$59y-13=282y-1128$$

or $$223y=1115,$$

$$y=5.$$

from (2) $x=30-24=6$

$$\left.\begin{array}{l}x=6\\y=5\end{array}\right\}$$

8 Multiplying both sides of the first equation by 20,

$$40x-5y-15=140+12y-8x$$

or $$48x=17y+155$$

$$x=\frac{17y+155}{48} \qquad (1)$$

Multiplying both sides of the second equation by 6,

$$24y+2x-4=159-6y-3$$

or $$2x=160-30y$$

$$x=80-15y \qquad (2)$$

Hence, from (1) and (2), we get

$$\frac{17y+155}{48}=80-15y$$

or $$17y+155=3840-720y$$

or $\qquad 737y = 3685, \qquad y = 5,$

from (2) $x = 80 - 75 = 5,$

$$\left.\begin{array}{l} x = 5 \\ y = 5 \end{array}\right\}$$

9 Multiplying both sides of the first equation by 24,

$$48x - 16y + 8 = 77 + 18x - 12y$$

or $\qquad 30x = 4y + 69$

$$x = \frac{4y+69}{30} \qquad (1)$$

Multiplying both sides of the second equation by 20,

$$80y - 25 + 10x = 120 - 12 + 8y$$

or $\qquad 10x = 133 - 72y$

$$x = \frac{133-72y}{10} \qquad (2)$$

Hence, from (1) and (2), we get

$$\frac{4y+69}{30} = \frac{133-72y}{10}$$

or $\qquad 4y + 69 = 3(133 - 72y)$

$$= 399 - 216y$$

or $\qquad 220y = 330, \qquad y = \frac{3}{2} = 1\frac{1}{2},$

from (1) $\qquad x = \frac{6+69}{30} = \frac{75}{30} = 2\frac{1}{2},$

$$\left.\begin{array}{l} x = 2\frac{1}{2} \\ y = 1\frac{1}{2} \end{array}\right\}$$

10 From the first equation, we have

$$12x(23 - x) - 2(2y - x) = 40(23 - x) - (59 - 12x)(23 - x)$$

or $\quad 276x - 12x^2 - 4y + 2x = 920 - 40x - (1357 - 335x + 12x^2)$

or $\qquad 278x - 4y = 295x - 437$

or $\qquad 4y = -17x + 437$

$$y = \frac{-17x + 437}{4} \qquad (1)$$

From the second equation, we have

$$9y(x - 18) + 3(y - 3) = 60(x - 18) - (x - 18)(43 - 9y)$$

or $\quad 9xy - 162y + 3y - 9 = 60x - 1080 - (43x - 9xy - 774 + 162y)$

or $\qquad 3y - 9 = 60x - 1080 - 43x + 774$

or $\qquad 3y = 17x - 297,$

$$y=\frac{17x-297}{3} \qquad (2)$$

From (1) and (2), we get

$$\frac{-17x+437}{4}=\frac{17x-297}{3}$$

or $$-51x+1311=68x-1188$$

or $$119x=2499$$

or $$17x=357\ , \qquad x=21\ ,$$

$$\text{from (2) } y=\frac{357-297}{3}=\frac{60}{3}=20\ ,$$

$$\left.\begin{aligned} x &= 21 \\ y &= 20 \end{aligned}\right\}$$

Exercise 72

1 Multiplying the 1st equation by 3, and the 2nd by 7, we have

$$21x-15y=33$$

$$\text{and } 21x+14y=91$$

Hence, by subtraction,

$$-29y=-58,$$

$$y=2$$

Substituting this value of y in the 1st equation, we have

$$7x=11+10=21\ , \qquad x=3$$

Thus we have $$\left.\begin{aligned} x &= 8 \\ y &= 4 \end{aligned}\right\}$$

2 Multiplying the 1st equation by 11, and the 2nd by 6, we have

$$143x+66y=638$$

$$\text{and } 30x-66y=54$$

Hence, by addition, $173x=692$, $\qquad x=4$

Substituting this value of x in the 2nd equation, we have

$$11y=20-9=11$$

$$y=1$$

Thus we have $$\left.\begin{aligned} x &= 4 \\ y &= 1 \end{aligned}\right\}$$

3 Multiplying the first equation by 10, and the second by 9, we have

$$80x - 90y = 200$$

and $$63x - 90y = 81$$

Hence, by subtraction, $17x = 119$, $x = 7$

Substituting this value of x in the second equation, we have

$$10y = 49 - 9 = 40, \qquad y = 4$$

Thus we have $x = 7$, $y = 4$

4 Keeping the first equation as it is, and multiplying the second by 2, we have

$$25x - 14y = 8$$

and $$24x + 14y = 90$$

Hence, by addition, $49x = 98$, $x = 2$

Substituting this value of x in the second equation, we have

$$7y = 45 - 24 = 21, \qquad y = 3$$

Thus we have $x = 2$, $y = 3$

5 Multiplying the 1st equation by 2, and the 2nd by 3,

we have $24x + 22y = 140$,

and $24x - 21y = 54$

Hence, by subtraction, $43y = 86$, $y = 2$

Substituting this value of y in the 2nd equation,

we have $8x = 18 + 14 = 32$, $x = 4$

Thus we have $x = 4$, $y = 2$

6 Multiplying the 1st equation by 3, and the 2nd by 2,

we have $39x - 42y = 66$

and $34x - 42y = 36$

Hence, by subtraction, $5x = 30$, $x = 6$

Substituting this value of x in the 1st equation,

we have $14y = 78 - 22 = 56$, $y = 4$

Thus we have $x = 6$, $y = 4$

7 Multiplying the first equation by 3, and second by 4, we have

$$84x - 45y = 123$$

and $$84x + 52y = 220$$

Hence, by subtraction, $-97y=-97$, $y=1$

Substituting this value of y in the second equation, we have

$$21x=55-13=42, \qquad x=2$$

Thus we have $x=2$, $y=1$

8 Multiplying the first equation by 3, and the second by 2, we have

$$57x+72y=102$$

and $$46x+72y=124$$

Hence, by subtraction, $11x=-22$, $x=-2$

Substituting this value of x in the first equation, we have

$$24y=34-(-38)=72, \qquad y=3$$

Thus we have $x=-2$, $y=3$

9 Multiplying the 1st equation by 3, and the 2nd by 2, we have

$$141x-168y=369$$

and $$50x+168y=586$$

Hence, by addition, $191x=955$,

$$x=5$$

Substituting this value of x in the 1st equation, we have

$$56y=235-123=112, \qquad y=2$$

Thus we have $x=5$, $y=2$

10 Multiplying the 1st equation by 4, and the 2nd by 3, we have

$$204x6-4y=12$$

and $$204x+69y=411$$

Hence, by subtraction,

$$-133y=-399, \qquad y=3$$

Substituting this value of y in the 1st equation, we have

$$51x=3+48=51, \qquad x=1$$

Thus we have $x=1$, $y=3$

11 Multiplying the 1st equation by 3, and the 2nd by 4, we have

$$156x-27y=102$$

and $$156x+56y=268$$

Hence, by subtraction, $-83y=-166$,

$$y=2$$

Substituting this value of y in the 1st equation, we have

$$52x=34+18=52, \qquad x=1$$

Thus we have $x=1$, $y=2$

12 Multiplying the 1st equation by 2, and the 2nd by 5, we have

$$24x+170y=-98$$

and $$95x-170y=455$$

Hence, by addition $119x=357$, $x=3$

Substituting this value of x in the 2nd equation, we have

$$34y=57-91=-34, \qquad y=-1$$

Thus we have $x=3$, $y=-1$

13 Multiplying the 1st equation by 7, and the 2nd by 5, we have

$$455x-98y=63$$

and $$455x-75y=155$$

Hence, by subtraction, $-23y=-92$, $y=4$

Substituting this value of y in the 2nd equation, we have

$$91x=31+60=91, \qquad x=1$$

Thus $x=1$, $y=4$

14 Multiplying the 1st equation by 3, and the 2nd equation by 2, we have

$$45x+138y=51$$

and $$26x+138y=146$$

Hence, by subtraction, $19x=-95$, $x=-5$

Substituting this value of x in the 1st equation, we have

$$46y=17-(-75)=92, \qquad y=2$$

Thus we have $x=-5$, $y=2$

15 Multiplying the 1st equation by 5, and the 2nd by 3, we have

$$70x+405y=265$$

and $$51x+405y=303$$

Hence, by subtraction, $19x=-38$, $x=-2$

Substituting this value of x in the 1st equation, we have

$$81y=53-(-28)=81, \qquad y=1$$

Thus we have $x=-2$, $y=1$

16 Multiplying the 1st equation by 5, and the 2nd by 11, we have

$$25x+55y=730$$

and $$121x+55y=1210$$

Hence, by subtraction, $-96x=-480$, $\quad x=5$

Substituting this value of x in the 1st equation, we have

$$11y=146-25=121, \qquad y=11$$

Thus we have $x=5$, $\quad y=11$

17 Multiplying the 1st equation by b, and keeping the 2nd as it is, we have

$$abx+b^2y=bc$$

and $$a^2x+b^2y=c^2$$

Hence, by subtraction, $x(ab-a^2)=bc-c^2$,

$$x=\frac{bc-c^2}{ab-a^2}$$

Substituting this value of x in the 1st equation, we have

$$\frac{a(bc-c^2)}{ab-a^2}+by=c$$

or $$by=c-\frac{a(bc-c^2)}{ab-a^2}$$

$$=\frac{abc-a^2c-abc+ac^2}{ab-a^2}$$

$$=\frac{c^2-ac}{b-a},$$

$$y=\frac{ac-c^2}{ab-b^2}$$

Thus we have $\quad x=\dfrac{bc-c^2}{ab-a^2}$, $\; y=\dfrac{ac-c^2}{ab-b^2}$

18 Multiplying the 1st equation by 4, and the second by 126, we have

$$2x+2y+3x-5y=8$$

or $$5x-3y=8 \qquad (1)$$

and $$9x+7y=126 \qquad (2)$$

Now by multiplying (1) by 7, and (2) by 3, we have

$$35x-21y=56$$

and $$27x+21y=378$$

Hence, by addition, we have $62x = 434$,

$$x = 7$$

Substituting this value of x in the 2nd equation,

we have $\frac{y}{18} = 1 - \frac{1}{2} = \frac{1}{2}$,

$$y = 9$$

Thus we have $\quad x = 7, \; y = 9$

19 From the 1st equation, we have

$$4x + 5y = 40x - 40y$$

or $\quad -36x + 45y = 0$

or $\quad 4x - 5y = 0. \qquad (1)$

From the 2nd equation, we have

$$\frac{2x - y + 6y}{3} = \frac{1}{2}$$

or $\quad 4x + 10y = 3 \qquad (2)$

Hence, from (1) and (2), we have

$$15y = 3,$$

$$y = \tfrac{1}{5}$$

Substituting this value of y in (1), we have

$4x = 1, \qquad x = \frac{1}{4}$

Thus we have $x = \frac{1}{4}, \qquad y = \frac{1}{5}$

20 From the 1st equation, we have

$$\frac{4x - 3y - 7}{5} = \frac{9x - 4y - 25}{30}$$

or $\quad 4x - 3y - 7 = \dfrac{9x - 4y - 25}{6}$

or $\quad 24x - 18y - 42 = 9x - 4y - 25$

or $\quad 15x - 14y = 17 \qquad (1)$

From the 2nd equation, we have

$$\frac{20y - 20 + 30x - 9y}{60} = \frac{2y - 2x + 5x + 33}{30}$$

or $\quad \dfrac{11y + 30x - 20}{2} = 2y + 3x + 33$

or $\quad 11y + 30x - 20 = 4y + 6x + 66$

or $\quad 24x + 7y = 86 \qquad (2)$

Multiplying (2) by 2, we have

$$\left.\begin{aligned} 48x+14y&=172 \\ \text{also,}\qquad 15x-14y&=\ 17 \end{aligned}\right\}$$

Hence, by addition, $63x=189$,

$$x=3$$

Substituting this value of x in (1), we have

$$14y=45-17=28,$$

$$y=2$$

Thus we have $\quad x=3, \quad y=2$

21 From the 1st equation, we have

$$\frac{(2x-3)(6x+9)+2(3x+5y)}{4(2x-3)}=\frac{13+2(3x+4)}{4}$$

or $\quad (2x-3)(6x+9)+2(3x+5y)=13(2x-3)$
$$+(2x-3)(6x+8)$$

or $\quad 12x^2-27+6x+10y=26x-39+12x^2-2x-24$

or $\quad 18x-10y=36 \quad . \quad . \ (1)$

From the 2nd equation, we have

$$\frac{(8y+7)(2y-8)+10(6x-3y)}{10(2y-8)}=\frac{20+4y-9}{5}$$

or $\quad 16y^2-50y-56+60x-30y=(4y-16)(4y+11)$
$$=16y^2-20y-176$$

or $\quad 60x-60y=-120 \quad$ or $\quad 10x-10y=-20 \qquad (2)$

Hence, from (1) and (2), we have

$8x=65, \qquad x=7$

Substituting this value of x in (1), we have

$10y=126-36=90, \qquad y=9$

Thus we have $\quad x=7, \qquad y=9$

22 From the 1st equation, we have

$$\frac{12x-20y-2x+8y+33}{12}=\frac{6y+4x+3}{12}$$

or $\quad 12x-20y-2x+8y+33=6y+4x+3$

or $\quad 6x-18y=-30$

or $\quad x-3y=-5 \qquad (1)$

From the 2nd equation, we have

$$\frac{x}{2}+\frac{7y}{8}+\frac{14}{3}=\frac{40x}{3}-\frac{5y}{12}-80$$

or $$\frac{12x+21y+112}{24}=\frac{160x-5y-960}{12}$$

or $$12x+21y+112=320x-10y-1920$$

or $$308x-31y=2032 \qquad (2)$$

Multiplying (1) by 31 and (2) by 3, we have

$$31x-93y=-155, \quad \text{and} \quad 924x-93y=6096$$

Hence, by subtraction, $893x=6251$,

$$x=7$$

Substituting this value of x in (1), we have

$$3y=7+5=12, \qquad y=4$$

Thus we have $x=7, \quad y=4$

23 From the 1st equation, we have

$$\frac{6x+08y-18x-025}{25}=\frac{4x+5\ 2+01y}{5}$$

or $$42x+08y-025=2x+2\ 6+005y$$

or $$22x+075y=2\ 625$$

or $$88x+3y=10\ 3$$

or $$88x+30y=1030 \qquad (1)$$

From the 2nd equation, we have

$$\frac{2y+5}{15}=\frac{7x-1}{6}$$

or $$12y+30=10\ 5x-15$$

or $$24y+60=21x-30$$

or $$21x-24y=90 \qquad (2)$$

Multiplying (1) by 4, and (2) by 5, we have

$$352x+120y=4120$$

and $$105x-120y=450$$

Hence, by addition, $457x=4570$,

$$x=10$$

Substituting this value of x in (2) we have

$$24y=210-90=120, \qquad y=5$$

Thus we have $x=10, \quad y=5$

24 Multiplying the 1st equation by 3, and the 2nd by 4, we have

$$\frac{12}{x}+\frac{30}{y}=6$$

and $$\frac{12}{x}+\frac{8}{y}=\frac{19}{5}$$

Hence, by subtraction, $$\frac{22}{y}=\frac{11}{5}$$

or $$11y=110, \qquad y=10$$

Substituting this value of y in the 1st equation, we have

$$\frac{4}{x}=2-\frac{10}{10}=1,$$

$$x=4$$

Thus we have $x=4$, $y=10$

25 Multiplying the 1st equation by 5, and the 2nd by 2, we have

$$\frac{10}{x}+\frac{15}{y}=10$$

and $$\frac{10}{x}+\frac{20}{y}=11\frac{2}{3}$$

Hence, by subtraction, $$\frac{5}{y}=\frac{5}{3}, \qquad y=3$$

Substituting this value of y in the 1st equation, we have

$$\frac{2}{x}=2-\frac{3}{3}=1,$$

$$x=2$$

Thus we have $x=2$, $y=3$

26 Multiplying the 1st equation by a, and the 2nd equation by b, we have

$$\frac{a^2}{x}+\frac{ab}{y}=am$$

and $$\frac{b^2}{x}+\frac{ab}{y}=bn$$

Hence, by subtraction, $\frac{1}{x}(a^2-b^2)=am-bn,$

$$x=\frac{a^2-b^2}{am-bn}$$

Substituting this value of x in the 1st equation, we have

$$\frac{b}{y}=m-\frac{a(an-bm)}{a^2-b^2}$$

$$=\frac{a^2 n-b^2 n-a^2 m-abm}{a^2-b^2}$$

$$=\frac{b(an-bm)}{a^2-b^2}$$

Or $$\frac{1}{y}=\frac{an-bm}{a^2-b^2}$$

$$=\frac{a^2-b^2}{an-bm}$$

Thus we have $x=\dfrac{a^2-b^2}{an-bm}$ $\quad y=\dfrac{a^2-b^2}{an-bm}$

27 Multiplying the 1st equation by 3, and the 2nd by 5, we have

$$\frac{1}{x}+\frac{3}{5y}=3$$

and $$\frac{1}{x}+\frac{5}{3y}=\frac{17}{3}$$

Hence, by subtraction,

$$\frac{1}{y}\left(\frac{3}{5}-\frac{5}{3}\right)=3-\frac{17}{3}$$

or $$\frac{1}{y}\left(\frac{-16}{15}\right)=\frac{9-17}{3}=-\frac{8}{3}$$

or $$\frac{1}{y}\cdot\frac{2}{5}=1, \quad \therefore \quad y=\frac{2}{5}$$

Substituting this value of y in the 1st equation, we have

$$\frac{1}{3x}=1-\frac{1}{2}=\frac{1}{2}$$

or $$3x=2, \qquad x=\frac{2}{3}.$$

Thus we have $x=\dfrac{2}{3}$ $\quad y=\dfrac{2}{5}$

28 Multiplying the 1st equation by $\frac{2}{5}$ we have

$$\frac{6}{5y}-\frac{2}{5x}=\frac{2}{5},$$

also $$\frac{5}{2y}-\frac{2}{5x}=7 \qquad \text{(2nd equation)}$$

Hence, by addition,

$$\frac{1}{y}\left(\frac{6}{5}+\frac{5}{2}\right)=7\frac{2}{5}=\frac{37}{5}$$

or $$\frac{1}{y}\left(\frac{37}{10}\right)=\frac{37}{5}$$

or $$\frac{1}{2y}=1, \qquad y=\frac{1}{2}$$

Substituting this value of y in the 1st equation, we have

$$\frac{1}{x}=\frac{3}{\frac{1}{2}}-1=5, \qquad x=\frac{1}{5}$$

Thus we have $x=\frac{1}{5}$, $y=\frac{1}{2}$

29 Multiplying the 1st equation by $\frac{3}{4}$, we have

$$\frac{3x}{16}+\frac{3}{2y}=\frac{3}{2},$$

also, $$\frac{2x}{5}+\frac{3}{2y}=\frac{47}{20}$$

Hence, by subtraction,

$$x\left(\frac{3}{16}-\frac{2}{5}\right)=\left(\frac{3}{2}-\frac{47}{20}\right)$$

or $$x\left(\frac{15-32}{80}\right)=\frac{60-94}{40}$$

or $$\frac{17x}{2}=34, \qquad x=4$$

Substituting this value of x in the 1st equation, we have

$$\frac{2}{y}=2-\frac{4}{4}=1, \qquad y=2$$

Thus we have $x=4$, $y=2$

30 Multiplying the 1st equation by $\frac{1}{3}$, and the 2nd by $\frac{1}{5}$, we have

$$\frac{1}{15x}+\frac{y}{27}=\frac{5}{3}$$

and $$\frac{1}{15x}+\frac{y}{10}=\frac{14}{5}$$

Hence, by subtraction

$$y\left(\frac{1}{27}-\frac{1}{10}\right)=\frac{5}{3}-\frac{14}{5}$$

or $$y\left(\frac{10-27}{270}\right)=\frac{25-42}{15}$$

or $$y\frac{17}{270}=\frac{17}{15}$$

or $$15y=270, \qquad y=18$$

Substituting this value of y in the 1st eqation, we have

$$\frac{1}{5x}=5-\frac{18}{9}=3, \qquad x=\tfrac{1}{15}$$

Thus we have $x=\frac{1}{15}$, $y=18$

Exercise 73

1 $$\left.\begin{array}{l}2x+3y-8=0\\3x-4y+5=0\end{array}\right\}$$

Hence, $$\frac{x}{3\times5-(-8)(-4)}=\frac{y}{(-8)3-2\times5}=\frac{1}{2(-4)-3\times3}$$

or $$\frac{x}{15-32}=\frac{y}{-24-10}=\frac{1}{-8-9}$$

or $$\frac{x}{-17}=\frac{y}{-34}=\frac{1}{-17}$$

$$x=\frac{-17}{-17}=1, \text{ and } y=\frac{-34}{-17}=2$$

Thus we have $x=1$, and $y=2$

2 $$\left.\begin{array}{l}3x-5y+\ 9=0\\5x+2y-16=0\end{array}\right\}$$

Hence $$\frac{x}{(-5)(-16)-9\times2}=\frac{y}{9\times5-3(-16)}$$

$$=\frac{1}{3\times2-5(-5)}$$

or $$\frac{x}{80-18}=\frac{y}{45+48}=\frac{1}{6+25}$$

or $$\frac{x}{62}=\frac{y}{93}=\frac{1}{31}, \qquad x=\frac{62}{31}=2, \text{ and}$$

$y=\frac{93}{31}=3$ Thus we have $x=2$, and $y=3$

3 $$\left.\begin{array}{l}4x-5y+8=0\\2x-3y+6=0\end{array}\right\}$$

Hence, $$\frac{x}{(-5)6-8(-3)}=\frac{y}{8\times2-4\times6}$$
$$=\frac{1}{4(-3)-(-5)2}$$

or $$\frac{x}{-30+24}=\frac{y}{16-24}=\frac{1}{-12+10}$$

or $$\frac{x}{-6}=\frac{y}{-8}=\frac{1}{-2}$$

Hence, $x=3$, and $y=4$

4 $$\left.\begin{array}{l}-3x+2y+2=0\\5x-3y-5=0\end{array}\right\}$$

Hence, $$\frac{x}{2(-5)-2(-3)}=\frac{y}{2\times5-(-3)(-5)}$$
$$=\frac{1}{(-3)(-3)-2\times5}$$

or $$\frac{x}{-10+6}=\frac{y}{10-15}=\frac{1}{9-10}$$

or $$\frac{x}{-4}=\frac{y}{-5}=\frac{1}{-1}$$

Hence, $x=4$, and $y=5$

5 $$\left.\begin{array}{l}6x-7y+12=0\\-7x+4y+11=0\end{array}\right\}$$

Hence, $$\frac{x}{(-7)11-12\times4}=\frac{y}{12(-7)-6\times11}=\frac{1}{6\times4-(-7)(-7)}$$

or $$\frac{x}{-77-48}=\frac{y}{-84-66}=\frac{1}{24-49}$$

or $$\frac{x}{-125}=\frac{y}{-150}=\frac{1}{-25}$$

Hence, $x=5$, and $y=6$

6 From the 1st equation, $7x-8y+14=0$
and from the 2nd equation, $5x-3y-9=0$

Hence, $$\frac{x}{(-8)(-9)-14(-3)}=\frac{y}{14\times5-7(-9)}$$
$$=\frac{1}{7(-3)-5(-8)}$$

or $$\frac{x}{72+42}=\frac{y}{70+63}=\frac{1}{-21+40}$$

or $$\frac{x}{114}=\frac{y}{133}=\frac{1}{19}$$

Hence. $x=6$ and $y=7$

7 $$\left.\begin{aligned}-6x+5y+2=0\\13x-9y-19=0\end{aligned}\right\}$$ (from the 2nd equation)

Hence, $$\frac{x}{5(-19)-2(-9)}=\frac{y}{2\times 13-(-6)(-19)}=\frac{1}{(-6)(-9)-5\times 13}$$

or $$\frac{x}{-95+18}=\frac{y}{26-114}=\frac{1}{54-65}$$

or $$\frac{x}{-77}=\frac{y}{-88}=\frac{1}{-11}$$

Hence, $x=7$, and $y=8$

8 $-7x+5y-11=0$
$8x-5y-19=0$. (from the 2nd equation).

Hence, $$\frac{x}{5(-19)-11(-5)}=\frac{y}{11\times 8-(-7)(-19)}=\frac{1}{(-7)(-5)-8\times 5}$$

or $$\frac{x}{-95+55}=\frac{y}{88-133}=\frac{1}{35-40}$$

or $$\frac{x}{-40}=\frac{y}{-45}=\frac{1}{-5}$$

Hence. $x=8$, and $y=9$

9 $4x-11y+6=0$
$9x-13y-10=0$. (from the 2nd equation)

Hence $$\frac{x}{(-11)(-10)-6(-13)}=\frac{y}{6\times 9-4(-10)}=\frac{1}{4(-13)-(-11)9}$$

or $$\frac{x}{110+78}=\frac{y}{54+40}=\frac{1}{-52+99}$$

or $$\frac{x}{188}=\frac{y}{94}=\frac{1}{47}$$

Hence, $x=4$, and $y=2$

10 From the 1st equation, and from the 2nd ,,
$$\left.\begin{array}{r}8x-7y-19=0\\10x-9y-23=0\end{array}\right\}$$

Hence, $$\frac{x}{(-7)(-23)-(-19)(-9)}=\frac{y}{(-19)\,10-8\,(-23)}$$
$$=\frac{1}{8\,(-9)-(-7)\,10}$$

or $$\frac{x}{161-171}=\frac{y}{-190+184}=\frac{1}{-72+70}$$

or $$\frac{x}{-10}=\frac{y}{-6}=\frac{1}{-2}$$

Hence, $x=5$, and $y=3$

11. $$-12x+17y+16=0$$
$$9x-13y-11=0\quad\text{(from the 2nd equation)}$$

Hence, $$\frac{x}{17\,(-11)-16\,(-13)}=\frac{y}{16\times 9-(-12)\,(-11)}$$
$$=\frac{1}{(-12)(-13)-17\times 9}$$

or $$\frac{x}{-187+208}=\frac{y}{144-132}=\frac{1}{156-153}$$

or $$\frac{x}{21}=\frac{y}{12}=\frac{1}{3}$$

Hence $x=7$, and $y=4$

12 $$\left.\begin{array}{r}14x-11y+18=0\\11x-7y+1=0\end{array}\right\}$$

Hence, $$\frac{x}{(-11)\,1-18\,(-7)}=\frac{y}{18\times 11-14\times 1}$$
$$=\frac{1}{14\,(-7)-(-11)\,11}$$

or $$\frac{x}{-11+126}=\frac{y}{198-14}=\frac{1}{-98+121}$$

or $$\frac{x}{115}=\frac{y}{184}=\frac{1}{23}$$

Hence $x=5$, and $y=8$

13 From the 2nd equation, we have
$$\frac{x}{2}=\frac{y}{3}=k\ \text{(suppose)},$$
$$x=2k,\ \text{and}\ y=3k \qquad \text{(A)}$$

Substituting these values of x and y in the 1st equation, we have $\quad 34k-21k=52$

or $\quad 13k=52, \qquad k=4$

Hence, from A, $x=8$, and $y=12$,

14 From the 2nd equation, we have

$$\frac{x}{3}=\frac{y}{7}=k \text{ (suppose)},$$

$$x=3k, \text{ and } y=7k \qquad \text{(A)}$$

Substituting these values of x and y in the 1st equation, we have $\quad 27k+35k=124$

or $\quad 62k=124, \qquad k=2$

Hence, from A, $x=6$, and $y=14$

15 From the 2nd equation, we have

$$\frac{x}{4}=\frac{y}{9}=k \text{ (suppose)},$$

$$x=4k, \text{ and } y=9k \qquad \text{(A)}$$

Substituting these values of x, y in the 1st equation, we have

$$60k+63k=246$$

or $\quad 123k=246, \qquad k=2$

Hence, from A, $x=8$, and $y=18$

16 From the 1st equation, we have

$$\frac{x}{8}=\frac{y}{9}=k \text{ (suppose)},$$

$$x=8k, \text{ and } y=9k \qquad \text{(A)}$$

Substituting these values of x, y in the 2nd equation, we have

$$80k+207k=287, \qquad k=1$$

Hence, from A, $x=8$, and $y=9$

17 From the first equation, we have $\quad 4x=3y$

or $\quad \dfrac{x}{3}=\dfrac{y}{4}=k \quad$ (suppose),

$$x=3k, \text{ and } y=4k \qquad (A)$$

Substituting the values of x, y in the 2nd equation, we have

$$21k-44k+92=0$$

or $\quad -23k=-92, \qquad k=4$

Hence, from A, $x=12$, and $y=16$

Substituting these values of x, y in the 1st equation, we have

$$\tfrac{1}{3}(33k+5k)+\tfrac{1}{4}(33k-5k)=59$$

or $$11k+1\tfrac{2}{3}k+7k=59$$

or $$\tfrac{59}{3}k=59$$

$$k=3$$

Hence, from A, $x=90$ and $y=15$

22 From the 1st equation, we have

$$4x+5y=40x-40y$$

or $$36x=45y$$

or $$4x=5y$$

or $$\frac{x}{5}=\frac{y}{4}=k \quad \text{(suppose)},$$

$$x=5k, \text{ and } y=4k \qquad \cdot \qquad (A)$$

Substituting these values of x, y in the 2nd equation, we have

$$\frac{10k-4k}{3}+8k=20,$$

or $$10k=20, \qquad k=2$$

Hence, from A, $x=10$, and $y=8$

23 From the 1st equation we have

$$3y+xy=7x+xy$$

or $$3y=7x$$

or $$\frac{y}{7}=\frac{x}{3}=k \quad \text{(suppose)} \qquad (A)$$

$$x=3k, \text{ and } y=7k$$

Substituting these values of x, y in the 2nd equation, we have

$$12k+9=35k-14$$

or $$23k=23, \qquad k=1$$

Hence from A, $x=3$, and $y=7$

24 From the 1st equation we have

$$4y-6=2x+2y$$

or $$2x-2y+6=0$$

or $$x-y+3=0 \qquad (1)$$

From the 2nd equation, we have

$$8x-5=9y-9x$$

or $$17x-9y-5=0 \qquad (2)$$

Hence, from (1) and (2),

$$\frac{x}{(-1)(-5)-3(-9)}=\frac{y}{3\times 17-1(-5)}$$
$$=\frac{1}{1(-9)-(-1)17}$$

or $$\frac{x}{5+27}=\frac{y}{51+5}=\frac{1}{-9+17}$$

or $$\frac{x}{32}=\frac{y}{56}=\frac{1}{8}$$

Hence, $x=4$, and $y=7$

25 From the 1st equation, we have

$$xy+5y+7x+35=xy+y-9x-9+112$$

or $16x+4y-68=0$ (1)

From the 2nd equation, we have

$$2x-3y+9=0 \quad (2)$$

Hence, from (1) and (2),

$$\frac{x}{4\times 9-(-68)(-3)}=\frac{y}{(-68)2-16\times 9}$$
$$=\frac{1}{16(-3)-4\times 2}$$

or $$\frac{x}{36-204}=\frac{y}{-136-144}=\frac{1}{-48-8}$$

or $$\frac{x}{-168}=\frac{y}{-280}=\frac{1}{-56}$$

Hence, $x=3$, and $y=5$

26 From the 1st and the 2nd equations, we have

$$\frac{x}{(-5)4-(-7)2}=\frac{y}{2\times 2-4\times 4}$$
$$=\frac{z}{4(-7)-2(-5)}$$

or $$\frac{x}{-20+14}=\frac{y}{4-16}=\frac{z}{-28+10}$$

or $$\frac{x}{-6}=\frac{y}{-12}=\frac{z}{-18}$$

or $$x=\frac{y}{2}=\frac{z}{3}=k \quad \text{(suppose)}$$

$$x=k,\ y=2k,\ z=3k \qquad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$\lambda+2\lambda+3\lambda=6$$

or $\quad 6\lambda=6, \qquad \lambda=1$

Hence, from A, $x=1$, $y=2$ and $z=3$

27 From the 1st and the 2nd equations, we have

$$\frac{x}{6\times 6-4\times 8}=\frac{y}{8\times 3-6\times 5}=\frac{z}{5\times 4-3\times 6}$$

or $$\frac{x}{36-32}=\frac{y}{24-30}=\frac{z}{20-18}$$

or $$\frac{x}{4}=\frac{y}{-6}=\frac{z}{2}$$

or $$\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}=\lambda \quad \text{(suppose)}$$

$$x=2\lambda,\ y=-3\lambda,\ z=\lambda \qquad . \qquad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$2\lambda-15\lambda+16\lambda=3$$

or $\quad 3\lambda=3, \qquad \lambda=1$

Hence, from A, $x=2$, $y=-3$, $z=1$

28 From the 1st two equations, we have

$$\frac{x}{(-7)7-(-8)11}=\frac{y}{11\times 6-2\times 7}=\frac{z}{2(-8)-6(-7)}$$

or $$\frac{x}{-49+88}=\frac{y}{66-14}=\frac{z}{-16+42}$$

or $$\frac{x}{39}=\frac{y}{52}=\frac{z}{26}$$

or $$\frac{x}{3}=\frac{y}{4}=\frac{z}{2}=\lambda \quad \text{(suppose)}$$

$$x=3\lambda,\quad y=4\lambda \text{ and } z=2\lambda \qquad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$9\lambda+16\lambda+10\lambda=35, \quad$ or, $\quad 35\lambda=35, \qquad \lambda=1$

Hence, from A, $x=3$, $y=4$, $z=2$

29 From the 1st. two equations, we have

$$\frac{x}{3\times 8-(-7)(-8)}=\frac{y}{(-8)5-8\times 7}=\frac{z}{7(-7)-5\times 3}$$

or $$\frac{x}{24-56}=\frac{y}{-40-56}=\frac{z}{-49-15}$$

or $$\frac{x}{-32}=\frac{y}{-96}=\frac{z}{-64}$$

or $$x=\frac{y}{3}=\frac{z}{2}=k \quad \text{(suppose)}$$

$$\therefore \quad x=k, \; y=3k, \; z=2k \qquad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$3k+15k-14k=64, \qquad \text{or,} \qquad 32k=64$$
$$k=2$$

Hence from A, $x=2$, $y=6$, $z=4$

30 From the 1st. two equations, we have

$$\frac{x}{(-2)3-(-8)1}=\frac{y}{1\times 9-3\times 1}=\frac{z}{1(-8)-9(-2)}$$

or $$\frac{x}{-6-8}=\frac{y}{9-3}=\frac{z}{-8-18}$$

or $$\frac{x}{2}=\frac{y}{6}=\frac{z}{10}$$

or $$x=\frac{y}{3}=\frac{z}{5}=k \quad \text{(suppose)}$$

$$x=k, \quad y=3k, \quad z=5k \qquad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$2k-9k+25k=36 \qquad \text{or,} \qquad 36k=36 \qquad k=1.$$

Hence, from A, $x=1$ $y=3$ $z=5$

31. From the 1st. equation, we have

$$8x-18y=14y-7z, \qquad \text{or,} \qquad 8x+4y-7z=0 \quad . \qquad (1)$$

From the 2nd equation, we have

$$7x-14y=8y-8z, \qquad \text{or} \qquad 7x-6y-8z=0 \qquad (2)$$

Hence, from (1) and (2)

$$\frac{x}{4(-8)-6(-7)}=\frac{y}{(-7)7-(-8)8}=\frac{z}{8\times 6-7\times 4}$$

or $\dfrac{x}{-32+42}=\dfrac{y}{-49+64}=\dfrac{z}{48-28}$

or $\dfrac{x}{10}=\dfrac{y}{15}=\dfrac{z}{20}$

or $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{4}=k$ (suppose)

$x=2k, \quad y=3k, \quad z=4k \qquad (A)$

Substituting these values of x, y, z in the 3rd equation, we have

$6k+12k+20k=38, \quad$ or, $\quad 38k=38, \qquad k=1$

Hence, from A, $x=2$, $y=3$, $z=4$

32 From the 1st equation, we have

$4x+4y=6z-3y, \quad$ or, $\quad 4x+7y-6z=0 \qquad (1)$

From the 2nd equation, we have

$5x-10y=6y-9z, \quad$ or, $\quad 5x-16y+9z=0 \qquad (2)$

Hence, from (1) and (2),

$$\frac{x}{7\times9-(-16)(-6)}=\frac{y}{(-6)5-9\times4}=\frac{z}{4(-16)-5\times7}$$

or $\dfrac{x}{63-96}=\dfrac{y}{-30-36}=\dfrac{z}{-64-35}$

or $\dfrac{x}{-33}=\dfrac{y}{-66}=\dfrac{z}{-99}$

or $x=\dfrac{y}{2}=\dfrac{z}{3}=k$ (suppose)

$x=k, \; y=2k, \; z=3k \qquad (A)$

Substituting these values of x, y, z in the 3rd equation, we have

$6(k-2)+7(2k-3)+8(3k-4)=67$

or $6k-12+14k-21+24k-32=67$

or $44k=67+12+21+32=132, \qquad k=3$

Hence, from A, $x=3$, $y=6$, $z=9$

33 From the 1st equation, we have

$5x-2y=0$

From the 2nd equation, we have

$7y-5z=0$

Hence,

$$\frac{x}{(-2)(-5)-7\times 0}=\frac{y}{0-(-5)5}=\frac{z}{5\times 7-(-2)0}$$

or $$\frac{x}{10}=\frac{y}{25}=\frac{z}{35}$$

or $$\frac{x}{2}=\frac{y}{5}=\frac{z}{7}=k \quad \text{(suppose)}$$

$$x=2k, \quad y=5k, \quad z=7k \qquad (A)$$

Substituting these values of x, y, z in the 3rd equation, we have

$$8k+25k+42k=150$$

or $$75k=150, \qquad k=2$$

Hence, from A, $x=4$, $y=10$, $z=14$

34 We have $15x=10y=6z=k$ (suppose)

$$x=\frac{k}{15}, \quad y=\frac{k}{10}, \quad z=\frac{k}{6} \qquad (A)$$

Substituting these values of x, y, z in the last equation, we have

$$\frac{7k}{15}+\frac{4k}{5}+\frac{3k}{2}=332$$

or $$14k+24k+45k=9960$$

or $$83k=9960$$

$$k=120$$

Hence, from A, $x=8$, $y=12$, $z=20$

35 From the 1st two equations, we have

$$\frac{x}{(-13)(-9)-6\times 8}=\frac{y}{8\times 7-(-9)4}=\frac{z}{4\times 6-7(-13)}$$

or $$\frac{x}{117-48}=\frac{y}{56+36}=\frac{z}{24+91}$$

or $$\frac{x}{69}=\frac{y}{92}=\frac{z}{115}$$

or $$\frac{x}{3}=\frac{y}{4}=\frac{z}{5}=k \quad \text{(suppose)}$$

$$x=3k, \quad y=4k, \quad z=5k \qquad (A)$$

Substituting these values of x, y, z in the 3rd equation,

we have

$$\frac{5}{3k}+\frac{8}{4k}+\frac{15}{5k}=\frac{20}{3}$$

or $$\frac{5}{3k}+\frac{2}{k}+\frac{3}{k}=\frac{20}{3}$$

or $$5+6+9=20k, \quad \text{or,} \quad 20k=20, \qquad k=1$$

Hence, from A, $x=3$, $y=4$, $z=5$

Exercise (74)

(Note) In all examples of this exercise the 1st, 2nd and the 3rd equations will be denoted by (1), (2) and (3) respectively

1 Multiplying (3) by 5, we have

$$-20x+15y+5z=25,$$

also, $$2x-3y+5z=11 \qquad (1)$$

Hence, by subtraction, $22x-18y=-14$

or $$11x-9y+7=0 \qquad (4)$$

Again, Multiplying (3) by 7, we have

$$-28x+21y+7z=35,$$

also, $$5x+2y-7z=-12 \qquad (2)$$

Hence, by addition, $-23x+23y=23$

or $$x-y+1=0 \qquad (5)$$

Thus we have $$11x-9y+7=0 \qquad (4)$$

and $$x-y+1=0 \qquad (5)$$

Therefore, by cross multiplication,

$$\frac{x}{-9+7}=\frac{y}{7-11}=\frac{1}{-11+9}$$

or $$\frac{x}{-2}=\frac{y}{-4}=\frac{1}{-2}$$

$$x=1, \quad \text{and} \quad y=2$$

Substituting these values of x, y in (3), we have

$$-4+6+z=5, \qquad z=3$$

Thus we have $x=1, \quad y=2, \quad z=3$

2 Multiplying (1) by 2 and (3) by 5, we have

$$6x+4y+10z=64$$

and $$25x+15y+10z=135$$

Hence, by subtraction,

$$19x+11y=71$$

or $$19x+11y-71=0 \qquad (4)$$

Again, multiplying (2) by 2 and (3) by 3, we have

$$4x+10y+6z=62$$

and $$15x+9y+6z=81$$

Hence, by subtraction,

$$11x-y=19$$

or $$11x-y-19=0 \qquad (5)$$

Thus we have $$19x+11y-71=0 \qquad (4)$$

and $$11x-y-19=0 \qquad (5)$$

Therefore, by cross multiplication,

$$\frac{x}{-209-71}=\frac{y}{-781+361}=\frac{1}{-19-121}$$

or, $$\frac{x}{-280}=\frac{y}{-420}=\frac{1}{-140}$$

$$x=2, \quad \text{and } y=3$$

Substituting these values of x, y in (1), we have

$$6+6+5z=32$$

or $$5z=20,$$

$$z=4$$

Thus we have $x=2$, $y=3$, $z=4$

3 From (1) and (2) by subtraction, we have

$$7x+2y-5z=0 \qquad (4)$$

Again from (1) and (3) by subtraction, we have

$$5x+2y-4z=0 \qquad (5)$$

Thus we have $$7x+2y-5z=0 \qquad (4)$$

and $$5x+2y-4z=0 \qquad (5)$$

Therefore, by cross multiplication,

$$\frac{x}{-8+10}=\frac{y}{-25+28}=\frac{z}{14-10}$$

or $\qquad \frac{x}{2}=\frac{y}{3}=\frac{z}{4}=k$ (suppose)

$$x=2k, \quad y=3k, \quad z=4k \qquad (A)$$

Substituting these values of x, y, z in (1), we have

$$2k+3k-4k=1$$

or $\qquad k=1$

Hence, from A, $\quad x=2, \quad y=3, \quad z=4$

4 From (1) and (3) by subtraction, we have

$$2x-2z=-4,$$

or $\qquad x-z+2=0 \qquad (4)$

Again, by multiplying (1) by 2 and (2) by 3, we have

$$4x+6y+8z=58$$

and $\qquad 9x+6y+15z=96$

Hence, by subtraction,

$$5x+7z=38,$$

or $\qquad 5x+7z-38=0 \qquad (5)$

Thus we have $\qquad 5x+7z-38=0 \qquad (5)$

and $\qquad x-z+2=0 \qquad (4)$

Therefore, by cross multiplication,

$$\frac{x}{14-38}=\frac{z}{-38-10}=\frac{1}{-5-7}$$

or $\qquad \frac{x}{-24}=\frac{y}{-48}=\frac{1}{-12},$

$$x=2, \text{ and } x=4$$

Substituting these values of x, z in (1), we have

$$4+3y+16z=29$$

or $\qquad 3y=9, \qquad y=3$

Thus we have $\qquad x=2, y=3, z=4$

5 Multiplying (2) by 2, we have

$$6x+4y-10z=16$$

also, $\qquad 2x+3y+4z=16 \qquad (1)$

Hence, by subtraction,

$$4x+y-14z=0 \qquad (4)$$

Again, multiplying (2) by 3 and (3) by 4, we have

$$9x-6y-15z=24$$

and $$20x-24y+12z=24.$$

Hence, by subtraction,

$$11x-30y-27z=0 \qquad (5)$$

also, $$4x+y-14z=0 \qquad (4)$$

Therefore. $$\frac{x}{420-27}=\frac{y}{108-154}=\frac{z}{11-120}$$

or $$\frac{x}{393}=\frac{y}{262}=\frac{z}{131}$$

or $$\frac{x}{3}=\frac{y}{2}=z=k \quad \text{(suppose)}$$

$$\therefore \quad x=3k,\ y=2k,\ z=k. \qquad (A)$$

Substituting these values of x, y, z in (1) we have

$6k+6k-4k=16$, or $16k=16$, $\therefore k=1$.

Therefore, $x=3$, $y=2$, $z=1$

6. Multiplying (1) by 3 and (3) by 2 we have

$$12x-9y-6z=24$$

and $$-12x+10y-14z=-2$$

Hence, by addition,

$$y+20z=22$$

or $$y-20z-22=0 \qquad (4)$$

Again, multiplying (2) by 2, we have

$$6x-8y+10z=12$$

also, $$-6x+5y-7z=-1 \qquad (3)$$

Hence, by addition,

$$-3y+17z=11$$

or $$3y-17z+11=0 \qquad (5)$$

also $$y-20z-22=0 \qquad (4)$$

Therefore, $$\frac{y}{374-220}=\frac{z}{11-66}=\frac{1}{60-17}$$

or $$\frac{y}{154}=\frac{z}{77}=\frac{1}{77}$$

$\therefore y=2$ and $z=1$

Substituting these values of y, z in (1), we have

$$4x-6+2=4$$

or $4x=12$, $x=3$

Thus, we have $x=3$, $y=2$, $z=1$

7 By adding (1) and (2), we have

$$x-2y+z=0 \quad \ldots(4)$$

Multiplying (2) by 2, we have

$$-14x+10y+12z=-2$$

also, $12x-8y-11z = 2$ (3)

Hence, by addition,

$$-2x+2y+z=0 \quad (5)$$

also, $x-2y+z=0$. (4)

Therefore, $\dfrac{x}{2+2}=\dfrac{y}{1+2}=\dfrac{z}{4-2}$

or $\dfrac{x}{4}=\dfrac{y}{3}=\dfrac{z}{2}=k$ (suppose)

$$x=4k, \quad y=3k, \quad z=2k$$

Substituting these values of x, y, z in (1), we have

$32k-21k-10k=1$, $k=1$

Therefore, $x=4$, $y=3$, $z=2$

8 Multiplying (2) by 2, we have

$$6x-4y+4z=28$$

also, $x+5y-4z= 5$. (1)

Hence, by addition,

$$7x+y=33$$

or $7x+y-33=0$ (4)

Again, multiplying (3) by 2, we have

$$-20x+16y+2z=12,$$

also, $3x-2y+2z=14$

Hence, by subtraction,

$$-23x+18y=-2$$

or $23x-18y-2=0$ (5)

also, $7x+y-33=0$ (4)

Therefore, $\dfrac{x}{594+2}=\dfrac{y}{-14+759}=\dfrac{1}{23+126}$

or $\dfrac{x}{596}=\dfrac{y}{745}=\dfrac{1}{149}$

$x=4$ and $y=5$

Substituting these values of x, y in (3), we have

$-40+40+z=6,\qquad z=6$

Thus, we have $x=4,\quad y=5,\quad z=6$

9 Multiplying (1) by 3 and (2) by 2 we have

$6x+12y+15z=147$

and $6x+10y+12z=128$

Hence, by subtraction,

$2y+3z=19$

or $2y+3z-19=0 \qquad (4)$

Again, multiplying (1) by 2, we have

$4x+8y+10z=98,$

also, $4x+3y+4z=55 \qquad (3)$

Hence, by subtraction,

$5y+6z=43$

or $5y+6z-43=0 \qquad (5)$

also, $2y+3z-19=0 \qquad (4)$

Therefore $\dfrac{y}{-114+129}=\dfrac{z}{-86+95}=\dfrac{1}{15-12}$

or $\dfrac{y}{15}=\dfrac{z}{9}=\dfrac{1}{3}$

$y=5,\qquad z=3$

Substituting these values of y, z in (1), we have

$2x+20+15=49$

or $2x=14,\qquad x=7$

Thus, we have $x=7,\quad y=5,\quad z=3$

10 Multiplying (1) by 3, we have

$3x+9y+15z=30$

also, $3x+5y+7z=14$

Hence, by subtraction,

$$4y+8z=16$$

or $$y+2z-4=0 \qquad (4)$$

Again, multiplying (1) by 5, we have

$$5x+15y+25z=50,$$

also, $$5x+7y+8z=15$$

Hence, by subtraction,

$$8y+17z=35$$

or $$8y+17z-35=0 \qquad (5)$$

also, $$y+2z-4=0 \qquad (4)$$

Therefore, $$\frac{y}{-68+70}=\frac{z}{-35+32}=\frac{1}{16-17}$$

or $$\frac{y}{2}=\frac{z}{-3}=-1$$

$$y=-2, \qquad z=3$$

Substituting these values of y, z in (1), we have

$$x-6+15=10$$

or $$x=1$$

Thus, we have $x=1$, $y=-2$, $z=3$

11 Multiplying (1) by 2 and (2) by 3, we have

$$24x+16y-22z=-6$$

and $$33x-39y-3z=6$$

Hence, by addition,

$$57x-23y-25z=0 \qquad (4)$$

Again, by adding (2) and (3), we have

$$19x+4y-13z=0 \qquad (5)$$

also, $$57x-23y-25z=0 \qquad (4)$$

Therefore, by cross multiplication,

$$\frac{x}{-100-299}=\frac{y}{-741+475}=\frac{z}{-437-228}$$

or $$\frac{x}{-399}=\frac{y}{-266}=\frac{z}{-665}$$

or $$\frac{x}{3}=\frac{y}{2}=\frac{z}{5}=\lambda \quad \text{(suppose)}$$

$$x=3\lambda, \quad y=2\lambda, \quad z=5\lambda$$

Substituting these values of x, y, z in (1), we have

$$36k+16k-55k=-3$$

or $$-3k=-3, \qquad k=1$$

$$x=3, \quad y=2, \qquad z=5$$

12 Multiplying (1) by 3 and (2) by 2, we have

$$15x-12y+27z=57$$

and $$14x+12y-24z=32$$

Hence, by addition,

$$29x+3z=89 \qquad (4)$$

Again, multiplying (1) by 2, we have

$$10x-8y+18z=38,$$

also, $$-9x+8y+15z=-13 \qquad (3)$$

Hence, by addition, $x+33z=25$

or $$x+33z-25=0 \qquad (5)$$

also, $$29x+3z-89=0 \qquad (4)$$

Therefore, $$\frac{x}{-2937+75}=\frac{z}{-725+89}=\frac{1}{3-957}$$

or $$\frac{x}{-2862}=\frac{z}{-636}=\frac{1}{-954}$$

$$x=3, \qquad z=\tfrac{2}{3}$$

Substituting these values of x, z in (1), we have

$$15-4y+6=19, \text{ or, } 4y=2, \qquad y=\tfrac{1}{2}$$

Thus, we have $x=3$, $y=\frac{1}{2}$, $z=\frac{2}{3}$

13 Adding (1) and (2), we have

$$2x+z=25 \qquad (4)$$

Again, multiplying (2) by 6, we have

$$6y+6x+12z=240,$$

also, $$4z-5x-6y=-150 \quad (3)$$

Hence, by addition,

$$x+16z=90$$

or $$x+16z-90=0 \qquad (5)$$

also, $$2x+z-25=0 \qquad (4)$$

Therefore, $\frac{x}{-400+90}=\frac{z}{-180+25}=\frac{1}{1-32}$

or $\frac{x}{-310}=\frac{z}{-155}=\frac{1}{-31}$

or $x=10,\ z=5$

Substituting these values of x, z in (1), we have

$$10-y-5=-15$$

or $-y=-20$, $\qquad y=20$

Thus, we have $x=10,\quad y=20,\quad z=5$

14 From (1) and (2), we have

$$2x-2y-3z=-2$$

and $x-3y-3z=-1$

Hence, by subtraction,

$$x+y=-1$$

or $x+y+1=0 \qquad (4)$

Again, from (2) and (3), we have

$$x-3y-3z=-1$$

and $2x+4y+3z=4$

Hence, by addition,

$$3x+y=3$$

or $3x+y-3=0 \qquad (5)$

also, $x+y+1=0 \qquad (4)$

Therefore, $\frac{x}{1+3}=\frac{y}{-3-3}=\frac{1}{3-1}$

or $\frac{x}{4}=\frac{y}{-6}=\frac{1}{2}$, $\qquad x=2,\ y=-3$

Substituting these values of x, y in (2), we have

$$2-3z=-9-1$$

or $-3z=-12$, $\qquad z=4$

Thus, we have $x=2,\quad y=-3,\ z=4$

15 From (2) and (3), by subtraction, we have

$$x+4y=29$$

or $x+4y-29=0 \ .. \qquad (4)$

Again, multiplying (1) by 6, we have

$$18x+12y-6z=120,$$

also, $$x-y+6z=41 \qquad (3)$$

Hence, by addition,

$$19x+11y=161$$

or $$19x+11y-161=0 \qquad (5)$$

also, $$x+4y-29=0 \qquad (4)$$

Therefore, $$\frac{x}{-319+644}=\frac{y}{-161+551}=\frac{1}{76-11}$$

or $$\frac{x}{325}=\frac{y}{320}=\frac{1}{65}$$

$$x=5,\ y=6$$

Substituting these values of x, y in (1), we have

$$15+12-z=20, \qquad z=7$$

Thus, we have $x=5$, $y=6$, $z=7$

16 From (1) and (2), we have

$$-4x+4y-5z=-22$$

and $$4x-6y+3z=2$$

Hence, by addition,

$$-2y-2z=-20$$

or $$y+z-10=0 \qquad (4)$$

Again, multiplying (2) by 5 and (3) by 2, we have

$$20x-30y+15z=10$$

and $$20x-6y+2z=28$$

Hence, by subtraction,

$$-24y+13z=-18$$

or $$24y-13z-18=0 \qquad (5)$$

also, $$y+z-10=0 \qquad (4)$$

Therefore, $$\frac{y}{130+18}=\frac{z}{-18+240}=\frac{1}{24+13}$$

or $$\frac{y}{148}=\frac{z}{222}=\frac{1}{37}, \qquad y=4,\ z=6$$

Substituting these values of y, z in (2), we have

$$18+4x=24+2$$

or $\quad 4x=8, \quad x=2$

Thus, we have $x=2$, $y=4$, $z=6$

17 Multiplying (2) by 10, we have

$$5x+8y-z=40,$$

also, $\quad 5x+2y+z=30 \quad (1)$

Hence, by addition, $\quad 10x+10y=70$

or $\quad x+y-7=0 \quad (4)$

Again, multiplying (1) by 10, we have

$$50x+20y+10z=300$$

also, $\quad 2x+5y+10z=129$

Hence, by subtraction,

$$48x+15y=171$$

or $\quad 16x+5y-57=0 \quad (5)$

also, $\quad x+y-7=0 \quad (4)$

Therefore, $\quad \dfrac{x}{-35+57}=\dfrac{y}{-57+112}=\dfrac{1}{16-5}$

or $\quad \dfrac{x}{22}=\dfrac{y}{55}=\dfrac{1}{11},$

$$x=2, \; y=5$$

Substituting these values of x, y in (1), we have

$$10+10+z=30$$

$$z=30$$

Thus, we have $x=2$, $y=5$, $z=10$

18 From (1), we have

$\frac{1}{2}x+\frac{1}{3}y+\frac{1}{6}z=12$, and multiplying this by $\frac{3}{2}$, we have

$$\tfrac{3}{4}x+\tfrac{1}{2}y+\tfrac{1}{4}z=18$$

also, $\quad -\frac{1}{6}x+\frac{1}{2}y+\frac{1}{6}z=8 \quad (2)$

Hence, by subtraction,

$$\tfrac{11}{12}x-\tfrac{1}{12}z=10$$

or $\quad 11x-z-120=0 \quad (4)$

also, $\quad 3x+2z-60=0 \quad (5)$

(multiplying (3) by 6)

Therefore, $$\frac{x}{60+240}=\frac{z}{-360+660}=\frac{1}{22+3}$$

or $$\frac{x}{300}=\frac{z}{300}=\frac{1}{25}$$

$$x=12,\ z=12$$

Substituting these values of x, y in (2), we have

$\frac{1}{2}y+4-2=8$, or, $\frac{1}{2}y=6$, $y=12$

Thus, we have $x=y=z=12$

19 From (1) and (3), by subtraction, we have

$$\frac{5}{x}-\frac{10}{z}=-\frac{5}{12}$$

or $$\frac{1}{x}-\frac{2}{z}+\frac{1}{12}=0 \qquad (4)$$

Again, multiplying (1) by 4 and (2) by 5, we have

$$\frac{4}{x}+\frac{20}{y}-\frac{16}{z}=\frac{1}{3}$$

and $$\frac{15}{x}-\frac{20}{y}+\frac{25}{z}=\frac{95}{24}$$

Hence, by addition, we have

$$\frac{19}{x}+\frac{9}{z}=\frac{103}{24}$$

or $$\frac{19}{x}+\frac{9}{z}-\frac{103}{24}=0 \qquad (5)$$

also, $$\frac{1}{x}-\frac{2}{z}+\frac{1}{12}=0 \qquad (4)$$

Therefore, by cross multiplication,

$$\frac{\frac{1}{x}}{\frac{3}{4}-\frac{103}{12}}=\frac{\frac{1}{z}}{-\frac{103}{24}-\frac{19}{12}}=\frac{1}{-38-9}$$

or $$\frac{\frac{1}{x}}{\frac{9-103}{12}}=\frac{\frac{1}{z}}{\frac{-103-38}{24}}=\frac{1}{-47}$$

or $\quad \dfrac{-12}{94x}=\dfrac{-24}{141z}=\dfrac{-1}{47}$

or $\quad \dfrac{12}{2x}=\dfrac{24}{3z}=1, \qquad x=6, \quad z=8$

Substituting these values of x, z in 1, we have

$\dfrac{1}{6}+\dfrac{5}{y}-\dfrac{1}{2}=\dfrac{1}{12}, \quad$ or, $\quad \dfrac{5}{y}=\dfrac{5}{12}, \qquad y=12$

Thus, we have $\quad x=6, \quad y=12, \quad z=8$

20 Multiplying (1) by 2, we have

$$\frac{6}{x}-\frac{8}{5y}+\frac{2}{z}=15\tfrac{1}{5},$$

also, $\quad \dfrac{1}{3x}+\dfrac{1}{2y}+\dfrac{2}{z}=10\tfrac{1}{6} \qquad (2)$

Hence, by subtraction,

$$\frac{17}{3x}-\frac{21}{10y}=5\tfrac{1}{30}$$

or $\quad \dfrac{170}{x}-\dfrac{63}{y}-151=0 \qquad (4)$

Again, multiplying (2) by 2, we have

$$\frac{2}{3x}+\frac{1}{y}+\frac{4}{z}=20\tfrac{1}{3}$$

also, $\quad \dfrac{17}{5x}-\dfrac{1}{2y}+\dfrac{4}{z}=16\tfrac{1}{10} \qquad (3)$

Hence, by subtraction,

$$-\frac{2}{15x}+\frac{3}{2y}=4\tfrac{7}{30}$$

or $\quad \dfrac{4}{x}-\dfrac{45}{y}+127=0 \qquad (5)$

also, $\quad \dfrac{170}{x}-\dfrac{63}{y}-151=0 \qquad (4)$

Therefore, $\dfrac{\frac{1}{x}}{6795+8001}=\dfrac{\frac{1}{y}}{21590+604}$

$$=\frac{1}{-252+7650}$$

or $\quad \dfrac{1}{14796x}=\dfrac{1}{22194y}=\dfrac{1}{7398}$

$$x=\tfrac{1}{2}, \quad y=\tfrac{1}{3}$$

Substituting these values of x, y in 1, we have

$$6-\frac{12}{5}+\frac{1}{z}=7\frac{3}{5} \quad \text{or,} \quad \frac{1}{z}=4, \qquad z=\frac{1}{4}$$

Thus, we have $x=\frac{1}{2}$, $y=\frac{1}{3}$, $z=\frac{1}{4}$

21 Multiplying (2) by 4, we have

$$8y-4z=44,$$

also $$3x+4z=57$$

Hence, by addition

$$3x+8y=101$$

or $$3x+8y-101=0 \qquad (4)$$

also, $$5x+3y-65=0 \qquad (1)$$

Therefore. $$\frac{x}{-520+303}=\frac{y}{-505+195}=\frac{1}{9-40}$$

or $$\frac{x}{-217}=\frac{y}{-310}=\frac{1}{-31}$$

$$x=7 \quad y=10$$

Substituting the value of y in (2), we have

$$20-z=11, \quad \text{or} \quad z=9$$

Thus, we have $x=7$, $y=10$, $z=9$

22 Multiplying (1) by 2, we have

$$\frac{4}{x}+\frac{2}{y}=3$$

also $$\frac{3}{z}-\frac{2}{y}=2$$

Hence, by addition,

$$\frac{4}{x}+\frac{3}{z}=5$$

or $$\frac{4}{x}+\frac{3}{z}-5=0 \qquad (4)$$

also $$\frac{1}{x}+\frac{1}{z}-\frac{4}{3}=0 \qquad (3)$$

Therefore, $$\frac{\frac{1}{x}}{-4+5}=\frac{\frac{1}{z}}{-5+\frac{16}{3}}=\frac{1}{4-3}$$

or $$\frac{1}{x}=\frac{1}{\frac{1}{3}z}=1$$

$$x=1, \quad z=3$$

Substituting the value of x in (7), we have

$$2+\frac{1}{y}=\frac{3}{2}$$

or $$\frac{1}{y}=-\frac{1}{2}, \qquad y=-2$$

Thus, we have $x=1$, $y=-2$, $z=3$

23 Multiplying (2) by b and (3) by a, we have

$$bcx+abz=b^2$$

and $$abz+acy=a^2$$

Hence, by subtraction,

$$bcx-acy=b^2-a^2$$

or $$bcx-acy+(a^2-b^2)=0 \qquad (4)$$

also, $$bx+ay-c \qquad =0 \qquad (1)$$

Therefore, $$\frac{x}{ac^2-a^3+ab^2}=\frac{y}{a^2b-b^3+bc^2}$$

$$=\frac{1}{abc+abc}$$

or $$\frac{x}{a(b^2+c^2-a^2)}=\frac{y}{b(a^2-b^2+c^2)}=\frac{1}{2abc}$$

$$x=\frac{b^2+c^2-a^2}{2bc}, \quad y=\frac{a^2-b^2+c^2}{2ac}$$

Substituting the value of x in (2), we have

$$\frac{b^2+c^2-a^2}{2b}+az=b$$

or $$az=b-\frac{b^2+c^2-a^2}{2b}$$

$$=\frac{2b^2-b^2-c^2+a^2}{2b}=\frac{b^2-c^2+a^2}{2b},$$

$$z=\frac{b^2-c^2+a^2}{2ab}$$

Thus, we have $x=\dfrac{b^2+c^2-a^2}{2bc}$, $y=\dfrac{c^2+a^2-b^2}{2ac}$, $z=\dfrac{a^2+b^2-c^2}{2ab}$

24 Multiplying (1) by 5 and (2) by 4, we have

$$15x+20y-55=0$$

and $$20y-24z+32=0$$

Hence, by subtraction,

$$15x+24z-87=0 \qquad (4)$$

also, $$-8x+7z-13=0 \qquad (3)$$

therefore, $$\frac{x}{-312+609}=\frac{z}{696+195}=\frac{1}{105+192}$$

or, $$\frac{x}{297}=\frac{z}{891}=\frac{1}{297}$$

$$x=1,\ z=3$$

Substituting the value of x in (1), we have

$$3+4y-11=0$$

or $$4y=8$$

$$y=2$$

Thus we have $x=1,\ y=2,\ z=3$

25 Multiplying (1) by 4 and (2) by 3, we have

$$4x+12y-8=0$$

and $$-3x-12y+9z-45=0$$

Hence, by addition,

$$x+9z-53=0 \qquad (4)$$

also, $$2x+7z-7=0 \qquad (3)$$

therefore, $$\frac{x}{-63+371}=\frac{z}{-106+7}=\frac{1}{7-18}$$

or $$\frac{x}{308}=\frac{z}{-99}=\frac{1}{-11},$$

$$x=-28, \qquad z=9$$

Substituting the value of x in (1), we have

$$3y-28-2=0$$

or $$3y=30$$

$$y=10$$

Thus we have $x=-28,\ y=10,\ z=9$

Exercise 75.

1 Adding together the given equations, we have

$$2\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)=3$$

or $$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=\frac{3}{2} \qquad (A)$$

Subtracting the 3rd equation from (A) we have

$$\frac{x}{a}=\frac{1}{2}, \qquad x=\frac{a}{2}$$

Similarly, we have $\quad y=\frac{b}{2}, \quad$ and $z=\frac{c}{2}$

2 Adding together the given equations, we have

$$2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=a+b+c$$

or $$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{a+b+c}{2} \qquad \text{(A)}$$

Subtracting the 3rd equation from (A), we have

$$\frac{1}{x}=\frac{a+b-c}{2}$$

$$x=\frac{2}{a+b-c}$$

Similarly, $$y=\frac{2}{a-b+c},$$

and $$z=\frac{2}{c+b-a}$$

3 From (1), we have

$$\frac{y+z}{yz}=\frac{1}{a}$$

or $$\frac{1}{y}+\frac{1}{z}=\frac{1}{a} \qquad (4)$$

From (2), we have

$$\frac{z+x}{zx}=\frac{1}{b}$$

or $$\frac{1}{z}+\frac{1}{x}=\frac{1}{b} \qquad (5)$$

From (3) we have

$$\frac{x+y}{xy}=\frac{1}{c}$$

or $$\frac{1}{x}+\frac{1}{y}=\frac{1}{c} \qquad (6)$$

From (4), (5), (6), by addition, we have

$$2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

or $$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \qquad (7)$$

Subtracting (4) from (7),

$$\frac{1}{x}=\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{1}{a}=\frac{1}{2}\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right),$$

$$x=\frac{2abc}{ab+ac-bc}$$

Subtracting (5) from (7),

$$\frac{1}{y}=\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{1}{b}=\frac{1}{2}\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right)$$

$$y=\frac{2abc}{bc-ac+ab}$$

Subtracting (6) from (7),

$$\frac{1}{z}=\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{1}{c}=\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)$$

$$z=\frac{2abc}{bc+ac-ab}$$

4 From (1) and (2), we have

$$axy-bcx-acy=0$$

and $$bxy-acx+bcy=0$$

Therefore, by cross multiplication,

$$\frac{xy}{-b^2c^2-a^2c^2}=\frac{x}{-abc-abc}=\frac{y}{-a^2c+b^2c}$$

$$x=\frac{a^2c^2+b^2c^2}{a^2c-b^2c}=\frac{c(a^2+b^2)}{a^2-b^2}$$

and $$y=\frac{a^2c^2+b^2c^2}{2abc}=\frac{c(a^2+b^2)}{2ab}$$

5 From (1), we have $\dfrac{x+y}{xy}=\dfrac{3}{4}$

or $$\frac{1}{x}+\frac{1}{y}=\frac{3}{4} \qquad (4)$$

From (2), $\dfrac{x+z}{xz}=\dfrac{2}{3}$

or $$\frac{1}{x}+\frac{1}{z}=\frac{2}{3} \qquad (5)$$

From (3), $\dfrac{y+z}{yz}=\dfrac{5}{12}$

or $$\frac{1}{y}+\frac{1}{z}=\frac{5}{12} \qquad (6)$$

From (4), (5) and (6), by addition,

$$2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{22}{12}$$

or $$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{12} \qquad (7)$$

Subtracting (6), (5) and (4) respectively from (7), we have

$$\frac{1}{x}=\frac{1}{2},\ \frac{1}{y}=\frac{1}{4} \text{ and } \frac{1}{z}=\frac{1}{6} \text{ respectively ,}$$

$$x=2,\ y=4, \text{ and } z=6$$

(6) Adding together (1), (2), and (3), we have

$$2(x+y+z)=4+6+8=18\ , \qquad x+y+z=9 \qquad (4)$$

Subtracting (1) from (4), $x=5$

(2) „ $y=3$

(3) „ $z=1$

Thus we have $x=5,\quad y=3,\quad z=1$

7 Adding together the three equations, we have

$$x+y+z=6+10+14=30 \qquad (4)$$

Subtracting (1) from (4), we have

$$2x=24\ , \qquad x=12$$

Subtracting (2) from (4), we have

$$2y=20\ , \qquad y=10$$

Subtracting (3) from (4), we have

$$2z=16\ , \qquad z=8$$

8 Adding together the three equations, we have

$$-2(x+y+z)=-60\ , \qquad x+y+z=30 \qquad (4)$$

Subtracting (3) from (4), we have

$$5x=65\ , \qquad x=13$$

Subtracting (1) from (4), we have

$$5y=40\ , \qquad y=8$$

Subtracting (2) from (4), we have

$$5z=45\ , \qquad z=9$$

9 Adding together the three equations, we have

$$5(x+y+z)-80=0$$

or $$x+y+z-16=0 \qquad (4)$$

Subtracting (1) from (4), we have

$$8x-32=0\,, \qquad x=4$$

Subtracting (2) from (4), we have

$$8y-40=0\,, \qquad y=5$$

Subtracting (3) from (4), we have

$$8z-56=0\,, \qquad z=7$$

Thus we have $x=4$, $y=5$, and $z=7$

10 Adding together the two equations, we have

$$x(a^2+2ab+b^2)+(b^2+a^2+2ab)y$$
$$=2a^2b+2ab^2+a^3+a^2b+ab^2+b^3$$

or $$x(a+b)^2+y(a+b)^2=a^3+3a^2b+3ab^2+b^3$$
$$=(a+b)^3$$

or $$x+y=a+b \qquad (3)$$

From (3), we have

$$b^2x+b^2y=b^2(a+b)$$

also, $$a^2x+b^2y=2ab(a+b)$$

Hence, by subtraction,

$$x(a^2-b^2)=b(a+b)(2a-b)$$

or $$x(a-b)=b(2a-b)\,,$$

$$x=\frac{b(2a-b)}{a-b}$$

Substituting this value in 3, we have

$$y=a+b-\frac{b(2a-b)}{a-b}$$
$$=\frac{a^2-b^2-2ab+b^2}{a-b}=\frac{a^2-2ab}{a-b}=\frac{a(2b-a)}{b-a}.$$

11 Since $$ax+by+cz=0 \qquad (2)$$
and $$a^2x+b^2y+c^2z=0 \qquad (3)\,,$$

therefore, by cross multiplication,

$$\frac{x}{bc^2-b^2c}=\frac{y}{a^2c-ac^2}=\frac{z}{ab^2-a^2b}$$

or $$\frac{x}{bc(c-b)}=\frac{y}{ac(a-c)}=\frac{z}{ab(b-a)}=k \text{ (suppose)}$$

$$x=bc(c-b)k,\ y=ac(a-c)k, \text{ and}$$
$$z=ab(b-a)k \qquad (B)$$

Substituting these values of x, y, z in (1), we have

$$-k\{ab(a-b)+ac(c-a)+bc(b-c)\}=A$$

or
$$k(a-b)(b-c)(c-a)=A$$

$$k=\frac{A}{(a-b)(b-c)(c-a)},$$

from (B),

$$x=\frac{Abc}{(a-b)(a-c)},$$

$$y=\frac{Aac}{(b-a)(b-c)},$$

and
$$z=\frac{Aab}{(c-b)(c-a)}$$

12 Since
$$\left.\begin{aligned} x+y+z&=0 \qquad (1)\\ (a+b)x+(a+c)y+(b+c)z&=0 \qquad (2)\end{aligned}\right\},$$

therefore, by cross multiplication,

$$\frac{x}{(b+c)-(a+c)}=\frac{y}{(a+b)-(b+c)}=\frac{z}{(a+c)-(a+b)}$$

or
$$\frac{x}{b-a}=\frac{y}{a-c}=\frac{z}{c-b}=k \text{ (suppose)}$$

$$x=k(b-a),$$

$$y=k(a-c) \text{ and } z=k(c-b) \qquad (B)$$

Substituting these values of x, y, z in (3), we have

$$-k\{ab(a-b)+ac(c-a)+bc(b-c)\}=1$$

or
$$k(a-b)(b-c)(c-a)=1,$$

$$k=\frac{1}{(a-b)(b-c)(c-a)}$$

Therefore, from (B),

$$x=\frac{1}{(b-c)(a-c)},$$

$$y=\frac{1}{(a-b)(c-b)},$$

and
$$z=\frac{1}{(c-a)(b-a)}$$

13 Since
$$\left.\begin{aligned} x+y+z&=0 \qquad (1)\\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}&=0 \qquad (2)\end{aligned}\right\},$$

therefore, by cross multiplication,

$$\frac{x}{\frac{1}{c}-\frac{1}{b}}=\frac{y}{\frac{1}{a}-\frac{1}{c}}=\frac{z}{\frac{1}{b}-\frac{1}{a}}=k \text{ (suppose)}$$

$$x=k\left(\frac{1}{c}-\frac{1}{b}\right),\quad y=k\left(\frac{1}{a}-\frac{1}{c}\right) \text{ and}$$

$$z=k\left(\frac{1}{b}-\frac{1}{a}\right) \qquad (B)$$

Substituting these values of x, y, z in (3), we have

$$-k\left\{\frac{1}{a^2}\left(\frac{1}{b}-\frac{1}{c}\right)+\frac{1}{b^2}\left(\frac{1}{c}-\frac{1}{a}\right)+\frac{1}{c^2}\left(\frac{1}{a}-\frac{1}{b}\right)\right\}=1$$

or $$k\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{c}-\frac{1}{a}\right)=1\ ,$$

$$k=\frac{1}{\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{c}-\frac{1}{a}\right)}$$

Therefore, from (B),

$$x=\frac{1}{\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{a}-\frac{1}{c}\right)}=\frac{a^2bc}{(b-a)(c-a)},$$

$$y=\frac{1}{\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{c}-\frac{1}{b}\right)}=\frac{ab^2c}{(b-a)(b-c)},$$

and $$z=\frac{1}{\left(\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{a}-\frac{1}{c}\right)}=\frac{abc^2}{(c-b)(c-a)}$$

14 From (1) and (2), by subtraction, we have

$$y(a-b)-z(a^2-b^2)=-(a^3-b^3)$$

or $$y-z(a+b)=-(a^2+ab+b^2) \qquad (4)$$

From (2) and (3), by subtraction, we have

$$y(b-c)-z(b^2-c^2)=-(b^3-c^3)$$

or $$y-z(b+c)=-(b^2+bc+c^2) \qquad (5)$$

Again, from (4) and (5), by subtraction, we have

$$z(a-c)=a^2+ab-bc-c^2$$
$$=(a^2-c^2)+(ab-bc)$$
$$=(a+c)(a-c)+b(a-c)$$

$$=(a+b+c)(a-c),$$

$$z=a+b+c$$

Substituting this value of z in (4), we have

$$y=(a+b)(a+b+c)-(a^2+ab+b^2)$$

$$=a^2+2ab+b^2+ac+bc-a^2-ab-b^2$$

$$=ab+ac+bc$$

Hence, from (1),

$$x-a(ab+ac+bc)+a^2(a+b+c)=a^3$$

or $$x=a^3+a^2b+a^2c+abc-a^3-a^2b-a^2c$$

$$=abc$$

Thus we have $x=abc$, $y=ab+ac+bc$,

and $z=a+b+c$

15 Since $$ax+by+cz=0 \qquad (1)$$

and $$(b+c)x+(c+a)y+(a+b)z=0 \qquad (2)$$

therefore, by cross multiplication,

$$\frac{x}{b(a+b)-c(c+a)}=\frac{y}{c(b+c)-a(a+b)}$$

$$=\frac{z}{a(c+a)-b(b+c)}$$

or $$\frac{x}{(b^2-c^2)+(ab-ac)}=\frac{y}{(c^2-a^2)+b(c-a)}$$

$$=\frac{z}{(a^2-b^2)+c(a-b)}$$

or $$\frac{x}{(b-c)(b+c+a)}=\frac{y}{(c-a)(c+a+b)}=\frac{z}{(a-b)(a+b+c)}$$

or $$\frac{x}{b-c}=\frac{y}{c-a}=\frac{z}{a-b}=k \text{ (suppose)}$$

$$x=k(b-c), y=k(c-a), z=k(a-b)$$

Substituting these values of x, y, z in (3), we have

$$k\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}$$

$$=a^2(b-c)+b^2(c-a)+c^2(a-b),$$

$$k=1$$

Therefore, $x=b-c$, $y=c-a$,

and $z=a-b$

16 Since $$a_1x+b_1y+c_1=0 \qquad (1)$$

and $$a_2x+b_2y+c_2=0 \qquad (2)$$

therefore, by cross multiplication,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{a_2c_1-a_1c_2}=\frac{1}{a_1b_2-b_1a_2}$$

$$x=\frac{b_1c_2-b_2c_1}{a_1b_2-b_1a_2}$$

and $$y=\frac{a_2c_1-a_1c_2}{a_1b_2-b_1a_2}$$

In order that the three equations may be consistent the 3rd equation will also be satisfied by the values of x, y

We have from (3),

$$\frac{a_3(b_1c_2-b_2c_1)}{a_1b_2-b_1a_2}+\frac{b_3(a_2c_1-a_1c_2)}{a_1b_2-b_1a_2}+c_3=0$$

$$a_3(b_1c_2-b_2c_1)+b_3(a_2c_1-a_1c_2)$$
$$+c_3(a_1b_2-b_1a_2)=0$$

is the required condition

17 Multiplying (2) by 3, we have

$$9x-3y+12z=60,$$

also, $$2x-3y+5z=18 \qquad (1)$$

Hence, by subtraction,

$$7x+7z=42$$

or $$x+z=6 \qquad (5)$$

Again, multiplying (2) by 2, we have

$$6x-2y+8z=40$$

also, $$4x+2y-z=5 \qquad (3)$$

Hence, by addition,

$$10x+7z=45 \qquad (6)$$

and from (5), $$7x+7z=42$$

Hence, by subtraction,

$$3x=3$$
$$x=1$$

Therefore, from (5), $$z=6-1=5$$

and from (2), $$3-y+20=20,$$
$$y=3$$

In order the four equations may be consistent the fourth equation also will be satisfied by these values of x, y, z

from (4), we have

$$(a+1)+(a-2)3-(a+3)5=76$$

or $$a-3a+5a=76-1-6-15$$

or $$9a=54$$

$$\therefore\quad a=6$$

18 Multiplying (3) by 4 and (4) by 3, we have

$$8x-12y=156$$

and $$12y-9z=123$$

Hence, by subtraction

$$8x-9z=33 \qquad (5)$$

Multiplying (2) by 8 and (5) by 5, we have

$$40x-56z=88,$$

and $$40x-45z=165$$

Hence, by subtraction,

$$-11z=-77$$

$$z=7$$

Substituting this value of z in (2), we have

$$5x=11+49=60$$

$$x=12$$

Substituting this value of x in (3), we have

$$3y=39-24=15$$

$$y=5$$

Substituting this value of y in (1), we have

$$3w=2-10=12, \qquad w=4$$

Thus we have $x=12$, $y=5$, $z=7$ and $w=4$

19 Multiplying (3) by 3, we have

$$12y-9x+6w=15,$$

also, $$9x-2z+w=41 \qquad (1)$$

Hence, by addition,

$$12y-2z+7w=56 \qquad (6)$$

Again, multiplying (2) by 3, we have

$$21y-15z-3t=36$$

also $$3y-4w+3t=7 \qquad (4)$$

Hence, by addition,

$$24y-15z-4w=43 \qquad (7)$$

also, $24y-4z+14w=112$ (multiplying (6) by 2)

Hence by subtraction,

$$11z+18w=69 \qquad (8)$$

Again, multiplying (5) by 11 and (8) by 7 we have

$$77z-55w=121$$

and $$77z+126w=483$$

Hence, by subtraction

$$181w=362, \qquad w=2$$

Substituting this value of w in (8), we have

$$11z+36=69$$

or $$11z=33 \qquad z=3$$

Again substituting these values of w, z in (1), we have

$$9x-6+2=41$$

or $$9x=45 \qquad x=5$$

Substituting the values of x, w in (3) we have

$$4y-15+4=5$$

or $4y=16$, $y=4$

Substituting the values of y, w in (4) we have

$$12-8+3t=7$$

or $$3t=3 \qquad t=1$$

Thus we have $x=5$, $y=4$, $z=3$, $w=2$ and $t=1$

20 Multiplying both sides of (2) by abc and those of (1) by b, we have

$$cx+ay+bz=3abc$$

and $$bx+by+bz=ab^2+b^2c+abc$$

Hence, by subtraction,

$$(c-b)x+(a-b)y=2abc-ab^2-b^2c \qquad (4)$$

also, $(c-b)x+(a-b)y+(c-a)z=2abc-ab^2-b^2c+ac^2-a^2c$ (3)

Hence, by subtraction,

$$(c-a)z=ac^2-a^2c=ac(c-a)$$

$$z=ac$$

Substituting this value of z in (1), we have

$$x+y+ac=ab+bc+ac$$

or $$x+y=ab+bc \qquad (5)$$

and in (2) we have

$$\frac{x}{ab}+\frac{z}{bc}+1=3$$

or $\qquad cx+az=2abc \qquad (6)$

also $\qquad cx+cy=abc+bc^2$ (multiplying (5) by c)

Hence by subtraction,

$$y(a-c)=abc-bc^2=bc(a-c),$$
$$y=bc$$

Therefore, from (5), we have

$$x+bc=ab+bc,$$
$$x=ab$$

Thus we have $x=ab$, $y=bc$ and $z=ac$

Exercise 76

1 Let $\frac{x}{y}$ represent the fraction

Then we have $\frac{2x}{y+7}=\frac{2}{3}$

or $\qquad x=\frac{y+7}{3} \qquad (1)$

and $\qquad \frac{x+2}{2y}=\frac{3}{5}$

or $\qquad x+2=\frac{6y}{5}$

or $\qquad x=\frac{6y}{5}-2=\frac{6y-10}{5} \qquad (2)$

Therefore, from (1) and (2),

$$\frac{y+7}{3}=\frac{6y-10}{5}$$

or $\qquad 5y+35=18y-30$

or $\qquad 13y=65, \qquad y=5$

Hence, $\qquad x=\frac{5+7}{3}=4$

Thus the fraction is $\frac{4}{5}$

2 Let x and y be the required numbers

Then we have $$x+5y=52 \qquad (1)$$
$$\text{and } y+8x=65 \qquad (2)$$

Multiplying (2) by 5, we have
$$5y+40x=325,$$
also, $$x+5y=52 \qquad (1)$$

Hence, by subtraction,
$$39x=273, \qquad x=7$$

Hence, $$y+56=65, \qquad y=9$$

Thus the numbers are 7 and 9

3 Let x, y be the two numbers of which x is the greater

Then we have $$5x-4y=22 \qquad (1)$$
and $$3x+7y=32 \qquad (2)$$

Multiplying (1) by 3 and (2) by 5, we have
$$15x-12y=66,$$
and $$15x+35y=160$$

Hence, by subtraction, $47y=94$, $\qquad y=2$

Hence, from (1), $$5x-8=22$$
or $$5x=30, \qquad x=6$$

Thus the numbers are 6 and 2

4 Let x and y be the numbers of which x is the greater

Then we have $$x-y=45 \qquad (1)$$
and $$\frac{x}{y}=4 \qquad (2)$$

From (2), we have $x=4y$

Hence, from (1), we have
$$4y-y=45$$
or $$3y=45, \qquad y=15$$

Therefore, $$\frac{x}{15}=4, \text{ or } x=60$$

Thus the numbers are 60 and 15

5 Let x and y be the numbers of which x is the greater

Then we have $$\frac{x}{4}+\frac{y}{3}=11 \qquad (1)$$
and $$\frac{x}{8}-\frac{y}{5}=0 \qquad (2)$$

Multiplying (2) by 2, we have

$$\frac{x}{4} - \frac{2y}{5} = 0,$$

also,

$$\frac{x}{4} + \frac{y}{3} = 11 \qquad (1)$$

Hence, by subtraction,

$$-\frac{11}{15}y = -11, \qquad y = 15$$

Therefore, from (1), $\frac{x}{4} + 5 = 11$

or

$$\frac{x}{4} = 6, \qquad x = 24$$

Thus the numbers are 24 and 15

6 Let $\frac{x}{y}$ represent the fraction

Then we have $\frac{x}{y-1} = \frac{1}{2}$ (1)

and $\frac{x+7}{y} = 1$ (2)

From (1), we have

$$2x = y - 1$$

or

$$2x - y = -1 \qquad (3)$$

From (2), we have $x + 7 = y$

or

$$x - y = -7 \qquad (4)$$

Subtracting (4) from (3), we have

$$x = 6$$

Therefore, from (1), we have

$$\frac{6}{y-1} = \frac{1}{2}$$

or

$$y - 1 = 12 \qquad y = 13$$

Thus the fraction is $\frac{6}{13}$

7 Let $\frac{x}{y}$ represent the fraction

Then we have $\frac{x+1}{y} = 1$ (1)

and $\frac{x}{y+1} = \frac{1}{2}$ (2)

From (1), we have $x+1=y$

From (2) we have $2x=y+1$

Hence, by subtraction,

$$x-1=1 \qquad x=2$$

Therefore, from (1), $\dfrac{2+1}{y}=1, \qquad y=3$

Thus the fraction is $\frac{2}{3}$

8 Let $\dfrac{x}{y}$ represent the fraction

Then we have $$\frac{x+1}{y}=\frac{1}{2} \qquad (1)$$

and $$\frac{x}{y+1}=\frac{1}{3} \qquad (2)$$

From (1) we have $2x+2=y$

From (2) we have $3x=y+1$

Hence by subtraction,

$$x-2=1, \qquad x=3$$

Therefore, form (1), $\dfrac{3+1}{y}=\dfrac{1}{2}$

or $$\frac{4}{y}=\frac{1}{2} \qquad y=8$$

Thus the fraction is $\frac{3}{8}$

9 Let A and B have Rs x and Rs y respectively

Then we have $$x+y=39 \quad . \quad (1)$$

and $$x\left(1-\frac{2}{3}\right)+y\left(1-\frac{3}{4}\right)=11 \qquad (2)$$

From (2), we have

$$\frac{x}{3}+\frac{y}{4}=11$$

or $4x+3y=132$,

also, $3x+3y=117$ (multiplying (1) by 3)

Hence by subtraction, $x=15$

Therefore. from (1),

$$15+y=39,$$

$$y=24$$

Thus A has Rs 15 and B has Rs 25

10 Let x and y be the numbers of which x is the greater

Then we have $\qquad y+7=2x \qquad (1)$

and $\qquad x+4=3y \quad . \quad (2)$

Multiplying (1) by 3 we have

$$3y+21=6x$$

or $\qquad 3y=6x-21$

also $\qquad 3y=x+4 \quad . \quad (2)$

Hence, $\qquad 5x-25=0, \qquad \therefore \quad x=5$

Therefore, from (1),

$$y+7=10, \qquad \therefore \quad y=3$$

Thus the numbers are 5 and 3

11 Let the rates be x and y miles per hour respectively

Then we have $\qquad 9(x-y)=27 \quad \ldots (1)$

and $\qquad 3(x+y)=27 \quad \ldots (2)$

From (1), we have $\qquad x-y=3.$

From (2), we have $\qquad x+y=9$

Hence, by addition $\qquad 2x=12 \qquad \therefore \quad x=6$

Also, $\qquad y=9-x$

$$=9-6$$

or $\qquad =3$

Thus the rates are 6 miles & 3 miles per hour

12 Let x be the number of sovereigns and $2x$ the number of half-crowns

Then we have $\qquad x+\left(\frac{5}{2\times 20}\right)\times 2x=10$

or $\qquad x+\frac{x}{4}=10$

or $\qquad \frac{5}{4}x=10 \qquad \therefore \quad x=8$

Thus the number of sovereigns is 8 and of half crowns is 16

13 Let one man can do the work in x days and one boy

can do it in y days Therefore, in one day one man can do $\frac{1}{x}$ of the work and one boy can do $\frac{1}{y}$ of the work

Then we get $\qquad \frac{1}{x}+\frac{1}{y}=\frac{1}{15} \qquad (1)$

$$\frac{7}{x}+\frac{9}{y}=\frac{1}{2} \qquad (2)$$

From (1), we have $\qquad \frac{9}{x}+\frac{9}{y}=\frac{3}{5}$

also, $\qquad \frac{7}{x}+\frac{9}{y}=\frac{1}{2} \qquad (2)$

Hence, by subtraction,

$$\frac{2}{x}=\frac{1}{10}, \qquad x=20$$

Thus one man would do the work in 20 days

14 Let x and y yards be the length and breadth respectively of the rectangle

Then we have the area of the rectangle

$$=xy=(x+6)(y-4)=(x+8)(y-5)$$

Hence, $\qquad xy=xy+6y-4x-24 \qquad (1)$

and $xy=xy+8y-5x-40 \qquad (2)$

From (1), we have $\qquad 6y-4x-24=0$

From (2), we have $\qquad 8y-5x-40=0$

Therefore, $\frac{y}{160-120}=\frac{x}{192+240}=\frac{1}{-30+32}$

or $\qquad \frac{y}{40}=\frac{x}{48}=\frac{1}{2}$

$$y=20, \qquad x=24$$

Thus the area $\qquad =xy=20\times 24=480$ sq yds

15 Let £x and £y be the price of tea and coffee respectively per pound

Then we have

$$15x+17y=3\frac{66}{240}=\frac{131}{40} \qquad (1)$$

and $$25x+13y=4\frac{74}{240}=\frac{517}{120} \qquad (2)$$

Multiplying (1) by 5 and (2) by 3, we have

$$75x+85y=\frac{131}{8},$$

and $$75x+39y=\frac{517}{40}$$

Hence by subtraction,

$$46y=\frac{655-517}{40}=\frac{138}{40}$$

$$y=\frac{3}{40}$$

Therefore, from (1),

$$15x+\frac{51}{40}=\frac{131}{40}$$

or $$15x=\frac{131}{40}-\frac{51}{40}=\frac{80}{40}=2,$$

$$x=\frac{2}{15}$$

Thus the price of tea per lb is £$\frac{2}{15}$ or 2s 8d and the price of coffee per lb is £$\frac{3}{40}$ or 1s 6d

16 Let the rates of A and B be x and y miles respectively per hour

Then we have

$$\frac{30}{x}-3=\frac{30}{y} \qquad (1)$$

Also, we have $\frac{30}{2x}+2=\frac{30}{y}$ (2)

Hence, by subtraction,

$$\frac{30}{2x}-5=0$$

or $$\frac{30}{2x}=5$$

or $$10x=30$$

$$x=3$$

Therefore, from (1),

$$\frac{30}{3}-3=\frac{30}{y}$$

or $$10-3=\frac{30}{y}$$

or $$7=\frac{30}{y}$$

$$y=\frac{30}{7}=4\frac{2}{7}$$

Thus A's rate is 3 miles and B's rate is $4\frac{2}{7}$ miles

17 Let x and y be the number of marbles belonging to Charles and William respectively

Then we have

$$x+10=2(y-10) \qquad (1)$$

and $$y+10=3(x-10) \qquad (2)$$

From (1), we have $\qquad x-2y+30=0$

From (2), we have $\qquad 3x-y-40=0$

Therefore,

$$\frac{x}{80+30}=\frac{y}{90+40}=\frac{1}{-1+6}$$

or $$\frac{x}{110}=\frac{y}{130}=\frac{1}{5}$$

$$x=22 \text{ and } y=26$$

Thus Charles had 22 marbles and William has 26 marbles

18 Let A, B, C receive x, y, z rupees respectively

Then we have

$$x+y+z=1100 \qquad (1)$$

$$y+200=2(x-200)=3z \qquad (2)$$

Multiplying (1), by 2 we have

$$2x+2y+2z=2200$$

and from (2), $\qquad 2x-y=\ 600$

Hence, by subtraction

$$3y+2z=1600\ ,$$

also, $\qquad -3y+9z=600$ [from (2)]

Hence, by addition $\qquad 11z=2200\ , \qquad z=200$

Therefore, from (2), $y+200=600$,

$$y=400$$

Therefore, from (1)

$$x+400+200=1100,$$

$$x=500$$

Thus A receives Rs 500, B Rs 400 and C Rs 200

19 Let x and y be the digits in the tens and units place respectively

Evidently the number $10x+y$

Then we have

$$10x+y=6(x+y)+3$$

or $\quad 4x-5y-3=0 \quad (1)$

and $\quad 10y+x=4(x+y)+9$

or $\quad -3x+6y-9=0$

or $\quad x-2y+3=0 \quad (2)$

Multiplying (2) by 4, we have

$$4x-8y+12=0$$

also, $\quad 4x-5y-3=0 \quad (1)$

Hence, by subtraction

$$3y=15,$$

$$y=5$$

Therefore, from (2),

$$x-10+3=0,$$

$$x=7$$

Thus the number is 75

20 Let the digits be x and y in the tens and units place respectively

Evidently the number $=10x+y$

Then we have

$$(10x+y)+(10y+x)=121$$

or $\quad 11x+11y=121$

or $\quad x+y=11 \quad (1)$

and $\quad (10x+y)-(10y+x)=9$

or $\quad 9x-9y=9$

or $\quad x-y=1 \quad (2)$

Adding (1) and (2), we have

$$2x=12,$$
$$x=6$$

Therefore, from (1),

$$6+y=11, \qquad y=5$$

Thus the number is 65

21 Let x and y be the numbers of crowns and half-guineas respectively

Then we have

$$5x+\frac{21y}{2}=21\times 25$$

or $$10x+21y=1050 \qquad (1)$$

and $$2y=3x+17$$

or $$3x-2y=-17 \qquad (2)$$

Multiplying (1) by 3, and (2) by 10, we have

$$30x+63y=3150,$$

and $$30x-20y=-170$$

Hence, by subtraction,

$$83y=3320, \qquad y=40$$

Therefore, from (2),

$$3x-80=-17$$

or $$3x=63, \qquad x=21$$

Thus there were 21 crowns and 40 half-guineas

22 Let the price of each horse be £x and that of each cow £y

Then we have

$$9x+7y=300 \qquad (1)$$

and $$6x+13y=300 \qquad (2)$$

Multiplying (1) by 2, and (2) by 3, we have

$$18x+14y=600,$$

and $$18x+39y=900$$

Hence, by subtraction,

$$25y=300, \qquad y=12$$

Therefore, from (1),

$$9x+84=300$$

or $$9x=216, \qquad x=24$$

Thus the price of each horse $=£24$,
and cow $=£12$

23 Let x s and y s be the respective wages per day of A and B

Now since $£5\ 17s = 217s$

we have $15x + 14y = 217$ (1)

also, $4x - 3y = 11$ (2)

Multiplying (1) by 4 and (2) by 15, we have

$$60x + 56y = 868,$$

$$\text{and } 60x - 45y = 165$$

Hence, by subtraction,

$$101y = 303, \qquad y = 3$$

Therefore, from (2), $4x - 9 = 11$

or $4x = 20, \qquad x = 5$

Thus the wages of A and B are $5s$ and $3s$ respectively

24 Let A and B do the work in x and y days respectively

Evidently they can do

$$\left(\frac{1}{x}+\frac{1}{y}\right) \text{ of the work in one day}$$

Then we have $\frac{1}{x}+\frac{1}{y}=\frac{1}{16}$ (1)

and $\frac{4}{x}+\frac{4}{y}+\frac{36}{y}=1$

or $\frac{4}{x}+\frac{40}{y}=1$

or $\frac{1}{x}+\frac{10}{y}=\frac{1}{4}$ (2)

Subtracting (1) from (2), we have

$$\frac{9}{y}=\frac{3}{16}$$

or $3y = 144, \qquad y = 48$

Therefore, from (1), $\frac{1}{x}+\frac{1}{48}=\frac{1}{16}$

or $$\frac{1}{x}=\frac{1}{16}-\frac{1}{48}=\frac{3-1}{48}=\frac{2}{48}=\frac{1}{24},$$

$$x = 24$$

Thus x and y would each separately do the work in 24 and 48 days respectively

25 Let the fraction be represented by $\dfrac{x}{y}$

Then we have

$$\frac{x+2}{y+1}=\frac{5}{8}$$

or $\quad 8x+16=5y+5$

or $\quad 8x-5y=-11 \qquad (1)$

and $\quad \dfrac{x-1}{y-1}=\dfrac{1}{2}$

or $\quad 2x-2=y-1$

or $\quad 2x-y=1 \qquad (2)$

Multiplying (2) by 5, we have

$$10x-5y=5$$

also, $\quad 8x-5y=-11 \qquad (1)$

Hence, by subtraction

$$2x=16\,, \qquad x=8$$

Therefore, from (2), $\quad 16-y=1\,, \qquad y=15$

Thus the fraction is $\frac{8}{15}$

26 Let x miles be the distance and y hours the time taken by the traveller to walk the distance Evidently his rate is $\dfrac{x}{y}$ miles per hour

Then we have

$$\frac{x}{\dfrac{x}{y}+\dfrac{1}{2}}=\frac{4y}{5}$$

or $\quad 5x=4x+2y$

or $\quad x-2y=0 \qquad (1)$

and $\quad \dfrac{x}{\dfrac{x}{y}-\dfrac{1}{2}}=y+2\frac{1}{2}$

or $\quad \dfrac{2xy}{2x-y}=\dfrac{2y+5}{2}$

or $\quad 4xy=4xy-2y^2+10x-5y$

or $\quad 10x-5y-2y^2=0 \qquad (2)$

From (1), we have $x = 2y$

Therefore, from (2),

$$20y - 5y - 2y^2 = 0$$

or $$15y = 2y^2$$

or $$15 = 2y\,; \qquad y = \frac{15}{2} = 7\frac{1}{2}$$

Therefore $$x = 2y = 15$$

Thus 15 miles is the distance

27 As the number lies between 10 and 100, the number of digits must be two and let them be x and y in the tens and units place respectively

Then we have

$$10x + y = 8(x + y)$$

or $$2x - 7y = 0 \qquad (1)$$

and $$10x + y - 45 = 10y + x$$

or $$9x - 9y = 45$$

or $$x - y = 5 \qquad (2)$$

Multiplying (2) by 2, we have

$$2x - 2y = 10,$$

also, $$2x - 7y = 0 \qquad (1)$$

Hence, by subtraction,

$$5y = 10\,; \qquad y = 2$$

Therefore, from (2), $x - 2 = 5$, $\qquad x = 7$

Thus the number is 72

28 Let A and B have x and y shillings respectively

Then we have

$$x - 10 = 2(y + 10) - 25$$

or $$x - 10 = 2y + 20 - 25$$

or $$x - 2y = 5 \qquad (1)$$

and $$\frac{5}{17}(x + 10) = y - 10$$

or $$5x + 50 = 17y - 170$$

or $$5x - 17y = -220 \qquad (2)$$

Multiplying (1) by 5, we have

$$5x - 10y = 25,$$

also, $$5x - 17y = -220 \qquad (2)$$

Hence, by subtraction,

$$7y = 245\,, \qquad y = 35$$

Therefore, from (1),

$$x-70=5 \qquad x=75$$

Thus A and B have 75s and 35s respectively

29 £2 2s $=$ £$2\frac{1}{10}=$ £$\frac{21}{10}$, and £1 8s $=$ £$1\frac{2}{5}=$ £$\frac{7}{5}$

Let x be the number of sheep and y the number of pounds the farmer had

Then we have

$$y=\frac{21x}{10}-\frac{7}{5} \qquad (1)$$

and

$$y=2x+2 \qquad (2)$$

Therefore, from (1) and (2), we have

$$\frac{21x}{10}-\frac{7}{5}=2x+2$$

or

$$x\left(\frac{21}{10}-2\right)=2+\frac{7}{5}=\frac{17}{5}$$

or

$$\frac{x}{10}=\frac{17}{5}, \qquad x=34$$

Therefore, from (2), $y=68+2=70$

Thus there were 34 sheep and the farmer had 70 pounds

30 Let the digits be x and y in the tens and units place respectively

The number is evidently $10x+y$

Then we have

$$10x+y=3(x+y)$$

or

$$7x=2y$$

or

$$x=\tfrac{2}{7}y \qquad (1)$$

and

$$3(10x+y)=(x+y)^2 \qquad (2)$$

Substituting in (2), the value of x as found in (1), we have

$$3(\tfrac{20}{7}y+y)=(\tfrac{2}{7}y+y)^2$$

or

$$3y(\tfrac{20}{7}+1)=y^2(\tfrac{2}{7}+1)^2$$

or

$$3(\tfrac{27}{7})=y(\tfrac{9}{7})^2$$

or

$$\tfrac{81}{7}=y\,\tfrac{81}{49},$$

$$y=\tfrac{81}{7}\times\tfrac{49}{81}=7$$

Therefore, from (1), $x=\frac{2}{7}\times7=2$

Thus the number is 27

Exercise 77.

1 Let x, y and z be the digits in the hundreds, tens and units place respectively.

$$\text{the required number} = 100x + 10y + z$$

Then we have

$$100x + 10y + z = 25(x + y + z)$$

or $$75x - 15y - 24z = 0$$

or $$25x - 5y - 8z = 0 \qquad (1)$$

Again $$100x + 10y + z + 198 = 100z + 10y + x$$

or $$99x - 99z + 198 = 0$$

or $$x - z + 2 = 0 \qquad (2)$$

Also $$x + z = y + 1$$

or $$x - y + z = 1 \qquad (3)$$

Multiplying (3) by 5, we have

$$5x - 5y + 5z = 5,$$

also $$25x - 5y - 8z = 0 \qquad (1)$$

by subtraction

$$20x - 13z = -5 \qquad (4)$$

Multiplying (2) by 13, we have

$$13x - 13z = -26,$$

also $$20x - 13z = -5 \qquad (4)$$

by subtraction,

$$7x = 21 \qquad x = 3$$

From (2) $$3 - z + 2 = 0 \qquad z = 5$$

Therefore from (3)

$$3 - y + 5 = 1 \qquad y = 7$$

Thus the number is 375.

2 Let x lbs. be the weight and y s. per pound be the prime cost of the article.

xy s. is the cost of the whole.

We have $$30x = xy + 100 \qquad (1)$$

and $$22x = xy - 300 \qquad (2)$$

by subtraction $8x = 400$, $\qquad x = 50$

From (1), $1500 = 50y + 100$

or $30 = y + 2$, $y = 28$

Thus the weight is 50 lbs and the cost per pound is 28 s

3 Let A and B have x and y shillings respectively

Then we have

$$x + \frac{x}{2} + 15 = 3\left(y - \frac{x}{2}\right)$$

or $3x - 3y + 15 = 0$

or $x - y + 5 = 0$ (1)

and $y - \frac{x}{2} + 10 = 2\left(x + \frac{x}{2} - 10\right)$

or $y - \frac{7x}{2} + 30 = 0$

or $7x - 2y - 60 = 0$ (2)

Multiplying (1) by 2, we have

$2x - 2y + 10 = 0$,

also, $7x - 2y - 60 = 0$

by subtraction,

$5x - 70 = 0$, $x = 14$

From (1), $14 - y + 5 = 0$, $y = 19$

Thus A had 14s and B had 19s

4 Suppose A can do the work in x days, B in y days and C in z days

Therefore, in one day A, B and C can respectively do $\frac{1}{x}$, $\frac{1}{y}$ and $\frac{1}{z}$ of the work

Then we have $\frac{12}{x} + \frac{12}{y} = 1$ (1)

$\frac{15}{y} + \frac{30}{z} = 1$ (2)

and $\frac{10}{x} + \frac{10}{y} + \frac{10}{z} = 1$ (3)

Multiplying (3) by 3, we have

$$\frac{30}{x} + \frac{30}{y} + \frac{30}{z} = 3,$$

A has 3*s* 6*d*,

and B has 4*s* 2*d*

6 Suppose A can do the work in x days and B in y days in one day A can do $\frac{1}{x}$ of the work and B $\frac{1}{y}$ of the work

Then we have $$\frac{m}{x}+\frac{m}{y}=1 \qquad (1)$$

and $$\frac{n}{x}+\frac{n+p}{y}=1 \qquad (2)$$

Multiplying (1) by n and (2) by m, we have

$$\frac{mn}{x}+\frac{mn}{y}=n,$$

and $$\frac{mn}{x}+\frac{mn+mp}{y}=m$$

Hence, by subtraction,

$$\frac{mp}{y}=m-n$$

$$y=\frac{mp}{m-n}$$

From (1), $$\frac{m}{x}+\frac{m-n}{p}=1$$

or $$\frac{m}{x}=1-\frac{m-n}{p}=\frac{p-m+n}{p},$$

$$x=\frac{mp}{p-m+n}$$

Thus A can do the work in $\frac{mp}{p-m+n}$ days and B in $\frac{mp}{m-n}$ days

7 Let A have Rs x, B Rs y and C Rs z

Then we have

$$x+700=2(y-700) \qquad (1)$$
$$y+1400=3(z-1400) \qquad (2)$$
$$z+420=5(x-420) \qquad (3)$$

From (1), we have

$$x-2y+2100=0,$$

also, $$2y-6z+11200=0 \qquad \text{[multiplying (2) by 2]}$$

Hence, by addition,

$$x-6z+13300=0 \qquad (4)$$

Multiplying (4) by 5, we have

$$5x-30z+66500=0$$

Also, $\qquad 5x-z-2520=0 \qquad$ [from (3)]

by subtraction,

$$29z-69020=0,$$

$$z=2380\ ,$$

∴ from (3), $\qquad 2380+420=5x-2100,$

or, $\qquad 5x=4900,$

$$\therefore\ x=980\ ,$$

∴ from (1), $\qquad 980+700=2y-1400$

or, $\qquad 2y=3080, \qquad y=1540$

Thus A has Rs 980, B Rs 1540 and C Rs 2380

8 Suppose the man walked x hours at the rate of 4 miles an hour and y hours at the rate of 5 miles an hour,

he walked for $(x+y)$ hours,

We have $\qquad 4x+5y=35 \qquad (1)$

and $\qquad 5x+4y=37 \qquad (2)$

or, $\qquad 4x+5y-35=0,$

and $\qquad 5x+4y-37=0$

$$\frac{x}{-185+140}=\frac{y}{-175+148}=\frac{1}{16-25},$$

or, $$\frac{x}{-45}=\frac{y}{-27}=\frac{1}{-9},$$

$$\therefore\quad x=5, \text{ and } y=3\ ,$$

$$x+y=8$$

Thus he walked for 8 hours

9 Let x miles be the required distance and y miles per hour the rate at which the train travelled

Then the journey occupied $\dfrac{x}{y}$ hours

We have $$\frac{x}{y+6}=\frac{x}{y}-4 \qquad (1)$$

Multiplying (2) by 3, we have

$$\frac{3x}{4}+\frac{5y}{2}=\frac{21}{2},$$

also, $$\frac{3x}{4}+\frac{y}{6}=\frac{7}{2} \qquad (1)$$

Hence, by subtraction, $\frac{14y}{6}=7,$

$$y=3,$$
$$x=4$$

Therefore, 4 gallons and 3 gallons were drawn out from the 1st and the 2nd vessels respectively

11 Let x be the digit in the hundreds' place and y the digit in the units' place

the digit in the tens' place $=(x+y)$

The required number $=100x+10(x+y)+y$

$=110x+11y$

We have $x+(x+y)+y=10$

or, $2x+2y=10$

or, $x+y=5 \qquad (1)$

Also we have $100y+10(x+y)+x=99+110x+11y$

or, $110y+11x=99+110x+11y$

or, $99x-99y=-99$

or, $x-y=-1 \qquad (2)$

by adding (1) and (2), $2x=64,$

$x=2,$

from (1), $y=3$

Thus the number is 253

12 Let the man have x half-crowns, y shillings and z six pences

Then we have $x+y+z=20 \qquad (1)$

$$\frac{5x}{2}+y+\frac{z}{2}=20 \qquad (2)$$

and $x+2y+6z=73 \qquad (3)$

From (1) and (2) by subtraction, we have

$$\frac{3x}{2}-\frac{z}{2}=0,$$

or, $$3x-z=0 \qquad (4)$$

Multiplying (1) by 2, we have

$$2x+2y+2z=40,$$

also, $$x+2y+6z=73 \qquad (3)$$

by subtraction,

$$x-4z=-33 \qquad (5)$$

and from (4), $$12x-4z=0\,,$$

by subtraction,

$$-11x=-33,$$
$$x=3\,.$$

from (5), $$3-4z=33,$$

or, $$-4z=-36,$$
$$z=9\,.$$

from (1), $$3+y+9=20$$
$$y=8$$

Thus the man has 3 half-crowns, 8 shillings and 9 six-pences

13 Let there be x men and let each receive y shillings

the sum divided is xy shillings

Then we have

$$(x+4)(y-1)=xy,$$

or $$4y-x-4=0 \qquad (1)$$

and $$(x-5)(y+2)=xy,$$

or, $$2x-5y-10=0 \qquad (2)$$

From (1), $$8y-2x-8=0,$$

by addition, $$3y-18=0$$
$$y=6\,,$$

from (1), $$24-x-4=0,$$

or, $$-x=-20\,,$$
$$x=20$$

Thus there are 20 persons and each receives 6*s*

14 Let each of the equal stop cocks fill the cistern in x hours and the remaining one in y hours. Evidently each of the equal cocks can fill $\frac{1}{x}$ and the third cock $\frac{1}{y}$ of the cistern in one hour.

Then we have $$\frac{8}{x}+\frac{4}{y}=\frac{5}{12} \qquad (1)$$

and $$\frac{10\frac{2}{3}}{x}+\frac{10\frac{2}{3}}{y}=\frac{7}{9}$$

or, $$\frac{32}{x}+\frac{32}{y}=\frac{7}{3}$$

Multiplying (1) by 4, we have
$$\frac{32}{x}+\frac{16}{y}=\frac{5}{3}$$

also $$\frac{32}{x}+\frac{32}{y}=\frac{7}{3} \qquad (2)$$

by subtraction,
$$\frac{16}{y}=\frac{2}{3}$$

or $$2y=48,$$
$$y=24$$

from (1), $$\frac{8}{x}+\frac{1}{6}=\frac{5}{12}$$

or $$\frac{8}{x}=\frac{1}{4},$$
$$x=32$$

Therefore, each of the equal cocks can fill the cistern in 32 hours and the remaining cock in 24 hours.

15 Let x shillings and y shillings be the price per bushel of wheat and barley respectively, also let the person offer to sell m bushels of wheat.

Then we have

$$12x=8y+56 \qquad (1)$$

and $$xm=200, \qquad (2)$$

also, $$ym=200-75=125 \qquad (3)$$

Dividing (2) by (3), we have

$$\frac{x}{y}=\frac{200}{125}=\frac{8}{5}$$

or, $5x=8y$

from (1),

$$12x=5x+56$$

or, $7x=56$

$$x=8$$

from (1),

$$96=8y+56$$

or, $8y=40$

$$y=5$$

Thus there were 8 bushels of wheat and 5 bushels of barley

16 Let him take x quarts from the 1st sort and y quarts from the 2nd

Then we have

$$x+y=100 \qquad (1)$$

and $2x+3\frac{1}{3}y=2\frac{1}{3}\times 100$

or, $6x+10y=700$

or, $3x+5y=350 \qquad (2)$

also, from (1), $3x+3y=300$,

by subtraction

$$2y=50,$$

$$y=25,$$

from (1) $x+25=100$,

$$x=75$$

Therefore, he must take 75 quarts from the 1st sort and 25 quarts from the 2nd sort

17 Let the corn rent be x quarters of wheat and y quarters of barley

Then we have

$$55x=33y$$

or, $5x=3y \qquad (1)$

and $65x+41y=55x+33y+140$

or, $10x+8y=140$

or, $5x+4y=70$,

also, $5x-3y=0$ (1)

by subtraction,

$7y=70$,

$y=10$,

from (1) $5x=30$,

$x=6$

Thus the corn rent is 6 quarters of wheat and 10 quarters of barley

18 Let x and y miles per hour be the rates of the larger and the smaller trains respectively

Then we have

$$\frac{60+72}{1760(x-y)}=\frac{12}{60\times 60}$$

or $$\frac{132}{1760(x-y)}=\frac{12}{3600}$$

or, $$\frac{11}{176(x-y)}=\frac{1}{360}$$

or, $$\frac{1}{16(x-y)}=\frac{1}{360}$$

or, $$\frac{1}{2(x-y)}=\frac{1}{45}$$

or, $2x-2y=45$ (1)

and $$\frac{60+72}{1760\left(x-\frac{3y}{2}\right)}=\frac{24}{60\times 60}$$

or, $$\frac{1}{2\left(x-\frac{3y}{2}\right)}=\frac{2}{45}$$

or, $4x-6y=45$ (2)

also, $4x-4y=90$ (1)

by subtraction, $2y=45$,

$y=22\frac{1}{2}$,

and from (1), $2x-45=45$ or, $2x=90$, $x=45$

Thus the rates are 45 and $22\frac{1}{2}$ miles per hour

19. Let x bushels of rye and y bushels of wheat be mixed with the 28 bushels of barley.

Then $x+y=100-28=72$. . (1)

and $28\times\frac{7}{3}+3x+4y=100\times\frac{10}{3}$

or, $9x+12y=1000-196=804$

or, $3x+4y=268$ (2)

also, from (1), $3x+3y=216$,

. by subtraction, $y=52$,

from (1) $x=20$

Thus he must mix 20 bushels of rye and 52 bushels of wheat

20 Suppose the person had x guineas and y crowns at first, and z guineas and w crowns after he had paid off the debt

The amount left with the person after paying off the debt

$=£27\ 6s-£14\ 17s=546s-297s=249s$

Then we have

$$21x+5y=546 \quad (1)$$

$$21z+5w=249 \quad (2)$$

$$z=y-w \quad (3)$$

and $$w=x-z \quad (4)$$

From (3) and (4), we have

$$y=z+w=x$$

Hence, from (1),

$$21x+5x=546$$

or, $$26x=546$$

$$x=21,$$

and $$y=21,$$

Again, since $z+w=21$,

we have $5z+5w=105$,

also, $21z+5w=249$

Hence, $16z=144$,

$z=9$

Hence $w=21-9=12$

Thus the man had 21 guineas and 21 crowns at first and had 9 guineas and 12 crowns left

21 Let x miles per hour be the rate of the tide in the middle of the river and y miles per hour the rate of the man independent of the tide

Then we have $\quad 1\frac{1}{2}(x+y)=18$

or, $\quad x+y=12 \qquad (1)$

and $\quad 2\frac{1}{4}(y-\frac{3}{5}x)=18$

or $\quad 5y-3x=40 \qquad (2)$

Multiplying (1) by 5, we have

$$5x+5y=60$$

also $\quad 5y-3x=40 \qquad (2)$

by subtraction,

$$8x=20, \qquad \therefore x=2\frac{1}{2}$$

Thus the rate of the tide in the middle is $2\frac{1}{2}$ miles per hour

22 Let A run a mile in x minutes and B in y minutes

Then we have

$$y-\frac{44y}{1760}-\frac{51}{60}=x$$

or, $\quad 120y-3y-102=120x$

or, $\quad 117y-120x=102$

or $\quad 39y-40x=34 \qquad (1)$

and $\quad y-1\frac{1}{4}=x-\frac{88}{1760}x=x-\frac{1}{20}$

or, $\quad 20y-25=20x-x$

or, $\quad 20y-19x=25 \qquad (2)$

From (1) and (2), we have

$$40x-39y+34=0$$

and $\quad 19x-20y+25=0$,

$$\therefore \frac{x}{-975+680}=\frac{y}{646-1000}=\frac{1}{-800+741}$$

or, $\quad \frac{x}{-295}=\frac{y}{-354}=\frac{1}{-59}$,

$\therefore x=5$ and $y=6$

Thus A can run a mile in 5 minutes and B in 6 minutes

23. Let A run x miles an hour and B y miles an hour.

Then we have

$$\frac{2}{x}-\frac{2}{y}=\frac{2}{60}$$

or, $$\frac{1}{x}-\frac{1}{y}=\frac{1}{60}. \qquad (1)$$

and $$\frac{2}{y-2}-\frac{2}{x-2}=\frac{2}{60}$$

or $$\frac{1}{y-2}-\frac{1}{x-2}=\frac{1}{60} \qquad (2)$$

From (1), we have $60(y-x)=xy$

From (2) we have $60(x-2)-60(y-2)=(x-2)(y-2)$

or $$60(x-y)-240=xy-2(y+x)+4$$

or, $$62(x-y)=xy-244.$$

Hence, by subtraction

$$122(y-x)=244.$$

$$\therefore \quad y-x=2 \quad . \qquad (3)$$

$\therefore$ from (1), $$\frac{y-x}{xy}=\frac{1}{60}$$

or $$\frac{2}{xy}=\frac{1}{60} \qquad xy=120$$

From (3) $$xy-x^2=2x$$

or $$120-x^2=2x$$

or $$x^2+2x+1=121$$

or $$(x+1)^2=121$$

or, $$x+1=11, \qquad \therefore x=10$$

from (3),

$$y-10=2$$

$$y=12$$

Thus A ran 10 miles an hour and B 12 miles an hour.

24. Let x miles per hour be the required rate, and y miles the distance from Cambridge where the accident happened. Therefore, with the original speed the train could have reached Cambridge in $\frac{y}{x}$ from the place of the accident.

Then we have $$\frac{y}{\frac{1}{n}\times x}=\frac{y}{x}+a$$

or, $$\frac{ny}{x}=\frac{y}{x}+a$$

or, $$y(n-1)=ax \qquad (1)$$

and $$\frac{y-b}{\frac{1}{n}\times x}=\frac{y-b}{x}+c$$

or, $$\frac{n(y-b)}{x}=\frac{y-b}{x}+c$$

or, $$(n-1)(y-b)=cx \qquad (2)$$

Now dividing (1) by (2), we have

$$\frac{y}{y-b}=\frac{a}{c}$$

or, $$cy=ay-ab$$

or, $$y(a-c)=ab,$$

$$y=\frac{ab}{a-c}$$

from (1),

$$\frac{ab}{a-c}(n-1)=ax$$

$$x=\frac{b(n-1)}{a-c}$$

Thus $\frac{b(n-1)}{a-c}$ miles per hour is the rate of the train before the accident.

25. Let x miles be the distance of the place of accident from the terminus and y miles per hour be the rate of travelling before the accident.

Then we have $$\frac{x}{\frac{3y}{5}}=\frac{x}{y}+(3-1)$$

or, $$\frac{5x}{3y}=\frac{x}{y}+2$$

or, $\frac{2}{3}\cdot\frac{x}{y}=2$

or, $x=3y \qquad (1)$

and $\frac{x-50}{\frac{3y}{5}}=\frac{x-50}{y}+(2-1\frac{1}{3})$

or, $\frac{2}{3}\cdot\frac{x-50}{y}=\frac{2}{3}$

or, $x-50=y$

or, $x=y+50 \qquad (2)$

from (1) and (2),

$$3y=y+50$$

or, $2y=50$,

$$y=25$$

$$x=75$$

Thus the train travelled at first at the rate of 25 miles per hour

Therefore, the length of the journey before the accident $=25$ miles and the total length of the journey is $(25+75)$ or 100 miles

Miscellaneous Exercises (3).

I.

1 $$(a^2+ax+x^2)(a^2-ax+x^2)$$
$$=\{(a^2+x^2)+ax\}\{(a^2+x^2)-ax\}$$
$$=(a^2+x^2)^2-a^2x^2$$
$$=a^4+a^2x^2+x^4$$

2 $$(ac-bd)^2+(ad+bc)^2$$
$$=a^2c^2-2acbd+b^2d^2+a^2d^2+2adbc+b^2c^2$$
$$=a^2(c^2+d^2)+b^2(c^2+d^2)$$
$$=(a^2+b^2)(c^2+d^2)$$

Each of these factors being equal to 1 the given expression is equal to 1

3 $$x^3+y^3+3xy-1$$
$$=x^3+y^3+(-1)^3-3xy(-1)$$

$$=\{x+y+(-1)\}\{x^2+y^2+(-1)^2-xy-x(-1)-y(-1)\}$$
$$=(x+y-1)(x^2+y^2+1-xy+x+y)$$

4
$$\frac{2bc+(b^2+c^2-a^2)}{2bc-(b^2+c^2-a^2)}$$
$$=\frac{(b^2+c^2+2bc)-a^2}{a^2-(b^2+c^2-2bc)}$$
$$=\frac{(b+c)^2-a^2}{a^2-(b-c)^2}$$
$$=\frac{(b+c+a)(b+c-a)}{(a+b-c)(a-b+c)}$$
$$=\frac{(b+c+a)(b+c+a-2a)}{(a+b+c-2c)(a+b+c-2b)}$$
$$=\frac{2s(2s-2a)}{(2s-2c)(2s-2b)}=\frac{s(s-a)}{(s-c)(s-b)}$$

5 The given expression
$$=\left(\frac{x^6}{x^2-1}-\frac{1}{x^2-1}\right)-\left(\frac{x^4}{x^2+1}-\frac{1}{x^2+1}\right)$$
$$=\frac{x^6-1}{x^2-1}-\frac{x^4-1}{x^2+1}$$
$$=(x^4+x^2+1)-(x^2-1)$$
$$=x^4+2$$

6
$$\frac{x+4a+b}{x+a+b}+\frac{4x+a+2b}{x+a-b}=5$$
or, $$\frac{x+4a+b}{x+a+b}-1+\frac{4x+a+2b}{x+a-b}-4=0$$
or, $$\frac{x+4a+b-x-a-b}{x+a+b}+\frac{4x+a+2b-4x-4a+4b}{x+a-b}=0$$
or, $$\frac{3a}{x+a+b}+\frac{6b-3a}{x+a-b}=0$$
or, $$\frac{a}{x+a+b}=\frac{a-2b}{x+a-b}$$
or, $$ax+a^2-ab=ax+a^2+ab-2bx-2ab-2b^2$$
or, $$-2bx-2b^2=0,$$
$$x=-b$$

2 Putting a for $x+y$, b for $-(y+z)$ and c for $(z-x)$,

we have $a+b+c=x+y-y-z+z-x=0$,

$$(x+y)^3-(y+z)^3+(z-x)^3$$
$$=a^3+b^3+c^3$$
$$=(a^3+b^3+c^3-3abc)+3abc$$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)+3abc$$
$$=3abc \qquad [\qquad a+b+c=0\ ]$$
$$=-3(x+y)(y+z)(z-x)$$
$$=3(x+y)(y+z)(x-z)\,.$$

3 We have $\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right)$

$$=2(a+m)\,2(c+n)\,,$$

$$xy+\frac{x}{y}+\frac{y}{x}+\frac{1}{xy}=4(a+m)(c+n) \qquad (1)$$

Again $\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right)=2b\,2d\,,$

$$xy-\frac{x}{y}+\frac{y}{x}+\frac{1}{xy}=4bd \qquad (2)$$

By adding (1) and (2) we have

$$2xy+\frac{2}{xy}=4(a+m)(c+n)+4bd\,,$$

$$xy+\frac{1}{xy}=2(a+m)(c+n)+2bd$$

4 $$\frac{1}{x-1}+\frac{2}{x-2}=\frac{3}{x-3}$$

or, $$\frac{1}{x-1}-\frac{1}{x-3}=\frac{3}{x-3}-\frac{2}{x-2}$$

or, $$\frac{x-3-x+1}{(x-1)(x-3)}=\frac{2(x-2-x+3)}{(x-3)(x-2)}$$

or, $$-2(x-2)=2(x-1)$$

or, $$x-2-x+1=0$$

or, $$2x-3=0\,,$$
$$x=\tfrac{3}{2}$$

5 $$ax+1=by+1=ay+bx$$

From the first two, we have

$$ax=by$$

or, $$y=\frac{ax}{b}.$$

Then we have $$ax+1=a\,\frac{ax}{b}+bx$$

$$=\frac{a^2x+b^2x}{b}$$

or, $$abx+b=a^2x+b^2x$$

or, $$x(a^2+b^2-ab)=b\ ,$$

$$x=\frac{b}{a^2+b^2-ab}.$$

$$y=\frac{a}{b}\ \frac{b}{a^2+b^2-ab}=\frac{a}{a^2+b^2-ab}$$

6 We have $x+y+z=1$ (1)

and $2x+3y+z=4$ (2)

Hence, $x+2y=3$ (4)

Again, $2x+3y+z=4$ (2)

also, $4x+9y+z=16$ (3)

Hence, $2x+6y=12$

or, $x+3y=6$ (5)

from (4) and (5), $y=3$,

from (4), $x=3-6=-3$

and from (1), $z=1+3-3=1$

Thus $x=-3$, $y=3$ and $z=1$

7 In one hour two pipes separately will fill $\frac{1}{a}$ and $\frac{1}{b}$ of the cistern

if x hours be the time required to fill it with the two pipes together we have

$$\frac{x}{a}+\frac{x}{b}=1$$

or, $$\frac{x(a+b)}{ab}=1\ , \qquad x=\frac{ab}{a+b}$$

The third pipe can empty $\frac{1}{c}$ of the cistern in 1 hour

if y hours be the time required to empty the cistern while the three pipes are working,

we have $$\frac{y}{c}-\frac{y}{a}-\frac{y}{b}=1$$

or, $$\frac{y(ab-bc-ac)}{abc}=1\,, \qquad y=\frac{abc}{ab-ac-bc}$$

Thus the cistern will be filled in $\frac{ab}{a+b}$ hours and emptied in $\frac{ab-ac-bc}{abc}$ hours

7 Let the fraction be represented by $\frac{x}{y}$

Then we have $$\frac{x+6}{y}=\frac{3}{4}$$

or, $$4x+24=3y$$

or, $$4x-3y+24=0 \qquad (1)$$

and $$\frac{x}{y-2}=\frac{1}{2}$$

or, $$2x=y-2$$

or, $$2x-y+2=0 \qquad (2)$$

From (1) and (2) we get by cross multiplication,

$$\frac{x}{-6+24}=\frac{y}{48-8}=\frac{1}{-4+6}$$

or, $$\frac{x}{18}=\frac{y}{40}=\frac{1}{2}\,,$$

$$x=9 \text{ and } y=20$$

Thus the fraction is $\frac{9}{20}$

III

1 Putting a for $x^{\frac{1}{3}}$, b for $y^{\frac{1}{3}}$ and c for $z^{\frac{1}{3}}$, the divisor becomes $a+b+c$ and the dividend $=a^3+b^3+c^3-3abc$

$$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)\,,$$

the quotient $= a^2+b^2+c^2-ab-bc-ca$

$$=(x^{\frac{1}{3}})^2+(y^{\frac{1}{6}})^2+(z^{\frac{1}{9}})^2-x^{\frac{1}{3}}y^{\frac{1}{6}}-x^{\frac{1}{3}}z^{\frac{1}{9}}-y^{\frac{1}{6}}z^{\frac{1}{9}}$$

$$=x^{\frac{2}{3}}+y^{\frac{1}{3}}+z^{\frac{2}{9}}-x^{\frac{1}{3}}y^{\frac{1}{6}}-x^{\frac{1}{3}}z^{\frac{1}{9}}-y^{\frac{1}{6}}z^{\frac{1}{9}}$$

2 The first expression

$$=a^2x^3-2abx^3+b^2x^3+a^5+a^3b^2-2a^4b$$
$$=x^3(a^2-2ab+b^2)+a^3(a^2+b^2-2ab)$$
$$=(x^3+a^3)(a^2-2ab+b^2)$$
$$=(x+a)(x^2-ax+a^2)(a-b)^2$$

The second expression

$$=a^2(2x^4-5a^2x^2+3a^4)-b^2(2x^4-5a^2x^2+3a^4)$$
$$=(a^2-b^2)(2x^4-5a^2x^2+3a^4)$$
$$=(a^2-b^2)\{2x^2(x^2-a^2)-3a^2(x^2-a^2)\}$$
$$=(a+b)(a-b)(2x^2-3a^2)(x^2-a^2)$$
$$=(a+b)(a-b)(2x^2-3a^2)(x+a)(x-a)$$

Thus the H C F is $(a-b)(x+a)$

3 The given expression

$$=\frac{1}{bc(a-b)(a-c)}-\frac{1}{ca(b-c)(a-b)}+\frac{1}{ab(a-c)(b-c)}$$
$$=\frac{a(b-c)-b(a-c)+c(a-b)}{abc(a-b)(b-c)(a-c)}$$
$$=0$$

4 We have $x-1=\frac{a}{a+b}-1=\frac{-b}{a+b}$

and $\qquad y+1=\frac{b}{a-b}+1=\frac{a}{a-b}$

$$\frac{x}{y}+\frac{x-1}{y+1}=\frac{a(a-b)}{b(a+b)}-\frac{b(a-b)}{a(a+b)}$$
$$=\frac{a^2(a-b)-b^2(a-b)}{ab(a+b)}$$
$$=\frac{(a-b)(a^2-b^2)}{ab(a+b)}=\frac{(a-b)^2}{ab}.$$

5 From the given equation

$$\frac{7x-29}{5x-12}=\frac{8x+19}{18}-\frac{4x+3}{9}$$

$$=\frac{8x+19-8x-6}{18}=\frac{13}{18},$$

or, $18(7x-29)=13(5x-12)$

or, $126x-65x=522-156$

or, $61x=366$, $\therefore\ x=6$

6 $$\frac{3}{x}+\frac{2}{y}-13=0$$

and $$\frac{7}{x}+\frac{3}{y}-27=0,$$

$$\frac{1}{x(-54+39)}=\frac{1}{y(-91+81)}=\frac{1}{9-14}$$

or, $$\frac{1}{x(-15)}=\frac{1}{y(-10)}=\frac{1}{-5},$$

$$\therefore\ x=\tfrac{1}{3}\text{ and }y=\tfrac{1}{2}$$

7 Let x minutes past 5 o'clock be the required time. At 5 o'clock the hands are 25 minute divisions apart, $\therefore$ they will coincide when the minute-hand will travel 25 minute-divisions more than the hour-hand, and we also know that the minute-hand travels 12 times faster than the hour-hand

$\therefore$ we have $12(x-25)=x$

or, $11x=300$

$\therefore\ x=\frac{300}{11}=27\frac{3}{11}$

Thus the hands will coincide at $27\frac{3}{11}$ minutes past 5 o'clock

8 Let x be the digit in the hundreds' place. Then the digit in the tens' place is $\dfrac{5+x}{2}$,

$\therefore$ the required number $=100x+5(5+x)+5=105x+30$

Then we have $105x+30+108=100\dfrac{(5+x)}{2}+50+x$

$=250+50x+50+x$

or $54x=300-138=162$, $\therefore\ x=3$

Thus the number is 345

IV.

1 The given expression

$$=\{(a^2+b^2)+ab\sqrt{2}\}\{(a^2+b^2)-ab\sqrt{2}\}$$
$$=(a^2+b^2)^2-(ab\sqrt{2})^2$$
$$=a^4+2a^2b^2+b^4-2a^2b^2$$
$$=a^4+b^4$$

2 (i) The 1st expression

$$=(x^3-ax^2)-(px^2-apx)+(qx-aq)$$
$$=x^2(x-a)-px(x-a)+q(x-a)$$
$$=(x-a)(x^2-px+q)$$

The 2nd expression

$$=(x^3-a^2x)+(ax^2-2a^2x+a^3)$$
$$=x(x^2-a^2)+a(x^2-2ax+a^2)$$
$$=x(x+a)(x-a)+a(x-a)^2$$
$$=(x-a)\{x(x+a)+a(x-a)\}$$
$$=(x-a)(x^2+2ax-a^2)$$

Thus the H C F is $x-a$

(ii) The 1st expression

$$=x^3+(-y)^3+(-z)^3-3x(-y)(-z)$$
$$=(x-y-z)(x^2+y^2+z^2+xy+xz-yz)$$

The 2nd expression

$$=x^2+(-y)^2+(-z)^2+2x(-y)+2x(-z)+2(-y)(-z)$$
$$=(x-y-z)^2$$

Thus the H C F is $x-y-z$

3 (i) The given expression

$$=\frac{(a+b+c)^2}{a^2-(b^2+c^2+2bc)}$$
$$=\frac{(a+b+c)^2}{a^2-(b+c)^2}=\frac{(a+b+c)^2}{(a+b+c)(a-b-c)}=\frac{a+b+c}{a-b-c}$$

(ii)

$$\frac{\dfrac{a}{a+b}+\dfrac{b}{a-b}}{\dfrac{a}{a-b}-\dfrac{b}{a+b}}=\frac{\dfrac{a^2-ab+ab+b^2}{a^2-b^2}}{\dfrac{a^2+ab-ab+b^2}{a^2-b^2}}$$
$$=\frac{a^2+b^2}{a^2-b^2}\times\frac{a^2-b^2}{a^2+b^2}=1$$

4 The given expression

$$=\frac{(a+b)^2+(a-b)^2}{a^2-b^2}\div\frac{\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)^2-\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)^2}{a-b}$$

$$=\frac{2(a^2+b^2)}{a^2-b^2}\times\frac{a-b}{4a^{\frac{1}{2}}b^{\frac{1}{2}}}=\frac{a^2+b^2}{2a^{\frac{1}{2}}b^{\frac{1}{2}}(a+b)}$$

5 (i) From the given equation

$$56+x-x^2-2\tfrac{1}{3}x=17x+1-x^2$$

or, $18\tfrac{1}{3}x=55$ $\quad\therefore x=3$

(ii) $ax+y=x+by=\frac{1}{2}(x+y)+1$

We have $2ax+2y=x+y+2$

or $x(2a-1)+y-2=0$ (1)

Again we have $2x+2by=x+y+2$

or, $x+y(2b-1)-2=0$ (2)

From (1) and (2) by cross multiplication, we have

$$\frac{x}{-2+4b-2}=\frac{y}{-2+4a-2}=\frac{1}{4ab-2a-2b+1-1}$$

or, $$\frac{x}{4(b-1)}=\frac{y}{4(a-1)}=\frac{1}{2(2ab-a-b)},$$

$$\therefore x=\frac{2(b-1)}{2ab-a-b}\text{ and }y=\frac{2(a-1)}{2ab-a-b}$$

6 Putting x for $(a+b)$, y for $(b+c)$ and z for $(c+a)$ we have

$$x+y+z=2(a+b+c),$$
$$\therefore (x+y+z)^3=8(a+b+c)^3$$

Thus the left-hand expression becomes

$$=(x+y+z)^3-(x^3+y^3+z^3)$$
$$=\{x^3+y^3+z^3+3(x+y)(y+z)(z+x)\}-(x^3+y^3+z^3)$$
$$=3(x+y)(y+z)(z+x)$$
$$=3(a+b+b+c)(b+c+c+a)(c+a+a+b)$$
$$=3(2a+b+c)(a+2b+c)(a+b+2c)$$

7 Let x minutes past 10 o'clock be the required time Therefore, the minute-hand was x minute-divisions and the hour-

hand $50+\frac{x}{12}$ minute-divisions apart from the 12th mark. Now because the person mistook the time to be 54 minutes earlier

we must have $\quad 50+\frac{x}{12}-x=60-54,$

or, $\quad 50-\frac{11}{12}x=6$

or, $\quad \frac{11}{12}x=44, \qquad \therefore x=48$

Thus the required time is 48 minutes past 10 o'clock.

8 Let £x and £y be the required sums.

Then we have

$$\frac{4x}{100}+\frac{7y}{100}=196$$

or, $\quad 4x+7y-19600=0 \qquad (1)$

and $\quad \frac{5x}{100}+\frac{6y}{100}=212$

or, $\quad 5x+6y-21200=0 \qquad (2)$

From (1) & (2), by cross multiplication,

$$\frac{x}{400(-371+294)}=\frac{y}{400(-245+212)}=\frac{1}{24-35}$$

or, $\quad \frac{x}{400\times(-77)}=\frac{y}{400\times(-33)}=\frac{1}{-11},$

$\therefore x=2800$ and $y=1200$.

Thus the required sums are £2800 and £1200.

V

1 (i) $\quad 15x^2-41x+14$

$=15x^2-35x-6x+14$

$=5x(3x-7)-2(3x-7)$

$=(3x-7)(5x-2)$

(ii) $\quad (2b^2+a^2-ac)(2b^2+c^2-ac)-b^2(a+c)^2$

$=\{(b^2+a^2)+(b^2-ac)\}\{(b^2+c^2)+(b^2-ac)\}-b^2(a+c)^2$

$=(b^2-ac)^2+(b^2-ac)(b^2+a^2+b^2+c^2)$

$\quad +(b^2+a^2)(b^2+c^2)-b^2(a+c)^2$

$$=(b^2-ac)^2+(b^2-ac)(a^2+2b^2+c^2)$$
$$\qquad+b^4+b^2(a^2+c^2)+a^2c^2-b^2(a^2+c^2+2ac)$$
$$=(b^2-ac)^2+(b^2-ac)(a^2+2b^2+c^2)+b^4+a^2c^2-2acb^2$$
$$=(b^2-ac)^2+(b^2-ac)(a^2+2b^2+c^2)+(b^2-ac)^2$$
$$=(b^2-ac)(b^2-ac+a^2+2b^2+c^2+b^2-ac)$$
$$=(b^2-ac)\{4b^2+(a-c)^2\}$$

2 We have evidently $xy=ab$

Now $$x^3+y^3=(x+y)^3-3xy(x+y)$$
$$=\left(ma+\frac{b}{m}\right)^3-3ab\left(ma+\frac{b}{m}\right)$$
$$=\left(ma+\frac{b}{m}\right)^3-3ma\frac{b}{m}\left(ma+\frac{b}{m}\right)$$
$$=(ma)^3+\left(\frac{b}{m}\right)^3=m^3a^3+\frac{b^3}{m^3}$$

3 The given expression

$$=\left\{\frac{y^2-yz+z^2}{x}+\frac{x^2}{y+z}-\frac{3yz}{y+z}\right\}\frac{\dfrac{2y+2z}{yz}}{\dfrac{x+y+z}{xyz}}+(x+y+z)^2$$
$$=\frac{y^3+z^3+x^3-3xyz}{x(y+z)}\,\frac{2x(y+z)}{x+y+z}+(x+y+z)^2$$
$$=\frac{(x+y+z)(x^2+y^2+z^2-xy-xz-yz)}{x(y+z)}\,\frac{2x(y+z)}{x+y+z}+(x+y+z)^2$$
$$=2(x^2+y^2+z^2-xy-xz-yz)+(x^2+y^2+z^2+2xy+2yz+2zx)$$
$$=3x^2+3y^2+3z^2=3(x^2+y^2+z^2)$$

4 $$A=x^3+(5m-3)x^2+3m(2m-5)x-18m^2,$$
$$B=x^3+(m-3)x^2-m(2m+3)x+6m^2\ ,$$
$$A-B=4mx^2+(8m^2-12m)x-24m^2$$
$$=4m\{x^2+(2m-3)x-6m\}=4mC \text{ (Suppose)}\ ,$$
again, $B=x^3+(m-3)x^2-m(2m+3)x+6m^2$

and $mC=mx^2+m(2m-3)x-6m^2$,

$\therefore$ $$B+mC=x^3+(2m-3)x^2-6mx$$
$$=x\{x^2+(2m-3)x-6m\}\quad[=xC]$$

Thus the required H C F $=x^2+(2m-3)x-6m$

5 $x^3 - ax^2 + a^2x - a^3$

$$= x^2(x - a) + a^2(x - a) = (x - a)(x^2 + a^2)$$

$x^3 + ax^2 + a^2x + a^3$

$$= x^2(x + a) + a^2(x + a) = (x^2 + a^2)(x + a)$$

$x^3 + ax^2 - a^2x - a^3$

$$= x^2(x + a) - a^2(x + a) = (x + a)(x + a)(x - a)$$

Thus the required L C M

$$= (x - a)(x + a)(x^2 + a^2)(x + a)$$
$$= (x + a)(x^2 - a^2)(x^2 + a^2)$$
$$= (x + a)(x^4 - a^4)$$
$$= x^5 + ax^4 - a^4x - a^5$$

6 (i) $\frac{3}{7}(6x - 7) + \frac{1 - 7x}{6} = x$

or, $108x - 126 + 7 - 49x = 42x$

or, $108x - 91x = 119$

or, $17x = 119,$

$x = 7$

(ii) By adding the three equations we have

$10x = 10, \qquad x = 1$

Hence, from (1) & (2), we have

$2y + 5z = 1 - 3 = -2$

and $3y - 2z = 2 - 5 = -3$

or, $4y + 10z = -4$

and $15y - 10z = -15$

Hence, $19y = -19 \qquad y = -1$

Therefore, from (1),

$3 - 2 + 5z = 1$

or, $5z = 0, \qquad z = 0$

Thus $x = 1, y = -1$ and $z = 0$

7 Suppose the man bought x eggs at two a penny Then he bought $4x$ and $5x$ eggs respectively at $5d$ a dozen and at $8d$ a score Therefore, on the whole he bought $(x + 4x + 5x)$ or $10x$ eggs

His total cost of buying $=\left(\frac{x}{2}+\frac{5}{12}\times 4x+\frac{8}{20}\times 5x\right)d$

$$=\left(\frac{x}{2}+\frac{5x}{3}+2x\right)d=\frac{25x}{6}d$$

and by selling he got $\frac{101}{100}\times 44d=\frac{22x}{5}d$

Then we have $\frac{25x}{6}+42=\frac{22x}{5}$

or, $x\left(\frac{22}{5}-\frac{25}{6}\right)=42$

or, $\left(\frac{132-125}{30}\right)x=42$

or, $\frac{7x}{30}=42$, $\quad x=180$

Therefore, he bought $(180\times 10)=1800$ eggs

8 Let x yds be the length of the course, y yds per minute the speed of A and z yds per minute the speed of B

Then we have $\frac{y}{z}=\frac{3}{2}$

or, $2y=3z$ (1)

Also we have $\frac{x-30-20}{z}=\frac{x}{y}=5$

$x=5y$ (2)

and $x=50-5z$ (3)

From (2) and (3), $5y-50=5z$

or, $y-10=z$

or, $y=z+10$

or, $2y=2z+20=3z$; $\quad z=20$,

and from (1), $2y=60$, $\quad y=30$,

also from (2), $x=150$

Thus the length of the course was 150 yds, A's speed was 30 yds per minute and B's speed 20 yds per minute

VI

1 (i) Putting x for $\left(\frac{a}{b}+\frac{b}{a}\right)$ and y for $\left(\frac{a}{b}-\frac{b}{a}\right)$, the given expression becomes

$$\begin{aligned}
&= x^4-2x^2y^2+y^4\\
&=(x^2-y^2)^2\\
&=\left\{\left(\frac{a}{b}+\frac{b}{a}\right)^2-\left(\frac{a}{b}-\frac{b}{a}\right)^2\right\}^2=(4)^2=16
\end{aligned}$$

(ii)
$$\frac{x^6-y^6}{x^3+2x^2y+2xy^2+y^3}$$

$$\begin{aligned}
&=\frac{(x^3+y^3)(x^3-y^3)}{(x^3+y^3)+2xy(x+y)}\\
&=\frac{(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}{(x+y)(x^2-xy+y^2+2xy)}\\
&=\frac{(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}{(x^2+xy+y^2)}\\
&=(x^2-xy+y^2)(x-y)
\end{aligned}$$

2 (i) $x^5-y^5-(x-y)^5$

$$\begin{aligned}
&=x^5-y^5-(x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5)\\
&=5xy(x^3-2x^2y+2xy^2-y^3)\\
&=5xy\{(x^3-y^3)-2xy(x-y)\}\\
&=5xy\{(x-y)(x^2+xy+y^2-2xy)\}\\
&=5xy(x-y)(x^2-xy+y^2)
\end{aligned}$$

(ii) See Example I Art 9, Ch VIII

$$\begin{array}{l|l|l}
x+p+q & x^3+(p+q)x+a & x^2-(p+q)x+(p+q)\{(p+q)+1\}\\
 & \underline{x^3+px^2+qx^2} & \\
 & -(p+q)x^2+(p+q)x+a & \\
 & \underline{-(p+q)x^2-(p+q)^2x} & \\
 & x(p+q)\{(p+q)+1\}+a & \\
 & \underline{x(p+q)\{(p+q)+1\}+(p+q)^2\{(p+q)+1\}} & \\
 & -(p+q)^2\{(p+q)+1\}+a &
\end{array}$$

Therefore, the condition of divisibility is evidently

$$-(p+q)^2\{(p+q)+1\}+a=0$$

or $$(p+q)^3+(p+q)^2-a=0$$

3 The given expression

$$=\frac{(x-y)\times(x^2-xy+y^2)\times(x^3-y^3)\times(x^4-y^4)}{(x+y)\times(x^2+xy+y^2)+(x^3+y^3)+(x-y)^4}$$

$$=\frac{(x-y)(x^2-xy+y^2)(x^3-y^3)(x^2+y^2)(x+y)(x-y)}{(x+y)(x^2+xy+y^2)(x^3+y^3)(x-y)(x-y)^2(x-y)}$$

$$=\frac{(x^3+y^3)(x^3-y^3)(x^2+y^2)}{(x^2-y^2)(x^3-y^3)(x^3+y^3)}=\frac{x^2+y^2}{x^2-y^2}$$

4 The first expression $=x^3(x^2+x+1)+(x^2+x+1)$

$=(x^3+1)(x^2+x+1)$

$=(x^2-x+1)(x+1)(x^2+x+1)$

The second expression $=x^3(x^2-x+1)-(x^2-x+1)$

$=(x^3-x+1)(x^3-1)$

$=(x^2-x+1)(x-1)(x^2+x+1)$

Thus their L. C. M. $=(x^2-x+1)(x+1)(x^2-x+1)(x+1)$

$=(x^3-1)(x^3+1)=x^6-1$

5 $$\frac{x-1}{4}-\frac{2(x+1)}{9}+\frac{5(x-5)}{12}-4=\frac{x+1}{18}$$

or, $9(x-1)-8(x+1)+15(x-5)-144=2(x+1)$

or, $9x-8x+15x-2x=2+9+8+75+144$

or, $14x=238$, $\quad x=17$

6 From (2), we have

$12x-9z=42$

also, $5y-9z=18$ (1)

$12x-5y=24$

or, $12x-5y-24=0$ (4)

and from (3), $7x-6y-14=0$

$$\frac{x}{70-144}=\frac{y}{-168+168}=\frac{1}{-72+35}$$

or, $$\frac{x}{-74}=\frac{y}{0}=\frac{1}{-37},$$

$x=2$ and $y=0$

from (1), $-9z=18$, $\quad z=-2$

Thus $x=2$, $y=0$ and $z=-2$

7 Let x miles be the distance between them after t days and let them come together after y days

Each day their distance dimineshes by $(a-b)$ miles,

after t days the distance will diminish by $t(a-b)$ miles

Then we have $x=d-t(a-b)$ and $d=y(a-b)$

or, $y=\dfrac{d}{a-b}$

Thus the distance between them after t days is $\{d-t(a-b)\}$ miles and they will come together after $\left(\dfrac{d}{a-b}\right)$ days

(1) When $a=b$, their rates are equal and $x=d$, i e, the distance will remain the same

(2) When $a=b$ and $d=0$, the rates being equal and they being in the same place they will walk together all the time and y becomes indeterminate and is capable of assuming any value whatever

8 Let it be x minutes past 6 o'clock when the hands are at right angles Then at the required instant the minute-hand is at a distance of x minute-divisions from the 12th mark and the hour-hand which was 30 minute divisions apart at 6 o'clock is at a distance of $\left(30+\dfrac{x}{12}\right)$ minute-divisions from the 12th mark In order that the hands may be at right angles the minute hand must be 15 minute divisions apart from the hour-hand on either side

Then we have $x=\left(30+\dfrac{x}{12}\right)\pm 15$

or, $12x=360+x\pm 180$

or, $11x=540$ or 180,

$x=\dfrac{540}{11}$ or $\dfrac{180}{11}$,

$=49\frac{1}{11}$ or $16\frac{4}{11}$

Thus the hands are at right angles at $49\frac{1}{11}$ and $16\frac{4}{11}$ minutes past 6 o'clock

VII

1 (i)

$$
\begin{array}{llll}
 & a - a^{\frac{1}{2}}b^{\frac{1}{3}} - a^{\frac{1}{2}}c^{\frac{1}{4}} + b^{\frac{2}{3}} - b^{\frac{1}{3}}c^{\frac{1}{4}} + c^{\frac{1}{2}} & & \\
 & a^{\frac{1}{2}} + b^{\frac{1}{3}} + c^{\frac{1}{4}} & & \\
\hline
a^{\frac{3}{2}} - ab^{\frac{1}{3}} - ac^{\frac{1}{4}} + a^{\frac{1}{2}}b^{\frac{2}{3}} - a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}} + a^{\frac{1}{2}}c^{\frac{1}{2}} & & & \\
\quad ab^{\frac{1}{3}} & - a^{\frac{1}{2}}b^{\frac{2}{3}} - a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}} & + b - b^{\frac{2}{3}}c^{\frac{1}{4}} + b^{\frac{1}{3}}c^{\frac{1}{2}} & \\
\quad\quad ac^{\frac{1}{4}} & - a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}} - a^{\frac{1}{2}}c^{\frac{1}{2}} & + b^{\frac{2}{3}}c^{\frac{1}{4}} - b^{\frac{1}{3}}c^{\frac{1}{2}} + c^{\frac{3}{4}} & \\
\hline
a^{\frac{3}{2}} & - 3a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}} & + b & + c^{\frac{3}{4}}
\end{array}
$$

Thus the product is $a^{\frac{3}{2}} - 3a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}} + b + c^{\frac{3}{4}}$

(ii) Quotient $= \dfrac{x^2}{2} - \dfrac{3x}{4} + 6$

$$
\begin{array}{l|l}
\dfrac{2x^2}{3} - \dfrac{5x}{6} + 1 & \dfrac{x^4}{3} - \dfrac{11x^3}{12} + \dfrac{41x^2}{8} - \dfrac{23x}{4} + 6 \\
 & \dfrac{x^4}{3} - \dfrac{5x^3}{12} + \dfrac{x^2}{2} \\
\hline
 & -\dfrac{x^3}{2} + \dfrac{37x^2}{8} - \dfrac{23x}{4} + 6 \\
 & -\dfrac{x^3}{2} + \dfrac{5x^2}{8} - \dfrac{3x}{4} \\
\hline
 & \qquad 4x^2 - 5x + 6 \\
 & \qquad 4x^2 - 5x + 6 \\
\hline
\end{array}
$$

2

$$
\left(1 + \frac{x^2+y^2-z^2}{2xy}\right) \div \left(1 + \frac{y^2+z^2-x^2}{2yz}\right)
$$

$$
= \frac{2xy + x^2 + y^2 - z^2}{2xy} \div \frac{2yz + y^2 + z^2 - x^2}{2yz}
$$

$$
= \frac{(x+y)^2 - z^2}{2xy} \div \frac{(y+z)^2 - x^2}{2yz}
$$

$$
= \frac{(x+y+z)(x+y-z)}{2xy} \times \frac{2yz}{(y+z+x)(y+z-x)}
$$

$$
= \frac{z(x+y-z)}{x(y+z-x)}
$$

3 The given expression

$$
= \frac{x^3(x-1) - (x-1)}{x^4 + x^3 + x^2 - 3x^3 - 3x^2 - 3x + x^2 + x + 1}
$$

$$=\frac{(x-1)(x^3-1)}{x^2(x^2+x+1)-3x(x^2+x+1)+(x^2+x+1)}$$
$$=\frac{(x-1)(x-1)(x^2+x+1)}{(x^2+x+1)(x^2-3x+1)}$$
$$=\frac{(x-1)^2}{x^2-3x+1}=\frac{x^2-2x+1}{x^2-3x+1}$$

4

$$\begin{array}{l}
\qquad\qquad\qquad\qquad 5x^4-36x^3+87x^2-90x+54 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2 \\
10x^3-54x^2+87x-45\,)\,10x^4-72x^3+174x^2-180x+108\,(x \\
\qquad\qquad\qquad\qquad 10x^4-54x^3+87x^2-45x \\
\qquad\qquad\qquad -3\,|\,-18x^3+87x^2-135x+108 \\
\qquad\qquad\qquad\qquad 6x^3-29x^2+45x-36 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 5 \\
\qquad\qquad\qquad\qquad 30x^3-145x^2+225x-180\,(3 \\
\qquad\qquad\qquad\qquad 30x^3-162x^2+261x-135 \\
\qquad\qquad\qquad\qquad\qquad 17x^2-36x-45
\end{array}$$

$$\begin{array}{l}
17x^2-36x-45\,)\,10x^3-54x^2+87x-45\,(1 \\
\qquad\qquad\qquad\qquad\quad 17x^2-36x-45 \\
\qquad\qquad x\,|\,10x^3-71x^2+123x \\
\qquad\qquad\quad 10x^2-71x+123 \\
\qquad\qquad\qquad\qquad\qquad\quad 17 \\
\qquad\qquad\quad 170x^2-1207x+2091\,(10 \\
\qquad\qquad\quad 170x^2-360x-450 \\
\qquad -847\,|\,-847x+2541 \\
\qquad\qquad\qquad\qquad x-3
\end{array}$$

$$\begin{array}{l}
x-3\,)\,17x^2-36x-45\,(17x+15 \\
\qquad\quad 17x^2-51x \\
\qquad\qquad\qquad 15x-45 \\
\qquad\qquad\qquad 15x-45
\end{array}$$

The H C F $=x-3$

5 The given expression

$$=\frac{a^2+bx}{(a-b)(a-x)}+\frac{b^2-ax}{(a-b)(b-x)}-\frac{x^2+ab}{(a-x)(b-x)}$$
$$=\frac{(a^2+bx)(b-x)+(b^2-ax)(a-x)-(a-b)(x^2+ab)}{(a-b)(a-x)(b-x)}$$
$$=\frac{a^2b+b^2x-a^2x-bx^2-ab^2+a^2x-b^2x-ax^2+ax^2+bx^2+a^2b-ab^2}{(a-b)(a-x)(b-x)}$$
$$=\frac{2a^2b-2ab^2}{(a-b)(a-x)(b-x)}=\frac{2ab(a-b)}{(a-b)(a-x)(b-x)}=\frac{2ab}{(a-x)(b-x)}$$

A and B can respectively do $\frac{1}{2(x-1)}$ and $\frac{1}{2(x+2)}$ of the work in one day. Also they can do together $\frac{1}{x}$ of the work in one day.

Thus we have $$\frac{1}{2(x-1)}+\frac{1}{2(x+2)}=\frac{1}{x}$$

or, $$\frac{1}{x-1}+\frac{1}{x+2}=\frac{2}{x}$$

or, $$\frac{x+2+x-1}{(x+2)(x-1)}=\frac{2}{x}$$

or, $$x(2x+1)=2(x^2+x-2)$$

or, $$2x^2+x=2x^2+2x-4,\ \therefore$$

$$x=4$$

Thus they can finish the work together in 4 days.

VIII.

1. The given expression

$$=\left\{\frac{(a^2-y^2)(a^2+y^2)}{(a-y)^2}\div\frac{a(a+y)}{a-y}\right\}$$

$$\times\left\{\frac{a^2(a^2-y^2)}{a^3+y^3}\div\frac{a^2(a^2-2ay+y^2)}{a^2-ay+y^2}\right\}$$

$$=\left\{\frac{(a-y)(a+y)(a^2+y^2)}{(a-y)^2}\times\frac{a-y}{a(a+y)}\right\}$$

$$\times\left\{\frac{a^2(a-y)(a+y)}{(a+y)(a^2-ay+y^2)}\times\frac{a^2-ay+y^2}{a^2(a-y)^2}\right\}$$

$$=\frac{a^2+y^2}{a}\times\frac{a}{a-y}=\frac{a^2+y^2}{a-y}$$

2.

$$\begin{array}{r|l|l}
3x^3+16ax^2+15a^2x-4a^3 & 6x^4-5ax^3+3a^2x^2-13a^3x-3a^4 & 2x-9a \\
 & 6x^4+32ax^3+30a^2x^2-8a^3x & \\ \hline
 & -27ax^3-27a^2x^2-21a^3x-3a^4 & \\
 & -27ax^3-144a^2x^2-135a^3x+36a^4 & \\ \hline
39a^2 & 117a^2x^2-156a^3x-39a^4 & \\ \hline
 & 3x^2-4ax-a^2 &
\end{array}$$

$$3x^2+4ax-a^2\,\Big)\,3x^3+16ax^2+15a^2x-4a^3\,\Big(\,x+4a$$
$$\underline{3x^3+4ax^2-a^2x}$$
$$12ax^2+16a^2x-4a^3$$
$$\underline{12ax^2+16a^2x-4a^3}$$

Thus the H C F is $3x^2+4ax-a^2$

3 The 1st expression $=2x^2-2x+x-1$

$$=2x(x-1)+(x-1)$$
$$=(x-1)(2x-1)$$

The 2nd expression $=2x^2+2x+x+1$

$$=2x(x+1)+(x+1)$$
$$=(2x+1)(x+1)$$

The 3rd expression $=(x+1)(x-1)$

Therefore their L C M $=(x-1)(2x+1)(x+1)$

$$=(2x+1)(x^2-1)$$

4 The given expression

$$=\frac{a^2+ab+ac+bc}{a^2+ab+2bc+2ca}\times\frac{a^3+(2c)^3}{a^4+8ac^3+a^2c^2-2ac^3+4c^4}$$
$$=\frac{a(a+b)+c(a+b)}{a(a+b)+2c(a+b)}\times\frac{(a+2c)(a^2-2ac+4c^2)}{a\{a^3+(2c)^3\}+c^2\{a^2-2ac+(2c)^2\}}$$
$$=\frac{(a+b)(a+c)}{(a+b)(a+2c)}\times\frac{(a+2c)(a^2-2ac+4c^2)}{a(a+2c)(a^2-2ac+4c^2)+c^2(a^2-2ac+4c^2)}$$
$$=\frac{(a+c)(a^2-2ac+4c^2)}{(a^2-2ac+4c^2)\{a(a+2c)+c^2\}}=\frac{a+c}{a^2+2ac+c^2}=\frac{1}{a+c}$$

5 Since $ap=bq=cr$,

we have $(ap)^2=bq\times cr$ (1)

$(bq)^2=ap\times cr$ (2)

and $(cr)^2=ap\times bq$ (3)

Now from (1), $a^2p^2=bqcr$ or, $\dfrac{p^2}{qr}=\dfrac{bc}{a^2}$,

from (2), $b^2q^2=apcr$ or, $\dfrac{q^2}{pr}=\dfrac{ac}{b^2}$,

and from (3), $c^2r^2=apbq$ or, $\dfrac{r^2}{pq}=\dfrac{ab}{c^2}$

Hence, $\dfrac{p^2}{qr}+\dfrac{q^2}{pr}+\dfrac{r^2}{pq}=\dfrac{bc}{a^2}+\dfrac{ac}{b^2}+\dfrac{ab}{c^2}$

6 $$x+y=\frac{a-b}{m-c}+\frac{b-c}{m-a}$$
$$=\frac{m(a-b)+m(b-c)-a(a-b)-c(b-c)}{(m-c)(m-a)}$$
$$=\frac{m(a-c)-(a^2-c^2)+b(a-c)}{(m-c)(m-a)}$$
$$=\frac{(a-c)\{m-(a+c-b)\}}{(m-c)(m-a)},$$

and $z+xyz=z(1+xy)$
$$=\frac{c-a}{m-b}\left\{1+\frac{(a-b)(b-c)}{(m-c)(m-a)}\right\}$$
$$=\frac{c-a}{m-b}\left\{\frac{m^2-(a+c)m+ac+ab-ac+bc-b^2}{(m-c)(m-a)}\right\}$$
$$=\frac{c-a}{m-b}\left\{\frac{m^2-(a+c)m+b(a+c-b)}{(m-c)(m-a)}\right\}$$
$$=\frac{c-a}{m-b}\times\frac{(m-b)\{m-(a+c-b)\}}{(m-c)(m-a)}$$
$$=\frac{-(a-c)\{m-(a+c-b)\}}{(m-c)(m-a)}$$

Thus $x+y+z+xyz=0$

7 From (2), we have $\frac{a}{b+y}=\frac{b}{a+x}$

or, $a^2+ax=b^2+by$

or, $ax-by=b^2-a^2$,

also, $ax+by=c^2$ (1)

Hence, $2ax=c^2+b^2-a^2$, $x=\frac{c^2+b^2-a^2}{2a}$,

from (1), $by=c^2-\frac{c^2+b^2-a^2}{2}$

$=\frac{a^2-b^2+c^2}{2}$, $y=\frac{a^2-b^2+c^2}{2b}$

8 Let £x be the sum Then each received £ $\frac{x}{15}$ Therefore, each would receive £$\frac{105x}{100\times 15}$ or £$\frac{7x}{100}$, if £2 more had been available

Thus we have $\dfrac{x+2}{15}=\dfrac{7x}{100}$

or, $100x+200=105x$

or, $5x=200, \qquad x=40$

Thus the sum is £40

IX

1 Since $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{b}{a}+\dfrac{d}{c}$

or $$\frac{ad+bc}{bd}=\frac{bc+ad}{ac}$$

$$bd=ac$$

Thus we must have

$$\frac{a^2}{b^2}+\frac{c^2}{d^2}=\frac{a^2d^2+b^2c^2}{b^2d^2}=\frac{a^2d^2+b^2c^2}{a^2c^2}=\frac{d^2}{c^2}+\frac{b^2}{a^2}$$

2 $$x-a=\frac{a^2+ab+b^2}{a+b}-a=\frac{a^2+ab+b^2-a^2-ab}{a+b}=\frac{b^2}{a+b},$$

$$x-b=\frac{a^2+ab+b^2}{a+b}-a=\frac{a^2+ab+b^2-a^2-ab}{a+b}=\frac{a^2}{a+b},$$

and $$x-a-b=\frac{a^2+ab+b^2}{a+b}-(a+b)$$

$$=\frac{a^2+ab+b^2-a^2-2ab-b^2}{a+b}=\frac{-ab}{a+b}.$$

$$\frac{(x-a)(x-b)}{(x-a-b)^2}=\frac{\dfrac{b^2}{(a+b)}\;\dfrac{a^2}{(a+b)}}{\dfrac{(-ab)^2}{(a+b)^2}}=\frac{a^2b^2}{(a+b)^2}\times\frac{(a+b)^2}{a^2b^2}=1$$

3 The given expression

$$=\frac{2x^2-2xy-x^2+2xy+2y^2}{x^2-y^2}\times\frac{x+y}{xy}-\frac{3xy-2xy+2y^2+x^2-xy}{xy(x-y)}$$

$$=\frac{x^2+2y^2}{x^2-y^2}\times\frac{x+y}{xy}-\frac{x^2+2y^2}{xy(x-y)}$$

$$=\frac{x^2+2y^2}{(x+y)(x-y)}\times\frac{x+y}{xy}\times\frac{xy(x-y)}{x^2+2y^2}=1$$

also $\qquad 7y + 2z = 20$

$\qquad 47z = 141 \qquad\qquad z = 3$

Therefore from (5), $y = 23 - 21 = 2$,
and from (1) $\quad x = 14 - 4 - 9 = 1$
Thus $\quad x = 1,\ y = 2$ and $z = 3$

7 Suppose the woman bought $2x$ apples on the whole,

her total cost of buying $=\left(\frac{x}{3}+\frac{x}{4}\right)$ pence

and by selling she got $\frac{2\times 2x}{7}$ i.e., $\frac{4x}{7}$ pence

Then we have $\quad \frac{x}{3}+\frac{x}{4}=\frac{4x}{7}+3$

or $\quad 28x+21x=48x+252$,

$x=252$,

$\frac{4x}{7}=\frac{4\times 252}{7}=144$ pence i.e. 12s

Thus the woman sold them for 12s

8 Let A run x miles per hour, B run y miles per hour and C run z miles per hour. When A runs 1760 yds B runs $(1760-160)$ yds or 1600 yds and when B runs 1760 yds C runs $(1760-76\frac{12}{23})$ yds or, $1683\frac{11}{23}$ yds or, $\frac{38720}{23}$ yds

Thus we have $\qquad \frac{1}{x}=\frac{1600}{1760y}=\frac{10}{11y}$

or, $\qquad 10x = 11y \qquad\qquad (1)$

Again $\qquad \frac{1}{y}=\frac{38720}{1760\times 23z}=\frac{22}{23z}$

or, $\qquad 22y = 23z \qquad\qquad (2)$

Also $\qquad \frac{1}{z}-\frac{1}{y}=\frac{1}{4\times 60} \qquad\qquad (3)$

Hence $\qquad \frac{1}{z}-\frac{22}{23z}=\frac{1}{240}$

or $\qquad \frac{1}{z}\cdot\frac{1}{23}=\frac{1}{240}, \qquad\qquad z=\frac{240}{23}=10\frac{10}{23}$

or, $$\frac{1}{x-1}-\frac{1}{x-2}=\frac{2}{x-2}-\frac{2}{x-3}$$

or, $$\frac{x-2-x+1}{(x-2)(x-2)}=2\left\{\frac{x-3-x+2}{(x-2)(x-3)}\right\}$$

or, $$\frac{-1}{x-1}=\frac{-2}{x-3}$$

or, $$x-3=2x-2, \qquad x=-1.$$

(ii) From (1) and (3) we have by addition

$$x+2y+z=x+z+4a$$

or, $$2y=4a, \qquad y=2a$$

Again from (2) and (3), we have

$$2x+y+z=y+z+2a$$

or, $$2x=2a \qquad x=a$$

Therefore from (3), we have $z=3a$

Thus $x=a$, $y=2a$ and $z=3a$

7 Subtracting (2) from (1), we have

$$\frac{a-b}{x}-\frac{a-b}{y}=\frac{1}{b}-\frac{1}{a}=\frac{a-b}{ab}$$

or, $$\frac{1}{x}-\frac{1}{y}=\frac{1}{ab} \qquad (3)$$

Again, by adding (1) and (2), we have

$$\frac{a+b-2}{x}-\frac{a+b-4}{y}=\frac{1}{b}+\frac{1}{a}=\frac{a+b}{ab},$$

and, from (3), $\frac{a+b}{x}-\frac{a+b}{y}=\frac{a+b}{ab}$

Hence $$\frac{2}{x}-\frac{4}{y}=0$$

or, $$\frac{1}{x}-\frac{2}{y}=0 \qquad (4)$$

from (3) and (4), we have

$$\frac{c}{x}-\frac{c}{y}-\left(\frac{1}{x}-\frac{2}{y}\right)=\frac{c}{ab}$$

Thus $$\frac{c-1}{x}-\frac{c-2}{y}=\frac{c}{ab}$$

8 Let x miles and y miles per hour be the respective rates of the faster and the slower trains When they are moving in opposite directions a distance of $(x+y)$ miles is covered per hour and when in the same direction $(x-y)$ miles per hour In passing each other they shall have to traverse their lengths $(82+84)$ feet or 176 feet

Thus we have $\quad \dfrac{176}{3\times1760}\times\dfrac{1}{x+y}=\dfrac{3}{2}\times\dfrac{1}{60\times60}$

or, $\quad \dfrac{1}{30(x+y)}=\dfrac{1}{2\times20\times60}$

or, $\quad x+y=80 \qquad (1)$

Also we have $\quad \dfrac{176}{3\times1760}\times\dfrac{1}{x-y}=\dfrac{6}{60\times60}$

or, $\quad \dfrac{1}{30(x-y)}=\dfrac{1}{10\times60}$

or, $\quad x-y=20 \qquad (2)$

Hence, from (1) and (2),

$2x=100 \qquad \therefore x=50$,

from (1) $y=50-20=30$

Thus the rates are 50 miles and 30 miles per hour

XI

1 The given expression

$$=\{(x+y)^2+z^2\}^2-2(x+y)^2z^2$$
$$=(x+y)^4+2(x^2+y^2)^2z^2+z^4-2(x+y)^2z^2$$
$$=(x+y)^4+z^4$$

2

$$\begin{array}{r|l|l}
 & 6x^5-9x^4+19x^3-12x^2+19x-15 & \\
 & \quad 2 & \\
4x^4-2x^3+10x^2 & 12x^5-18x^4+38x^3-24x^2+38x-30 & 3x-3 \\
+x+15 & \underline{12x^5-\ \ 6x^4+30x^3+\ \ 3x^2+45x} & \\
 & \quad -12x^4+\ 8x^3-27x^2-\ 7x-30 & \\
 & \quad \underline{-12x^4+\ 6x^3-30x^2-\ 3x-45} & \\
 & \qquad\qquad 2x^3+\ 3x^2-\ 4x+15 & \\
\end{array}$$

$$2x^3+3x^2-4x+15\,)\,4x^4-2x^3+10x^2+x+15\,(\,2x-4$$
$$4x^4+6x^3-8x^2+30x$$
$$-8x^3+18x^2-29x+15$$
$$-8x^3-12x^2+16x-60$$
$$15\,|\,30x^2-45x+75$$
$$2x^2-3x+5$$

$$2x^2-3x+5\,)\,2x^3+3x^2-4x+15\,(\,x+3$$
$$2x^3-3x^2+5x$$
$$6x^2-9x+15$$
$$6x^2-9x+15$$

Thus the H C F is $2x^2-3x+5$

3 Since $a+b+c=0$, we have $a+b=-c$

Therefore the given expression

$$=\frac{(a+b)^2-2ab+c^2}{(a+b)^3-3ab(a+b)+c^3}+\frac{2}{3}\left(\frac{ab+bc+ca}{abc}\right)$$

$$=\frac{2c^2-2ab}{-c^3+3abc+c^3}+\frac{2}{3}\left\{\frac{ab+c(a+b)}{abc}\right\}$$

$$=\frac{2}{3}\,\frac{(c^2-ab)}{abc}+\frac{2}{3}\,\frac{(ab-c^2)}{abc}=0$$

4 $$\left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{b}-\frac{y}{a}\right)=\frac{x^2}{ab}-\frac{xy}{a^2}-\frac{xy}{b^2}+\frac{y^2}{ab}$$

$$=\frac{x^2+y^2}{ab}-xy\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

$$=\frac{xy+a^3+b^3}{ab}-ab\,(a+b)\left(\frac{a^2+b^2}{a^2b^2}\right)$$

$$=\frac{ab(a+b)+(a+b)(a^2+b^2-ab)}{ab}-\frac{(a+b)(a^2+b^2)}{ab}$$

$$=\frac{a+b}{ab}\left(ab+a^2+b^2-ab-a^2-b^2\right)=0$$

5 The left hand expression

$$=\frac{\{(a+b)^3+(c+d)^3\}-\{(b+c)^3+(d+a)^3\}}{\{(a+b)^2+(c+d)^2\}-\{(b+c)^2+(d+a)^2\}}$$

$$[+3(b+c)(d+a)(b+c+d+a)$$

$$=\frac{\{(a+b)+(c+d)\}^3-3(a+b)(c+d)(a+b+c+d)-\{(b+c)+(d+a)\}^3}{\{(a+b)+(c+d)\}^2-2(a+b)(c+d)-\{(b+c)+(d+a)\}^2+2(b+c)(d+a)}$$

$$=\frac{3(a+b+c+d)\{(b+c)(d+a)-(a+b)(c+d)\}}{2\{(b+c)(d+a)-(a+b)(c+d)\}}=\frac{3}{2}(a+b+c+d)$$

6 (i) $27x^2-48x-512$

$$=\tfrac{1}{3}(81x^2-144x-1536)$$
$$=\tfrac{1}{3}\{(9x-8)^2-(40)^2\}$$
$$=\tfrac{1}{3}(9x+32)(9x-48)$$
$$=(9x+32)(3x-16)$$

(ii) $(a+b-c-d)^2-(a-b+c-d)^2$

$$=(a+b-c-d+a-b+c-d)(a+b-c-d-a+b-c+d)$$
$$=(2a-2d)(2b-2c)=4(a-d)(b-c)$$

(iii) $m^4+m^2n^2+n^4$

$$=m^4+2m^2n^2+n^4-m^2n^2$$
$$=(m^2+n^2)^2-(mn)^2$$
$$=(m^2+mn+n^2)(m^2-mn+n^2)$$

7 Multiplying (3) by 2, we have $2x-4y+2z=0$

Also, $x-y+2z=11$ (1)

$x-3y=-11$ (4)

From (2) and (3) we have $x+y=9$ (5)

Hence, from (4) and (5), $4y=20$, $\therefore$ $y=5$

From (5), $x=9-5=4$

from (3) $z=10-4=6$

Thus, $x=4$, $y=5$ and $z=6$

8 Let the grey hound take x leaps to catch the hare and by that time the hare will take $\frac{5x}{4}$ leaps Then the hound will have to pass over $\left(40+\frac{5x}{4}\right)$ leaps of the hare or $\frac{3}{4}\left(40+\frac{5x}{4}\right)$ leaps of his own

Thus, we have $\frac{3}{4}\left(40+\frac{5x}{4}\right)=x$

or, $480+15x=16x$, $\therefore$ $x=480$

Thus the grey-hound must take 480 leaps

XII

1 The given expression

$$=\frac{y(x^3-y^3)}{xy(y+x)}\div\left\{\frac{x^2(x^2+xy+y^2)}{(x+y)^3(x-y)^3}\times\frac{(x+y)^2}{x^2}\right\}$$

$$=\frac{(x-y)(x^2+xy+y^2)}{x(x+y)}\div\left\{\frac{(x^2+xy+y^2)}{(x+y)(x-y)^3}\right\}$$

$$=\frac{(x-y)(x^2+xy+y^2)}{x(x+y)}\times\frac{(x+y)(x-y)^3}{(x^2+xy+y^2)}=\frac{(x-y)^4}{x}$$

2 $$x^2+1=\frac{(a+1)^2}{(a-1)^2}+1=\frac{(a+1)^2+(a-1)^2}{(a-1)^2}=\frac{2(a^2+1)}{(a-1)^2}$$

Similarly, $y^2+1=\dfrac{2(b^2+1)}{(b-1)^2}$ and $z^2+1=\dfrac{2(c^2+1)}{(c-1)^2}$

Again $$xy+1=\frac{(a+1)(b+1)}{(a-1)(b-1)}+1$$

$$=\frac{ab+(a+b)+1+ab-(a+b)+1}{(a-1)(b-1)}=\frac{2(ab+1)}{(a-1)(b+1)}$$

Similarly, $yz+1=\dfrac{2(bc+1)}{(b-1)(c-1)}$ and $zx+1=\dfrac{2(ac+1)}{(c-1)(a-1)}$

Thus the left-hand expression

$$=\frac{\dfrac{8(a^2+1)(b^2+1)(c^2+1)}{(a-1)^2(b-1)^2(c-1)^2}}{\dfrac{8(ab+1)(bc+1)(ca+1)}{(a-1)(b-1)(b-1)(c-1)(c-1)(a-1)}}$$

$$=\frac{(a^2+1)(b^2+1)(c^2+1)}{(ab+1)(bc+1)(ca+1)}$$

3
$$\begin{array}{r|l}
2x^4+x^3-9x^2+8x-2 & 2x^4-7x^3+11x^2-8x+2 \quad (\,1 \\
 & \underline{2x^4+x^3-9x^2+8x-2} \\
-4 & \underline{-8x^3+20x^2-16x-4} \\
 & 2x^3-5x^2+4x-1
\end{array}$$

$$\begin{array}{r|l}
2x^3-5x^2+4x-1 & 2x^4+x^3-9x^2+8x-2 \quad (\,x+3 \\
 & \underline{2x^4-5x^3+4x^2-x} \\
 & 6x^3-13x^2+9x-2 \\
 & \underline{6x^3-15x^2+12x-3} \\
 & 2x^2-3x+1
\end{array}$$

$$2x^2-3x-1\,)\,\frac{2x^3-5x^2-2x-1\,(\,x-1}{\dfrac{2x^3-3x^2-x}{\dfrac{-2x^2-3x-1}{-2x^2-3x-1}}}$$

Thus the H C F is $2x^2-3x-1$

The L C M of the two expressions $=\dfrac{\text{their Product}}{\text{their H C F}}$

$$=\frac{(2x^4-x^3-9x^2-8x-2)(2x^4-7x^3+11x^2-8x-2)}{2x^2-3x-1}$$

$$=(x^2-2x-2)(2x^4-7x^3-11x^2-8x+2)$$

$$=2x^6-3x^5-7x^4+28x^3-36x^2-20x-4$$

4. The left hand expression

$$=a^4-b^4+c^4-d^4-2(a^2-b^2)(c^2-d^2)-2a^2b^2+2c^2d^2$$
$$-2a^2c^2-2c^2d^2-8abcd$$
$$=(a^2-b^2-c^2-d^2)^2-4a^2b^2-4c^2d^2-8abcd$$
$$=(a^2-b^2-c^2-d^2)^2-4(a^2b^2-c^2d^2-2abcd)$$
$$=(a^2-b^2-c^2-d^2)^2-\{2(ab-cd)\}^2$$
$$=(a^2-b^2-c^2-d^2-2ab-2cd)(a^2+b^2-c^2-d^2-2ab-2cd)$$
$$=\{(a-b)^2-(c+d)^2\}\{(a-b)^2-(c-d)^2\}$$

5. $$x-yz=\frac{b^2+c^2-a^2}{2bc}-\frac{(c^2-a^2-b^2)(a^2-b^2-c^2)}{4a^2bc}$$

$$=\frac{2a^2b^2-2a^2c^2-2a^4-a^4-(b^2-c^2)^2}{4a^2bc}$$

$$=\frac{2a^2b^2-2a^2c^2+2b^2c^2-a^4-b^4-c^4}{4a^2bc}$$

$$y-zx=\frac{c^2-a^2-b^2}{2ac}+\frac{(a^2+b^2-c^2)(b^2-c^2-a^2)}{4ab^2c}$$

$$=\frac{2b^2c^2-2a^2b^2-2b^4-b^4-(a^2-c^2)^2}{4ab^2c}$$

$$=\frac{2b^2c^2+2a^2b^2-2a^2c^2-a^4-b^4-c^4}{4ab^2c}.$$

$$\frac{x+yz}{y+zx}=\frac{4ab^2c}{4a^2bc}=\frac{b}{a}$$

6
$$\frac{1}{a+bc}+\frac{1}{b+ca}+\frac{1}{c+ab}$$
$$=\frac{1}{1-b-c+bc}+\frac{1}{1-a-c+ca}+\frac{1}{1-a-b+ab}$$
$$[\quad a+b+c=1$$
$$=\frac{1}{(1-b)(1-c)}+\frac{1}{(1-a)(1-c)}+\frac{1}{(1-a)(1-b)}$$
$$=\frac{1-a+1-b+1-c}{(1-a)(1-b)(1-c)}=\frac{3-(a+b+c)}{1-(a+b+c)+(ab+ac+bc)-abc}$$
$$=\frac{3-1}{1-1+[illegible]-[illegible]}\qquad\left[\begin{array}{r}\text{Since } ab+ac+bc=[illegible]\\ \text{and } abc=[illegible].\end{array}\right]$$
$$=\frac{2}{[illegible]}=[illegible]$$

7 By adding (2) and (3), we have
$$\frac{20}{x}-\frac{29}{4y}=\frac{77}{12}$$
or, $$\frac{80}{x}-\frac{29}{y}=\frac{77}{3}\qquad(4)$$
Multiplying (1) by 2 and (3) by 3, we have
$$\frac{8}{x}+\frac{12}{y}+\frac{3}{z}=\frac{48}{5}$$
and $$\frac{75}{x}-\frac{15}{y}-\frac{3}{z}=8\ ,$$
hence, $$\frac{83}{x}-\frac{3}{y}=\frac{88}{5}\qquad(5)$$
Now multiplying (4) by 3 and (5) by 29, we have
$$\frac{240}{x}-\frac{87}{y}=77$$
and $$\frac{2407}{x}-\frac{87}{y}=\frac{2552}{5}\ ,$$
hence, $$\frac{2167}{x}=\frac{2167}{5},\qquad x=5$$
Therefore from (4) $\frac{29}{y}=16-\frac{77}{3}=\frac{-29}{3},\qquad y=-3$

Therefore from (2) $-1+\frac{3}{4}+\frac{1}{z}=3$ or, $\frac{1}{z}=4,\qquad z=\frac{1}{4}$

Thus $x=5$, $y=-3$ and $z=\frac{1}{4}$

8 Suppose A and B travelled at x and y miles per hour respectively Then in 5 hours A walked $5x$ miles and B, $5y$ miles

The distence is therefore $5x + 5y = 5(x + y)$

Then we have $\quad \dfrac{5x}{x+1} = 4 \qquad (1)$

and $\quad \dfrac{5y}{y-2} = 6 \qquad (2)$

From (1), $\quad 5x = 4x + 4, \quad x = 4$

From (2), $\quad 5y = 6y - 6, \quad y = 6$

$5(x + y) = 5(4 + 6) = 50$

Thus the distance between the two places is 50 miles

Exercise (78)

1 $\quad a^{\frac{6}{7}} = \sqrt[7]{a^6}$

2 $\quad x^{-\frac{8}{9}} = \dfrac{1}{x^{\frac{8}{9}}} = \dfrac{1}{\sqrt[9]{x^8}}$

3 $\quad \dfrac{3}{x^{-\frac{4}{5}}} = 3x^{\frac{4}{5}} = 3\sqrt[5]{x^4}$

4 $\quad x^{-\frac{2}{5}} \times 3a^{-\frac{1}{2}} = \dfrac{1}{x^{\frac{2}{5}}} \dfrac{3}{a^{\frac{1}{2}}} = \dfrac{3}{\sqrt[5]{x^2}\sqrt{a}}$

5 $\quad 8m^{-2} \times m^{-\frac{2}{3}} = 8m^{-(2+\frac{2}{3})} = 8m^{-\frac{8}{3}} = \dfrac{8}{m^{\frac{8}{3}}} = \dfrac{8}{\sqrt[3]{m^8}}$

6 $\quad x^{-\frac{4}{5}} - 3a^{-\frac{6}{4}} = \dfrac{1}{x^{\frac{4}{5}}} - \dfrac{3}{a^{\frac{6}{4}}} = \dfrac{1}{\sqrt[5]{x^4}} - \dfrac{3}{\sqrt[4]{a^6}} = \dfrac{\sqrt[4]{a^6}}{3\sqrt[5]{x^4}}$

7 $\quad x^{-\frac{2}{3}} - 2x^{-\frac{1}{2}} = \dfrac{1}{x^{\frac{2}{3}}} - \dfrac{2}{x^{\frac{1}{2}}} = \dfrac{1}{x^{\frac{2}{3}}} \times \dfrac{x^{\frac{1}{2}}}{2}$

$$= \frac{1}{2x^{\frac{2}{3}-\frac{1}{2}}} = \frac{1}{2x^{\frac{1}{6}}} = \frac{1}{2\sqrt[6]{x}}$$

8 $$\sqrt[5]{x^2} \div \sqrt[5]{x^{-a}} = \frac{\sqrt[5]{x^2}}{\sqrt[5]{x^{-a}}} = \sqrt[5]{\frac{x^2}{x^{-a}}} = \sqrt[5]{x^{2+a}}$$

9 $$\sqrt[2m]{a^{-5}} \times \sqrt[m]{a^8} = a^{\frac{-5}{2m}} \times a^{\frac{8}{m}} = a^{-\frac{5}{2m}+\frac{8}{m}} = a^{\frac{-5+16}{2m}} = a^{\frac{11}{2m}} = \sqrt[2m]{a^{11}}$$

10 $$\sqrt[4a]{x^6} \div \sqrt[2a]{x^{-5}} = x^{\frac{6}{4a}} \div x^{\frac{-5}{2a}} = x^{\frac{3}{2a}} \times x^{\frac{5}{2a}} = x^{\frac{3}{2a}+\frac{5}{2a}} = x^{\frac{8}{2a}} = x^{\frac{4}{a}} = \sqrt[a]{x^4}$$

11 $$\left(\sqrt[3]{x}\right)^7 = \left(x^{\frac{1}{3}}\right)^7 = x^{\frac{7}{3}}$$

12 $$\left(\sqrt[4]{a}\right)^{-6} = \left(a^{\frac{1}{4}}\right)^{-6} = a^{-\frac{3}{2}} = \frac{1}{a^{\frac{3}{2}}}$$

13 $$\frac{1}{\sqrt[3]{x^{-2}}} = \frac{1}{x^{-\frac{2}{3}}} = x^{\frac{2}{3}}.$$

14 $$\frac{1}{(\sqrt[5]{a})^{-2}} = \frac{1}{\left(a^{\frac{1}{5}}\right)^{-2}} = \frac{1}{a^{-\frac{2}{5}}} = a^{\frac{2}{5}}$$

15 $$\sqrt[4]{x^3} \div \left(\sqrt[6]{x}\right)^{-1} = x^{\frac{3}{4}} \div \left(x^{\frac{1}{6}}\right)^{-1}$$
$$= x^{\frac{3}{4}} \div x^{-\frac{1}{6}} = x^{\frac{3}{4}+\frac{1}{6}} = x^{\frac{11}{12}}$$

16 $$\sqrt[4]{a^{-3}} \div \left(\sqrt[8]{a}\right)^{-12} = a^{-\frac{3}{4}} \div \left(a^{\frac{1}{8}}\right)^{-12}$$
$$= a^{-\frac{3}{4}} \div a^{-\frac{12}{8}} = a^{-\frac{3}{4}+\frac{3}{2}} = a^{\frac{3}{4}}$$

17 $$4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{\sqrt{4^3}} = \frac{1}{\sqrt[2]{64}} = \frac{1}{8}$$

18 $$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

19 $$9^{\frac{3}{2}} = \sqrt{9^3} = \sqrt{9 \times 9 \times 9} = 3 \times 3 \times 3 = 27$$

20 $$16^{\frac{5}{4}} = \sqrt[4]{16^5} = \sqrt[4]{(2^4)^5} = 2^5 = 32$$

21 $$81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}} = \frac{1}{(\sqrt[4]{81})^3} = \frac{1}{3^3} = \frac{1}{27}$$

22 $$\frac{1}{6^{-2}} = 6^2 = 36$$

23 $(125)^{-\frac{2}{3}}=\frac{1}{(125)^{\frac{2}{3}}}=\frac{1}{(\sqrt[3]{125})^2}=\frac{1}{5^2}=\frac{1}{25}$

24 $\left(\frac{1}{27}\right)^{-\frac{4}{3}}=27^{\frac{4}{3}}=\left(\sqrt[3]{27}\right)^4=(3)^4=81$

25 $\left(\frac{1}{216}\right)^{-\frac{2}{3}}=(216)^{\frac{2}{3}}=\left(\sqrt[3]{216}\right)^2=(6)^2=36$

26 $\frac{x^{m+2n}x^{3m-8n}}{x^{5m-6n}}=\frac{x^{m+2n+3m-8n}}{x^{5m-6n}}=x^{4m-6n-(5m-6n)}=x^{-m}$

Exercise (79)

1 $\left(a^{-\frac{3}{4}}\right)^8=a^{\left(-\frac{3}{4}\right)\times 8}=a^{-(3\times 2)}=a^{-6}$

2 $\left(a^{-\frac{2}{3}}b^{\frac{5}{6}}\right)^{\frac{3}{4}}=a^{\left(-\frac{2}{3}\right)\times\frac{3}{4}}b^{\frac{5}{6}\times\frac{3}{4}}=a^{-\frac{1}{2}}b^{\frac{5}{8}}$

3 $\left(a^{-\frac{1}{2}}b^{-3}\right)^{-2}=a^{\left(-\frac{1}{2}\right)\times(-2)}b^{(-3)\times(-2)}=ab^6$

4 $\left(a^6b^{\frac{5}{4}}\right)^{-\frac{4}{3}}=a^{6\times\left(-\frac{4}{3}\right)}b^{\frac{5}{4}\times\left(-\frac{4}{3}\right)}=a^{-8}b^{-\frac{5}{3}}$

5 $\left(\sqrt[3]{a^4b^3}\right)^6=\left(a^{\frac{4}{3}}b\right)^6=a^8b^6$

6 $\left(\sqrt[6]{x^9y^{-8}}\right)^{-3}=\left(x^{\frac{9}{6}}y^{-\frac{8}{6}}\right)^{-3}=x^{-\frac{9}{2}}y^4$

7 $\sqrt[8]{x^2\sqrt[4]{x^{-3}}}=\sqrt[8]{x^2x^{-\frac{3}{4}}}=\sqrt[8]{x^{2-\frac{3}{4}}}=\sqrt[8]{x^{\frac{5}{4}}}=x^{\frac{5}{32}}$

8 $\sqrt{a^{-3}b^4}\times\sqrt[4]{a^2b^{-8}}=a^{-\frac{3}{2}}b^2\times a^{\frac{1}{2}}b^{-2}=a^{\frac{1}{2}-\frac{3}{2}}=a^{-1}$

9 $\sqrt[4]{x^{-2}\sqrt{y^5}}\times\sqrt{x\sqrt[4]{y^3}}=\sqrt[4]{x^{-2}y^{\frac{5}{2}}}\times\sqrt{xy^{\frac{3}{4}}}$

$=x^{-\frac{1}{2}}y^{\frac{5}{8}}\times x^{\frac{1}{2}}y^{\frac{3}{8}}=y^{\frac{5}{8}+\frac{3}{8}}=y$

10 $(8x^3-27a^{-3})^{\frac{2}{3}}=\left(\frac{8x^3}{27a^{-3}}\right)^{\frac{2}{3}}=\left(\frac{8}{27}x^3a^3\right)^{\frac{2}{3}}$

$=\left(\frac{2^3}{3^3}x^3a^3\right)^{\frac{2}{3}}=\frac{2^2}{3^2}x^2a^2=\frac{4}{9}x^2a^2$

11 $\left(64x^3-27a^{-3}\right)^{-\frac{2}{3}}=\left(\frac{4^3x^3}{3^3a^{-3}}\right)^{-\frac{2}{3}}=\left(\frac{4^3x^3a^3}{3^3}\right)^{-\frac{2}{3}}$

$=\left(\frac{3^3}{4^3}x^{-3}a^{-3}\right)^{\frac{2}{3}}=\frac{3^2}{4^2}x^{-2}a^{-2}=\frac{9}{16}x^{-2}a^{-2}$

12 $\sqrt[3]{a^6b^{-2}c^{-4}}\times\sqrt[4]{a^{-4}b^4c^8}=a^2b^{-\frac{2}{3}}c^{-\frac{4}{3}}\times a^{-1}bc^2$

$$=a^{2-1}b^{-\frac{2}{3}+1}c^{-\frac{4}{3}+2}=a^{1}b^{\frac{1}{3}}c^{\frac{2}{3}}$$

13 $\sqrt{a^{-\frac{2}{3}}b^4c^{-\frac{1}{3}}}\div\sqrt[3]{a^2b^4c^{-1}}=a^{-\frac{1}{3}}b^2c^{-\frac{1}{6}}\div a^{\frac{2}{3}}b^{\frac{4}{3}}c^{-\frac{1}{3}}$

$$=a^{-\frac{1}{3}-\frac{2}{3}}b^{2-\frac{4}{3}}c^{-\frac{1}{6}+\frac{1}{3}}=a^{-1}b^{\frac{2}{3}}c^{\frac{1}{6}}$$

14 $\sqrt{ab^{-2}c^3}\div(\sqrt[3]{a^3b^{-2}c^{-3}})^{-1}=\left(a^{\frac{1}{2}}b^{-1}c^{\frac{3}{2}}\right)\times\left(ab^{\frac{2}{3}}c^{-1}\right),$

$$=a^{\frac{1}{2}+1}b^{-1+\frac{2}{3}}c^{\frac{3}{2}-1}=a^{\frac{3}{2}}b^{-\frac{1}{3}}c^{\frac{1}{2}}$$

15 $\left(\dfrac{a^{-1}b^2}{a^2b^{-4}}\right)^7\div\left(\dfrac{a^3b^{-5}}{a^{-2}b^3}\right)^{-5}=\dfrac{a^{-7}b^{14}}{a^{14}b^{-28}}\times\dfrac{a^{15}b^{-20}}{a^{-10}b^{15}}=\dfrac{a^8b^{-11}}{a^4b^{-13}}=a^4b^2$

Exercise (80)

1
$$\begin{array}{l}
x^{\frac{2}{3}}+2x^{\frac{1}{2}}+3x^{\frac{1}{3}}+2x^{\frac{1}{6}}+1\\
\underline{x^{\frac{1}{3}}-2x^{\frac{1}{6}}+1\qquad\qquad\qquad}\\
x+2x^{\frac{5}{6}}+3x^{\frac{2}{3}}+2x^{\frac{1}{2}}+x^{\frac{1}{3}}\\
\quad-2x^{\frac{5}{6}}-4x^{\frac{2}{3}}-6x^{\frac{1}{2}}-4x^{\frac{1}{3}}-2x^{\frac{1}{6}}\\
\underline{\qquad\qquad\quad+x^{\frac{2}{3}}+2x^{\frac{1}{2}}+3x^{\frac{1}{3}}+2x^{\frac{1}{6}}+1}\\
x\qquad\qquad\quad-2x^{\frac{1}{2}}\qquad\qquad\qquad+1
\end{array}$$

2
$$\begin{array}{l}
a^{\frac{2}{3}}+3a^{\frac{1}{3}}b^{\frac{1}{3}}+9b^{\frac{2}{3}}\\
\underline{a^{\frac{1}{3}}-3b^{\frac{1}{3}}\qquad\qquad}\\
a+3a^{\frac{2}{3}}b^{\frac{1}{3}}+9a^{\frac{1}{3}}b^{\frac{2}{3}}\\
\underline{\quad-3a^{\frac{2}{3}}b^{\frac{1}{3}}-9a^{\frac{1}{3}}b^{\frac{2}{3}}-27b}\\
a\qquad\qquad\qquad\qquad-27b
\end{array}$$

3
$$\begin{array}{l}
1+ab^{-1}+a^2b^{-2}\\
\underline{1-ab^{-1}+a^2b^{-2}}\\
1+ab^{-1}+a^2b^{-2}\\
\quad-ab^{-1}-a^2b^{-2}-a^3b^{-3}\\
\underline{\qquad\qquad+a^2b^{-2}+a^3b^{-3}+a^4b^{-4}}\\
1\qquad\qquad+a^2b^{-2}\qquad\qquad+a^4b^{-4}
\end{array}$$

4

$$\begin{array}{l}
x+2y^{\frac12}+3z^{\frac13} \\
x-2y^{\frac12}+3z^{\frac13} \\
\hline
x^2+2xy^{\frac12}+3xz^{\frac13} \\
\quad -2xy^{\frac12} \qquad -4y-6y^{\frac12}z^{\frac13} \\
\qquad\qquad +3xz^{\frac13} \qquad +6y^{\frac12}z^{\frac13}+9z^{\frac23} \\
\hline
x^2 \qquad +6xz^{\frac13}-4y \qquad +9z^{\frac23}
\end{array}$$

5

$$\begin{array}{l}
x^{-1}+x^{-\frac12}y^{-\frac12}+y^{-1} \\
x^{-1}-x^{-\frac12}y^{-\frac12}+y^{-1} \\
\hline
x^{-2}+x^{-\frac32}y^{-\frac12}+x^{-1}y^{-1} \\
\quad -x^{-\frac32}y^{-\frac12}-x^{-1}y^{-1}-x^{-\frac12}y^{-\frac32} \\
\qquad\qquad +x^{-1}y^{-1}+x^{-\frac12}y^{-\frac32}+y^{-2} \\
\hline
x^{-2} \qquad +x^{-1}y^{-1} \qquad +y^{-2}
\end{array}$$

6

$$\begin{array}{l}
a^{\frac23}-a^{\frac13}+1-a^{-\frac13}+a^{-\frac23} \\
a^{\frac13}+1+a^{-\frac13} \\
\hline
a-a^{\frac23}+a^{\frac13}-1+a^{-\frac13} \\
\quad +a^{\frac23}-a^{\frac13}+1-a^{-\frac13}+a^{-\frac23} \\
\qquad +a^{\frac13}-1+a^{-\frac13}-a^{-\frac23}+a^{-1} \\
\hline
a \qquad +a^{\frac13}-1+a^{-\frac13} \qquad +a^{-1}
\end{array}$$

7

$$\begin{array}{l}
x^{\frac23}-x^{\frac13}y^{\frac13}-x^{\frac13}z^{\frac13}+y^{\frac23}-y^{\frac13}z^{\frac13}+z^{\frac23} \\
x^{\frac13}+y^{\frac13}+z^{\frac13} \\
\hline
x-x^{\frac23}y^{\frac13}-x^{\frac23}z^{\frac13}+x^{\frac13}y^{\frac23}-x^{\frac13}y^{\frac13}z^{\frac13}+x^{\frac13}z^{\frac23} \\
\quad +x^{\frac23}y^{\frac13} \qquad -x^{\frac13}y^{\frac23}-x^{\frac13}y^{\frac13}z^{\frac13} \qquad +y-y^{\frac23}z^{\frac13}+y^{\frac13}z^{\frac23} \\
\qquad\qquad +x^{\frac23}z^{\frac13} \qquad -x^{\frac13}y^{\frac13}z^{\frac13}-x^{\frac13}z^{\frac23} \quad +y^{\frac23}z^{\frac13}-y^{\frac13}z^{\frac23}+z \\
\hline
x \qquad\qquad -3x^{\frac13}y^{\frac13}z^{\frac13} \qquad +y \qquad +z
\end{array}$$

8

$$\begin{array}{l}
a^{m}+3b^{n}-2c^{p} \\
a^{m}-3b^{n}+2c^{p} \\
\hline
a^{2m}+3a^{m}b^{n}-2a^{m}c^{p} \\
\quad -3a^{m}b^{n} \quad -9b^{2n}+6b^{n}c^{p} \\
\quad\quad +2a^{m}c^{p} \quad +6b^{n}c^{p}-4c^{2p} \\
\hline
a^{2m} \quad -9b^{2n}+12b^{n}c^{p}-4c^{2p}
\end{array}$$

9

$$\begin{array}{l}
a^{\frac{5}{2}}+2a^{2}b^{\frac{1}{3}}+4a^{\frac{3}{2}}b^{\frac{2}{3}}+8ab+16a^{\frac{1}{2}}b^{\frac{4}{3}}+32b^{\frac{5}{3}} \\
a^{\frac{1}{2}}-2b^{\frac{1}{3}} \\
\hline
a^{3}+2a^{\frac{5}{2}}b^{\frac{1}{3}}+4a^{2}b^{\frac{2}{3}}+8a^{\frac{3}{2}}b+16ab^{\frac{4}{3}}+32a^{\frac{1}{2}}b^{\frac{5}{3}} \\
\quad -2a^{\frac{5}{2}}b^{\frac{1}{3}}-4a^{2}b^{\frac{2}{3}}-8a^{\frac{3}{2}}b-16ab^{\frac{4}{3}}-32a^{\frac{1}{2}}b^{\frac{5}{3}}-64b^{2} \\
\hline
a^{3} \quad -64b^{2}
\end{array}$$

10

[illegible]

$$a \quad +a^{\frac{1}{4}}x^{-\frac{1}{4}} \quad -a^{\frac{1}{4}}x^{\frac{1}{4}} \quad -x^{-1}$$

11

Quotient $= x-x^{\frac{1}{2}}$

$$\begin{array}{l}
x^{\frac{3}{2}}-4x^{\frac{1}{2}}+2\ \big)\ x^{\frac{5}{2}}-x^{2}-4x^{\frac{3}{2}}+6x-2x^{\frac{1}{2}} \\
\qquad\qquad x^{\frac{5}{2}} \quad -4x^{\frac{3}{2}}+2x \\
\hline
\qquad\qquad -x^{2} \quad +4x-2x^{\frac{1}{2}} \\
\qquad\qquad -x^{2} \quad +4x-2x^{\frac{1}{2}} \\
\hline
\end{array}$$

12

Quotient $=2+4x^{-1}+2x^{-2}$

$$\begin{array}{l}
4-2x^{-1}+x^{-2}\ \big)\ 8+12x^{-1}+2x^{-2}+2x^{-4} \\
\qquad\qquad 8-4x^{-1}+2x^{-2} \\
\hline
\qquad\qquad 16x^{-1} \quad +2x^{-4} \\
\qquad\qquad 16x^{-1}-8x^{-2}+4x^{-3} \\
\hline
\qquad\qquad 8x^{-2}-4x^{-3}+2x^{-4} \\
\qquad\qquad 8x^{-2}-4x^{-3}+2x^{-4} \\
\hline
\end{array}$$

13 Quotient $= y + x^{\frac{1}{2}}y^{\frac{1}{2}} + x$

$$x^{-1}+x^{-\frac{1}{2}}y^{-\frac{1}{2}}+y^{-1}\ \big)\ x^{-1}y+2x^{-\frac{1}{2}}y^{\frac{1}{2}}+3+2x^{\frac{1}{2}}y^{-\frac{1}{2}}+xy^{-1}$$
$$\underline{x^{-1}y+x^{-\frac{1}{2}}y^{\frac{1}{2}}+1}$$
$$x^{-\frac{1}{2}}y^{\frac{1}{2}}+2+2x^{\frac{1}{2}}y^{-\frac{1}{2}}+xy^{-1}$$
$$\underline{x^{-\frac{1}{2}}y^{\frac{1}{2}}+1+x^{\frac{1}{2}}y^{-\frac{1}{2}}}$$
$$1+x^{\frac{1}{2}}y^{-\frac{1}{2}}+xy^{-1}$$
$$\underline{1+x^{\frac{1}{2}}y^{-\frac{1}{2}}+xy^{-1}}$$

14 Quotient $= a + a^{\frac{1}{2}}b^{\frac{1}{2}} - b$

$$a^{\frac{3}{2}}-ab^{\frac{1}{2}}+a^{\frac{1}{2}}b-b^{\frac{3}{2}}\ \big)\ a^{\frac{5}{2}}-a^{\frac{3}{2}}b+ab^{\frac{3}{2}}-2a^{\frac{1}{2}}b^2+b^{\frac{5}{2}}$$
$$\underline{a^{\frac{5}{2}}-a^2b^{\frac{1}{2}}+a^{\frac{3}{2}}b-ab^{\frac{3}{2}}}$$
$$a^2b^{\frac{1}{2}}-2a^{\frac{3}{2}}b+2ab^{\frac{3}{2}}-2a^{\frac{1}{2}}b^2+b^{\frac{5}{2}}$$
$$\underline{a^2b^{\frac{1}{2}}-a^{\frac{3}{2}}b+ab^{\frac{3}{2}}-a^{\frac{1}{2}}b^2}$$
$$-a^{\frac{3}{2}}b+ab^{\frac{3}{2}}-a^{\frac{1}{2}}b^2+b^{\frac{5}{2}}$$
$$\underline{-a^{\frac{3}{2}}b+ab^{\frac{3}{2}}-a^{\frac{1}{2}}b^2+b^{\frac{5}{2}}}$$

15 Quotient $= x^{2n} - 1 + x^{-2n}$

$$5x^{n}-3x^{-n}\ \big)\ 5x^{3n}-8x^{n}+8x^{-n}-3x^{-3n}$$
$$\underline{5x^{3n}-3x^{n}}$$
$$-5x^{n}+8x^{-n}-3x^{-3n}$$
$$\underline{-5x^{n}-3x^{-n}}$$
$$5x^{-n}-3x^{-3n}$$
$$\underline{5x^{-n}-3x^{-3n}}$$

16 Putting a for $2x^{\frac{1}{3}}$, b for $y^{-\frac{1}{3}}$ and c for $-z^{\frac{1}{3}}$, we have

$$8x+y^{-1}-z+6x^{\frac{1}{3}}y^{-\frac{1}{3}}z^{\frac{1}{3}}$$
$$=a^3+b^3+c^3-3abc$$
$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$
$$=\left(2x^{\frac{1}{3}}+y^{-\frac{1}{3}}-z^{\frac{1}{3}}\right)\left\{\left(2x^{\frac{1}{3}}\right)^2+\left(y^{-\frac{1}{3}}\right)^2+\left(-z^{\frac{1}{3}}\right)^2\right.$$
$$\left.-2x^{\frac{1}{3}}y^{-\frac{1}{3}}+2x^{\frac{1}{3}}z^{\frac{1}{3}}+y^{-\frac{1}{3}}z^{\frac{1}{3}}\right\}$$
$$=\left(2x^{\frac{1}{3}}+y^{-\frac{1}{3}}-z^{\frac{1}{3}}\right)\left(4x^{\frac{2}{3}}+y^{-\frac{2}{3}}+z^{\frac{2}{3}}-2x^{\frac{1}{3}}y^{-\frac{1}{3}}+2x^{\frac{1}{3}}z^{\frac{1}{3}}+y^{-\frac{1}{3}}z^{\frac{1}{3}}\right)$$

Thus the quotient $=4x^{\frac{2}{3}}-2x^{\frac{1}{3}}y^{-\frac{1}{3}}+2x^{\frac{1}{3}}z^{\frac{1}{3}}+y^{-\frac{2}{3}}+y^{-\frac{1}{3}}z^{\frac{1}{3}}+z^{\frac{2}{3}}$

$$\text{Quotient}=x^{\frac{9}{4}}-x^{\frac{15}{8}}a^{\frac{3}{8}}+x^{\frac{9}{8}}a^{\frac{9}{8}}-x^{\frac{3}{8}}a^{\frac{15}{8}}+a^{\frac{9}{4}}$$

17 $\quad x^{\frac{3}{4}}+x^{\frac{3}{8}}a^{\frac{3}{8}}+a^{\frac{3}{4}} \Big)\ x^{3}+x^{\frac{3}{2}}a^{\frac{3}{2}}+a^{3}$

$$
\begin{array}{l}
\underline{x^{3}+x^{\frac{21}{8}}a^{\frac{3}{8}}+x^{\frac{9}{4}}a^{\frac{3}{4}}} \\
-x^{\frac{21}{8}}a^{\frac{3}{8}}-x^{\frac{9}{4}}a^{\frac{3}{4}}+x^{\frac{3}{2}}a^{\frac{3}{2}}+a^{3} \\
\underline{-x^{\frac{21}{8}}a^{\frac{3}{8}}-x^{\frac{9}{4}}a^{\frac{3}{4}}-x^{\frac{15}{8}}a^{\frac{9}{8}}} \\
x^{\frac{15}{8}}a^{\frac{9}{8}}+x^{\frac{3}{2}}a^{\frac{3}{2}}+a^{3} \\
\underline{x^{\frac{15}{8}}a^{\frac{9}{8}}+x^{\frac{3}{2}}a^{\frac{3}{2}}+x^{\frac{9}{8}}a^{\frac{15}{8}}} \\
-x^{\frac{9}{8}}a^{\frac{15}{8}}+a^{3} \\
\underline{-x^{\frac{9}{8}}a^{\frac{15}{8}}-x^{\frac{3}{4}}a^{\frac{9}{4}}-x^{\frac{3}{8}}a^{\frac{21}{8}}} \\
x^{\frac{3}{4}}a^{\frac{9}{4}}+x^{\frac{3}{8}}a^{\frac{21}{8}}+a^{3} \\
\underline{x^{\frac{3}{4}}a^{\frac{9}{4}}+x^{\frac{3}{8}}a^{\frac{21}{8}}+a^{3}}
\end{array}
$$

18 $$\left(x^{2^{n-1}}+a^{2^{n-1}}\right)\left(x^{2^{n-1}}-a^{2^{n-1}}\right)$$

$$=\left(x^{2^{n-1}}\right)^{2}-\left(a^{2^{n-1}}\right)^{2}$$

$$=x^{2\cdot 2^{n-1}}-a^{2\cdot 2^{n-1}}=x^{2^{1+n-1}}-a^{2^{1+n-1}}=x^{2n}+a^{2n}$$

$$\text{Quotient}=x^{2^{n-1}}-y^{2^{n-1}}$$

19 $\quad x^{2^{n-1}}+y^{2^{n-1}} \Big)\ x^{2^{n}}-y^{2^{n}}$

$$
\begin{array}{l}
\underline{x^{2^{n}}+x^{2^{n-1}}y^{2^{n-1}}} \\
-x^{2^{n-1}}y^{2^{n-1}}-y^{2^{n}} \\
\underline{-x^{2^{n-1}}y^{2^{n-1}}-y^{2^{n}}}
\end{array}
$$

20 $\left\{(a^n)^{n-\frac{1}{n}}\right\}^{\frac{1}{n+1}}$

$$=\left\{(a^n)^{\frac{n^2-1}{n}}\right\}^{\frac{1}{n+1}}$$

$$=(a^n)^{\frac{n^2-1}{n}\times\frac{1}{n+1}}=a^{n\times\frac{n-1}{n}}=a^{n-1}$$

21 Quotient $=x^{\frac{3}{4}}+3x^{\frac{1}{2}}-1$

$$
\begin{array}{r|l}
x^{\frac14}-2x^{-\frac14} & x+3x^{\frac34}-2x^{\frac12}-7x^{\frac14}+2x^{-\frac14} \\
 & \underline{x \qquad\quad -2x^{\frac12}} \\
 & 3x^{\frac34} \qquad -7x^{\frac14}+2x^{-\frac14} \\
 & \underline{3x^{\frac34} \qquad -6x^{\frac14}} \\
 & -x^{\frac14}+2x^{-\frac14} \\
 & \underline{-x^{\frac14}+2x^{-\frac14}}
\end{array}
$$

22

$$
\begin{array}{l}
x^{\frac34}-x^{\frac12}y^{-\frac14}+y^{\frac12} \\
\underline{x^{\frac34}-x^{\frac12}y^{-\frac14}+y^{\frac12}} \\
x^{\frac32}-x^{\frac54}y^{-\frac14}+x^{\frac34}y^{\frac12} \\
\qquad -x^{\frac54}y^{-\frac14}+xy^{-\frac12} \qquad -x^{\frac12}y^{\frac14} \\
\qquad\qquad\quad \underline{+x^{\frac34}y^{\frac12} \qquad -x^{\frac12}y^{\frac14}+y} \\
x^{\frac32}-2x^{\frac54}y^{-\frac14}+xy^{-\frac12}+2x^{\frac34}y^{\frac12}-2x^{\frac12}y^{\frac14}+y
\end{array}
$$

23 Quotient $=x^n+x^{\frac{n}{2}}a^{\frac{n}{2}}+a^n$

$$
\begin{array}{r|l}
x^{\frac n2}-a^{\frac n2} & x^{\frac{3n}{2}}-a^{\frac{3n}{2}} \\
 & \underline{x^{\frac{3n}{2}}-x^n a^{\frac n2}} \\
 & x^n a^{\frac n2}-a^{\frac{3n}{2}} \\
 & \underline{x^n a^{\frac n2}-x^{\frac n2}a^n} \\
 & x^{\frac n2}a^n-a^{\frac{3n}{2}} \\
 & \underline{x^{\frac n2}a^n-a^{\frac{3n}{2}}}
\end{array}
$$

24.

$$
\begin{array}{l}
x^{1} - 2x^{\frac{1}{2}} + x^{'} \\
x^{\frac{1}{2}} - 2x^{\frac{1}{4}} - x^{'} \\
\hline
x^{\frac{3}{2}} - 2x^{\frac{5}{4}} - x^{'} \\
\qquad - 2x^{'} - 4x \qquad\qquad - 2x^{\frac{1}{4}} \\
\qquad\qquad - x^{'} \qquad\qquad - 2x^{\frac{1}{4}} - x \\
\hline
x^{\frac{3}{2}} - 4x^{\frac{5}{4}} - 4x - 2x^{'} - 3x^{\frac{1}{2}} - x^{\frac{1}{4}}
\end{array}
$$

25. Quotient $= a^{\frac{2}{3}}x^{-\frac{2}{3}} - a^{\frac{1}{3}}x^{-\frac{1}{3}} - a^{-\frac{1}{3}}x^{\frac{1}{3}} - a^{-\frac{2}{3}}x^{\frac{2}{3}}$

$$
\begin{array}{l}
a^{\frac{1}{3}}x^{-\frac{1}{3}} - 1 - a^{-\frac{1}{3}}x^{\frac{1}{3}} \;\big)\; ax^{-1} - 2 - a^{-1}x \\
\qquad ax^{-1} - a^{\frac{2}{3}}x^{-\frac{2}{3}} - a^{\frac{1}{3}}x^{-\frac{1}{3}} \\
\hline
\qquad a^{\frac{2}{3}}x^{-\frac{2}{3}} - a^{\frac{1}{3}}x^{-\frac{1}{3}} + 2 - a^{-1}x \\
\qquad a^{\frac{2}{3}}x^{-\frac{2}{3}} - a^{\frac{1}{3}}x^{-\frac{1}{3}} + 1 \\
\hline
\qquad 1 \qquad\qquad - a^{-1}x \\
\qquad 1 - a^{-\frac{1}{3}}x^{\frac{1}{3}} - a^{-\frac{2}{3}}x^{\frac{2}{3}} \\
\hline
\qquad a^{-\frac{1}{3}}x^{\frac{1}{3}} - a^{-\frac{2}{3}}x^{\frac{2}{3}} + a^{-1}x \\
\qquad a^{-\frac{1}{3}}x^{\frac{1}{3}} - a^{-\frac{2}{3}}x^{\frac{2}{3}} - a^{-1}x
\end{array}
$$

26.

$$\left(\frac{a-b}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}-\frac{a^{\frac{3}{2}}-b^{\frac{3}{2}}}{a-b}\right)^{-1}$$

$$=\left\{\frac{(a-b)(a^{\frac{1}{2}}-b^{\frac{1}{2}})-(a^{\frac{3}{2}}-b^{\frac{3}{2}})}{a-b}\right\}^{-1}$$

$$=\left[\frac{(a^{\frac{1}{2}}-b^{\frac{1}{2}})\{(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{2}-(a+a^{\frac{1}{2}}b^{\frac{1}{2}}+b)\}}{(a^{\frac{1}{2}}-b^{\frac{1}{2}})(a^{\frac{1}{2}}-b^{\frac{1}{2}})}\right]^{-1}$$

$$=\left(\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}\right)^{-1}=\frac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a^{\frac{1}{2}}b^{\frac{1}{2}}}=\frac{1}{b^{\frac{1}{2}}}-\frac{1}{a^{\frac{1}{2}}}=\frac{1}{\sqrt{a}}-\frac{1}{\sqrt{b}}$$

27. The given expression

$$=\frac{(x^{\frac{1}{3}}+3y^{\frac{1}{3}})(x^{\frac{2}{3}}-3x^{\frac{1}{3}}y^{\frac{1}{3}}+9y^{\frac{2}{3}})-(x^{\frac{1}{3}}-3y^{\frac{1}{3}})(x^{\frac{2}{3}}-3x^{\frac{1}{3}}y^{\frac{1}{3}}-9y^{\frac{2}{3}})}{x-27y}$$

$$=\frac{(x+27y)+6x^{\frac{1}{3}}y^{\frac{1}{3}}\left(x^{\frac{1}{3}}+3y^{\frac{1}{3}}\right)+(x-27y)-6x^{\frac{1}{3}}y^{\frac{1}{3}}\left(x^{\frac{1}{3}}-3y^{\frac{1}{3}}\right)}{x-27y}$$

$$=\frac{2x+36x^{\frac{1}{3}}y^{\frac{2}{3}}}{x-27y}$$

28 The numerator

$$=a\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)+x\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)=\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)(a+x)$$

The denominator

$$=a^2\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)+3ax\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)+x^2\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)$$

$$=\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)\left(a^2+3ax+x^2\right)$$

Thus the given expression

$$=\frac{\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)(a+x)}{\left(a^{\frac{1}{2}}-x^{\frac{1}{2}}\right)\left(a^2+3ax+x^2\right)}=\frac{a+x}{a^2+3ax+x^2}$$

29 The given expression

$$=\frac{a^2+b^2-a^{-2}-b^{-2}+(ab-a^{-1}b-ab^{-1}+a^{-1}b^{-1})(ab-a^{-1}b^{-1})}{a^2b^2-a^{-2}b^{-2}}$$

$$=\frac{a^2+b^2-a^{-2}-b^{-2}+(a^2b^2-a^{-2}b^{-2})-(a^{-1}b+ab^{-1})(ab-a^{-1}b^{-1})}{a^2b^2-a^{-2}b^{-2}}$$

$$=\frac{a^2+b^2-a^{-2}-b^{-2}+(a^2b^2-a^{-2}b^{-2})-a^2-b^2+b^{-2}+a^{-2}}{a^2b^2-a^{-2}b^{-2}}$$

$$=\frac{a^2b^2-a^{-2}b^{-2}}{a^2b^2-a^{-2}b^{-2}}=1$$

30 The given expression

$$=\frac{\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)}{x^{\frac{1}{2}}\left(x^{\frac{1}{4}}+y^{\frac{1}{4}}\right)}\times\frac{x^{\frac{1}{4}}y^{\frac{1}{4}}\left(x^{\frac{1}{4}}+y^{\frac{1}{4}}\right)}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}$$

$$=\frac{\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)y^{\frac{1}{4}}}{x^{\frac{1}{4}}}=\left(x^{\frac{1}{2}}y^{\frac{1}{4}}-y^{\frac{3}{4}}\right)x^{-\frac{1}{4}}=x^{\frac{1}{4}}y^{\frac{1}{4}}-x^{-\frac{1}{4}}y^{\frac{3}{4}}$$

31 The 1st term

$$=1+ab^{-1}+ac^{-1}+a^{-1}b+1+bc^{-1}+a^{-1}c+b^{-1}c+1$$

$$=3+ab^{-1}+ac^{-1}+a^{-1}b+a^{-1}c+bc^{-1}+b^{-1}c$$

The 2nd. term

$$=a^{-1}b^{-1}c^{-1}\left(a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+2abc\right)$$
$$=ac^{-1}+ab^{-1}+bc^{-1}+b^{-1}c+a^{-1}c+2$$

the given expression $=1$

32 $$2^{x+7}=4^{x+2}$$

or, $$2^{x+7}=2^{2(x+2)}$$

or, $$x+7=2x+4\,,\qquad x=3$$

33 $$(\sqrt{3})^{x+5}=(\sqrt[3]{3})^{2x+5}$$

or, $$3^{\frac{x+5}{2}}=3^{\frac{2x+5}{3}}$$

or, $$\frac{x+5}{2}=\frac{2x+5}{3}$$

or, $$3x+15=4x+10\,,\qquad x=5$$

34. $$(\sqrt[5]{4})^{4x+7}=(\sqrt[11]{64})^{2x+7}$$

or, $$(4)^{\frac{4x+7}{5}}=(64)^{\frac{2x+7}{11}}$$

or, $$(4)^{\frac{4x+7}{5}}=\{(4)^3\}^{\frac{2x+7}{11}}$$

or, $$(4)^{\frac{4x+7}{5}}=(4)^{\frac{3(2x+7)}{11}}$$

or, $$\frac{4x+7}{5}=\frac{6x+21}{11}$$

or, $$44x+77=30x+105$$

or, $$14x=28\,,\qquad x=2$$

35 $$(\sqrt[3]{25})^{2x+1}=(\sqrt[5]{125})^{x+6}$$

or $$(25)^{\frac{2x+1}{3}}=(125)^{\frac{x+6}{5}}$$

or, $$(5^2)^{\frac{2x+1}{3}}=\{(5)^3\}^{\frac{x+6}{5}}$$

or, $$(5)^{\frac{2(2x+1)}{3}}=(5)^{\frac{3(x+6)}{5}}$$

or, $\dfrac{2(2x+1)}{3}=\dfrac{3(x+6)}{5}$

or, $20x+10=9x+54$

or, $11x=44$, $x=4$

36 From (1), we have

$2^{3x-1}=2^{2(y-1)}$ or, $3x-1=2y-2$ or, $3x=2y-1$

Also we have $3x-y=1$ (2) $2y-1-y=1$ $y=2$

Hence, from (2), $3x=1+2=3$, $x=1$

Thus $x=1, y=2$

37 From (1), we have $(3)^{2(2x-3)}=3^{\frac{2y-x}{2}}$

or, $4x-6=\dfrac{2y-x}{2}$

or, $8x-12=2y-x$

or, $9x-12=2y$ (3)

From (2), we have $2^{3x}=2^{2y}$ or, $3x=2y$

Hence, from (3), $9x-12=3x$ or, $6x=12$, $x=2$

Hence $2y=6$, $y=3$

Thus $x=2, y=3$

38 From (1), we have $4^{3y-1}=4^{2(x+y)}$

or, $3y-1=2(x+y)$

or, $y=2x+1$ (3)

From (2), we have $3^{x+3y}=3^{2(2x+3)}$

or, $x+3y=4x+6$

or, $3y=3x+6$

or, $y=x+2$ (4)

Hence, from (3) and (4), $2x+1=x+2$, $x=1$

from (1) $y=1+2=3$

Thus $x=1, y=3$

39 From (1), we have $2^{x+y+z}=2^{3(x+z-y)}$

or, $x+y+z=3x+3z-3y$

or, $2x-4y+2z=0$

or, $x-2y+z=0$ (4)

From (2), we have $5^{3y+2}=5^{2(x+z)}$

or, $3y+2=2x+2z$

or, $2x-3y+2z=2$ (5)

also, $2x-4y+2z=0$ .. . from (4)

$y=2$

From (3), we have

$$3^{2x+2z+y}=3^{2(3x+y)}$$

or, $2x+2z+y=6x+2y$

or, $4x-2z+y=0$

$4x-2z=-2$

or, $2x-z=-1$

also, $x+z=4$ (4)

$3x=3,\quad x=1$,

and $z=4-1=3$

Thus $x=1,\ y=2\ \ z=3$

40 From (1), we have $a^{\frac{x+y}{2}}=a^{\frac{y+z-1}{3}}$

or $\dfrac{x+y}{2}=\dfrac{y+z-1}{3}$

or, $3x+3y=2y+2z-2$

or, $3x+y-2z=-2$ (4)

From (2), we have $b^{\frac{x+z-2}{3}}=b^{\frac{y+z}{5}}$

or, $\dfrac{x+z-2}{3}=\dfrac{y+z}{5}$

or, $5x+5z-10=3y+3z$

or $5x-3y+2z=10$ (5)

From (3), we have $c^{\frac{y}{4}}=c^{\frac{x+y+1}{7}}$

or, $\dfrac{y}{4}=\dfrac{x+y+1}{7}$

or, $7y=4x+4y+4$

or, $4x-3y=-4$ (6)

Adding (4) and (5) we have $8x-2y=8$

or, $4x-y=4$ (7)

From (6) and (7), we have

$$2y=8\ , \qquad y=4$$

Hence, $\qquad 4x-4=4 \qquad$ or, $\quad x=2$

From (4) $\quad 6+4-2z=-2\ , \qquad z=6$

Thus $\qquad x=2,\ y=4,\ $ and $\ z=6$

Exercise (81)

1 $\quad 3\sqrt{5}=\sqrt{9}\sqrt{5}=\sqrt{9\times5}=\sqrt{45}$

2 $\quad 2\sqrt[3]{3}=\sqrt[3]{8}\times\sqrt[3]{3}=\sqrt[3]{24}$

3 $\quad 2\sqrt[4]{6}=\sqrt[4]{16}\sqrt[4]{6}=\sqrt[4]{96}$

4 $\quad 4\sqrt[4]{5}=\sqrt[4]{256}\sqrt[4]{5}=\sqrt[4]{1280}$

5 $\quad a\sqrt[n]{b}=\sqrt[n]{a^n}\sqrt[n]{b}=\sqrt[n]{a^nb}$

6 $\quad x^m\sqrt[n]{y}=\sqrt[n]{x^{mn}}\sqrt[n]{y}=\sqrt[n]{x^{mn}y}$

7 $\quad a^4\sqrt[5]{b^2}=\sqrt[5]{a^{20}}\sqrt[5]{b^2}=\sqrt[5]{a^{20}b^2}$

Exercise (82)

1 $\quad \sqrt{18}=\sqrt{9\times2}=\sqrt{3^2\times2}=3\sqrt{2}$

2 $\quad \sqrt{80}=\sqrt{16\times5}=\sqrt{4^2\times5}=4\sqrt{5}$

3 $\quad \sqrt[3]{250}=\sqrt[3]{125\times2}=\sqrt[3]{5^3\times2}=5\sqrt[3]{2}$

4 $\quad \sqrt[5]{128}=\sqrt[5]{32\times4}=\sqrt[5]{2^5\times4}=2\sqrt[5]{4}$

5 $\quad \sqrt[4]{405}=\sqrt[4]{81\times5}=\sqrt[4]{3^4\times5}=3\sqrt[4]{5}$

6 $\quad \sqrt[3]{1372}=\sqrt[3]{343\times4}=\sqrt[3]{7^3\times4}=7\sqrt[3]{4}$

7 $\quad \sqrt[4]{1875}=\sqrt[4]{625\times3}=\sqrt[4]{5^4\times3}=5\sqrt[4]{3}$

8 $\quad \sqrt[3]{a^6b}=\sqrt[3]{(a^2)^3b}=a^2\sqrt[3]{b}$

9 $\quad \sqrt[n]{x^{4n}a}=x^4\sqrt[n]{a}$

10 $\quad \sqrt[3]{-2560}=\sqrt[3]{-512\times5}=\sqrt[3]{(-8)^3\times5}=-8\sqrt[3]{5}$

11 $\quad \sqrt[3]{-192a^3b^4}=\sqrt[3]{(-64)\times3a^3b^4}=\sqrt[3]{(-4)^3\times3a^3b^3\times b}$

$$=-4ab\sqrt[3]{3b}$$

12 $\quad \sqrt[3]{500a^7x^4}=\sqrt[3]{125\times4a^7x^4}=\sqrt[3]{(5)^3\times4(a^2)^3a\,x^3x}$

$$=5a^2x\sqrt[3]{4ax}$$

Exercise (83).

1 $\sqrt{12}+\sqrt{75}=\sqrt{4\times 3}+\sqrt{25\times 3}=2\sqrt{3}+5\sqrt{3}=7\sqrt{3}$

2 $\sqrt{18}+\sqrt{32}=\sqrt{9\times 2}+\sqrt{16\times 2}=3\sqrt{2}+4\sqrt{2}=7\sqrt{2}$

3 $\sqrt{20}+\sqrt{180}=\sqrt{4\times 5}+\sqrt{36\times 5}=2\sqrt{5}+6\sqrt{5}=8\sqrt{5}$

4 $\sqrt{98}-\sqrt{50}=\sqrt{49\times 2}-\sqrt{25\times 2}=7\sqrt{2}-5\sqrt{2}=2\sqrt{2}$

5 $\sqrt[3]{128}-\sqrt[3]{54}=\sqrt[3]{64\times 2}-\sqrt[3]{27\times 2}=\sqrt[3]{4^3\times 2}-\sqrt[3]{3^3\times 2}$
$=4\sqrt[3]{2}-3\sqrt[3]{2}=\sqrt[3]{2}$

6 $\sqrt[4]{80}+\sqrt[4]{405}=\sqrt[4]{16\times 5}+\sqrt[4]{81\times 5}=2\sqrt[4]{5}+3\sqrt[4]{5}=5\sqrt[4]{5}$

7 $\sqrt[4]{768}-\sqrt[4]{243}=\sqrt[4]{256\times 3}-\sqrt[4]{81\times 3}=\sqrt[4]{4^4\times 3}-\sqrt[4]{3^4\times 3}$
$=4\sqrt[4]{3}-3\sqrt[4]{3}=\sqrt[4]{3}$

8 $2\sqrt{27}-\sqrt{75}+\sqrt{12}=2\sqrt{9\times 3}-\sqrt{25\times 3}+\sqrt{4\times 3}$
$=6\sqrt{3}-5\sqrt{3}+2\sqrt{3}=3\sqrt{3}$

9 $2\sqrt{405}-3\sqrt{125}+\sqrt{45}=2\sqrt{81\times 5}-3\sqrt{25\times 5}+\sqrt{9\times 5}$
$=18\sqrt{5}-15\sqrt{5}+3\sqrt{5}=21\sqrt{5}-15\sqrt{5}=6\sqrt{5}$

10 $4\sqrt[3]{192}-4\sqrt[3]{375}+2\sqrt[3]{24}=4\sqrt[3]{64\times 3}-4\sqrt{125\times 3}+2\sqrt[3]{8\times 3}$
$=16\sqrt[3]{3}-20\sqrt[3]{3}+4\sqrt[3]{3}=0$

11 $3\sqrt[3]{40}+2\sqrt[3]{625}-4\sqrt[3]{320}=3\sqrt[3]{8\times 5}+2\sqrt[3]{125\times 5}-4\sqrt[3]{64\times 5}$
$=6\sqrt[3]{5}+10\sqrt[3]{5}-16\sqrt{5}=0$

12 $5\sqrt[3]{-54}-2\sqrt{-16}+4\sqrt[3]{686}$
$=5\sqrt[3]{-27\times 2}-2\sqrt[3]{-8\times 2}+4\sqrt[3]{343\times 2}$
$=5\sqrt[3]{(-3)^3\times 2}-2\sqrt[3]{(-2)^3\times 2}+4\sqrt[3]{(7)^3\times 2}$
$=-15\sqrt{2}+4\sqrt[3]{2}+28\sqrt[3]{2}=17\sqrt[3]{2}$

13 $\sqrt{45x^3}+\sqrt{80x^3}+\sqrt{5xy^2}$
$=\sqrt{9\times 5\times x^2\times x}+\sqrt{16\times 5\times x^2\times x}+\sqrt{5xy^2}$
$=3x\sqrt{5x}+4x\sqrt{5x}+y\sqrt{5x}$
$=7x\sqrt{5x}+y\sqrt{5x}=(7x+y)\sqrt{5x}$

14 The given expression
$=x^2\sqrt[3]{a}+y\sqrt[3]{(-2y)^3a}-z\sqrt[3]{(-3z)^3a}$
$=x^2\sqrt[3]{a}-2y^2\sqrt[3]{a}+3z^2\sqrt[3]{a}=(x^2-2y^2+3z^2)\sqrt[3]{a}$

15 The given expression

$$= 2\sqrt[4]{2^4 a^4 2x} + 3\sqrt[4]{4^4 a^4 2x} - 4a\sqrt[4]{3^4 2x}$$
$$= 2 \cdot 2a\sqrt[4]{2x} + 3 \cdot 4a\sqrt[4]{2x} - 4a \cdot 3\sqrt[4]{2x}$$
$$= 4a\sqrt[4]{2x} + 12a\sqrt[4]{2x} - 12a\sqrt[4]{2x} = 4a\sqrt[4]{2x}$$

Exercise (84)

1 We have $\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{3}{6}} = \sqrt[6]{3^3} = \sqrt[6]{27}$,

and $\sqrt[3]{2} = 2^{\frac{1}{3}} = 2^{\frac{2}{6}} = \sqrt[6]{2^2} = \sqrt[6]{4}$

Thus the required surds are $\sqrt[6]{27}$ and $\sqrt[6]{4}$

2 We have $\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\frac{4}{12}} = \sqrt[12]{4^4} = \sqrt[12]{256}$,

and $\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{3}{12}} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Thus the required surds are $\sqrt[12]{256}$ and $\sqrt[12]{125}$

3 We have $\sqrt[5]{2} = 2^{\frac{1}{5}} = 2^{\frac{3}{15}} = \sqrt[15]{2^3} = \sqrt[15]{8}$,

and $\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{5}{15}} = \sqrt[15]{3^5} = \sqrt[15]{243}$

Thus the required surds are $\sqrt[15]{8}$ and $\sqrt[15]{243}$

4 We have $\sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27}$,

and $\sqrt[6]{5} = 5^{\frac{1}{6}} = 5^{\frac{2}{12}} = \sqrt[12]{5^2} = \sqrt[12]{25}$

Thus the required surds are $\sqrt[12]{27}$ and $\sqrt[12]{25}$

5 We have $\sqrt[6]{4} = 4^{\frac{1}{6}} = 4^{\frac{4}{24}} = \sqrt[24]{4^4} = \sqrt[24]{256}$,

and $\sqrt[8]{6} = 6^{\frac{1}{8}} = 6^{\frac{3}{24}} = \sqrt[24]{6^3} = \sqrt[24]{216}$

Thus the required surds $\sqrt[24]{256}$ and $\sqrt[24]{216}$

6 We have $\sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{3}{6}} = \sqrt[6]{2^3} = \sqrt[6]{8}$,

and $\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{2}{6}} = \sqrt[6]{3^2} = \sqrt[6]{9}$ $\qquad \sqrt[3]{3}$ is the greater

7 We have $\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{4}{12}} = \sqrt[12]{3^4} = \sqrt[12]{81}$,

and $\sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\frac{3}{12}} = \sqrt[12]{4^3} = \sqrt[12]{64}$ $\qquad \sqrt[3]{3}$ is the greater

8 We have $\sqrt[3]{6} = 6^{\frac{1}{3}} = 6^{\frac{4}{12}} = \sqrt[12]{6^4} = \sqrt[12]{1296}$

and $\sqrt[4]{10} = 10^{\frac{1}{4}} = 10^{\frac{3}{12}} = \sqrt[12]{10^3} = \sqrt[12]{1000}$ $\qquad \sqrt[3]{6}$ is the greater

9 We have $\sqrt[4]{6}=6^{\frac{1}{4}}=6^{\frac{3}{12}}=\sqrt[12]{6^3}=\sqrt[12]{216}$,

$\sqrt{2}=2^{\frac{1}{2}}=2^{\frac{6}{12}}=\sqrt[12]{2^6}=\sqrt[12]{64}$,

and $\sqrt[3]{4}=4^{\frac{1}{3}}=4^{\frac{4}{12}}=\sqrt[12]{4^4}=\sqrt[12]{256}$

the required arrangement is $\sqrt[3]{4}$, $\sqrt[4]{6}$ and $\sqrt{2}$

10 We have $\sqrt[4]{3}=3^{\frac{1}{4}}=3^{\frac{6}{24}}=\sqrt[24]{3^6}=\sqrt[24]{729}$,

$\sqrt[8]{10}=10^{\frac{1}{8}}=10^{\frac{3}{24}}=\sqrt[24]{10^3}=\sqrt[24]{1000}$,

and $\sqrt[12]{25}=25^{\frac{1}{12}}=25^{\frac{2}{24}}=\sqrt[24]{25^2}=\sqrt[24]{625}$

the required arrangement is $\sqrt[8]{10}$, $\sqrt[4]{3}$ and $\sqrt[12]{25}$

Exercise (85)

1 $\sqrt{5}\times\sqrt{10}=\sqrt{50}=\sqrt{25\times2}=5\sqrt{2}$

2 $\sqrt{8}\times\sqrt{6}=\sqrt{48}=\sqrt{16\times3}=4\sqrt{3}$

3 $\sqrt{27}\times\sqrt{3}=\sqrt{81}=9$

4 $\sqrt{15}\times\sqrt{6}=\sqrt{90}=\sqrt{9}\times\sqrt{10}=3\sqrt{10}$

5 $\sqrt{20}\times\sqrt{45}=\sqrt{4\times5}\times\sqrt{9\times5}=\sqrt{4\times9\times5\times5}=2\times3\times5=30$

6 $\sqrt[3]{5}\times\sqrt[3]{25}=\sqrt[3]{5\times25}=\sqrt[3]{5\times5\times5}=5$

7 $\sqrt[3]{6ax}\times\sqrt[3]{27a^2x^3}=\sqrt[3]{6ax}\times3x\sqrt[3]{a^2}=3x\sqrt[3]{6ax\times a^2}=3ax\sqrt[3]{6x}$

8 $\sqrt[3]{2}\times\sqrt{6}=\sqrt[6]{4}\times\sqrt[6]{216}=\sqrt[6]{864}$

9 $\sqrt{2}\times\sqrt[3]{6}=\sqrt[6]{2\times2\times2}\times\sqrt[6]{6\times6}=\sqrt[6]{288}$

10 $\sqrt[3]{4}\times\sqrt{8}=\sqrt[6]{4\times4}\times\sqrt[6]{8\times8\times8}=\sqrt[6]{4\times4\times4\times4\times4\times4\times2}$

$=4\sqrt[6]{2}$

11 $\sqrt[3]{9}\times\sqrt{27}=\sqrt[6]{9\times9}\times\sqrt[6]{27\times27\times27}=\sqrt[6]{9\times9\times9\times9\times9\times9\times3}$

$=9\sqrt[6]{3}$

12 $\sqrt[6]{2}\times\sqrt[9]{3}=\sqrt[18]{2\times2\times2}\times\sqrt[18]{3\times3}=\sqrt[18]{8\times9}=\sqrt[18]{72}$

13 $\sqrt[4]{3}\times\sqrt[8]{3}=\sqrt[8]{9}\times\sqrt[8]{3}=\sqrt[8]{27}$

14 $\sqrt[6]{2}\times\sqrt[9]{2}=\sqrt[18]{8}\times\sqrt[18]{4}=\sqrt[18]{32}$

15 $\sqrt[4]{4}\times\sqrt[6]{4}=\sqrt[12]{4\times4\times4}\times\sqrt[12]{4\times4}=\sqrt[12]{64\times16}=\sqrt[12]{1024}$.

16 $5\sqrt{8}\times2\sqrt{6}=5\sqrt{4\times2}\times2\sqrt{6}=10\sqrt{2}\times2\sqrt{6}=20\sqrt{12}$

$=20\sqrt{4\times3}=40\sqrt{3}$

17 $8\sqrt{12}\times 3\sqrt{24}=8\sqrt{12}\times 3\sqrt{12\times 2}=8\times 3\times 12\sqrt{2}=288\sqrt{2}$

18 $4\sqrt[3]{72}\times 5\sqrt[3]{576}=4\sqrt[3]{8\times 9}\times 5\sqrt[3]{64\times 9}=4\times 2\sqrt[3]{9}\times 5\times 4\sqrt[3]{9}$

$$=160\sqrt[3]{3\times 3\times 3\times 3}=480\sqrt[3]{3}$$

19 $7\sqrt[3]{8a^3x^2}\times 5\sqrt[3]{27b^3x^2}=7\times 2a\sqrt[3]{x^2}\times 5\times 3b\sqrt[3]{x^2}=210ab\sqrt[3]{x^4}$

$$=210abx\sqrt[3]{x}$$

20 $8\sqrt{10}\div 4\sqrt{15}=\dfrac{8\times\sqrt{5\times 2}}{4\times\sqrt{5\times 3}}=\sqrt[3]{\dfrac{2}{3}}$

21 $3\sqrt{12}\div 6\sqrt{27}=\dfrac{3\times 2\sqrt{3}}{6\times 3\sqrt{3}}=\dfrac{1}{3}$

22 $\sqrt[3]{36}\div\sqrt[3]{48}=\sqrt[3]{\frac{36}{48}}=\sqrt[3]{\frac{3}{4}}$

23 $\sqrt[9]{8}\div\sqrt[6]{6}=\sqrt[18]{8\times 8}\div\sqrt[18]{6\times 6\times 6}=\sqrt[18]{\frac{8\times 8}{6\times 6\times 6}}=\sqrt[18]{\frac{8}{27}}=\sqrt[18]{(\frac{2}{3})^3}$

$$=\sqrt[6]{\frac{2}{3}}$$

24 $\sqrt{2}\div\sqrt{6}=\sqrt{\frac{2}{6}}=\sqrt{\frac{1}{3}}=\dfrac{1}{1.732}=.577$

25 $\sqrt{72}\div\sqrt{40}=\sqrt{\frac{72}{40}}=\sqrt{\frac{9}{5}}=\dfrac{3\sqrt{5}}{5}=\dfrac{6\sqrt{5}}{10}=\dfrac{6\times 2.236}{10}=1.341$

26 $\sqrt{276}\div\sqrt{22}=\sqrt{\dfrac{275}{22}}=\sqrt{\dfrac{25}{2}}=\dfrac{5\sqrt{2}}{2}=\dfrac{5\times 1.414}{2}=3.536$

27 $10\sqrt{108}\div\sqrt{15}=10\sqrt{\dfrac{108}{15}}=\dfrac{10\sqrt{36}}{\sqrt{5}}=\dfrac{10\times 6}{\sqrt{5}}=\dfrac{10\times 6\times\sqrt{5}}{5}$

$$=2\times 6\times\sqrt{5}=12\sqrt{5}=12\times 2.236=26.832$$

Exercise (86).

1 $(\sqrt{a}+\sqrt{b})(\sqrt{ab})=\sqrt{a}\,\sqrt{ab}+\sqrt{b}\,\sqrt{ab}=a\sqrt{b}+b\sqrt{a}$

2 $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=(\sqrt{a})^2-(\sqrt{b})^2=a-b$

3 $(\sqrt{a}-5)\,2\sqrt{a}=3\sqrt{a}\,2\sqrt{a}-5\times 2\sqrt{a}=6a-10\sqrt{a}$

4 $(4\sqrt{x}+3\sqrt{y})(4\sqrt{x}-3\sqrt{y})=(4\sqrt{x})^2-(3\sqrt{y})^2=16x-9y$

5 $(2\sqrt{x-5}+4)(3\sqrt{x-5}-6)=2\sqrt{x-5}\;3\sqrt{x-5}+4\;3\sqrt{x-5}$

$$=6(x-5)-24=6x-30-24=6x-54$$

6 $(3\sqrt{5}-4\sqrt{2})(2\sqrt{5}+3\sqrt{2})$

$$=3\sqrt{5}\;2\sqrt{5}-4\sqrt{2}\;2\sqrt{5}+3\sqrt{5}\;3\sqrt{2}-4\sqrt{2}\;3\sqrt{2}$$

$$=6\;5+\sqrt{10}-12\;2=30+\sqrt{10}-24=6+\sqrt{10}$$

7. $\{(\sqrt{2}+2\sqrt{3})+\sqrt{7}\}\{(\sqrt{2}+2\sqrt{3})-\sqrt{7}\}=(\sqrt{2}+2\sqrt{3})^2-(\sqrt{7})^2$
$=2+4\sqrt{6}+4\cdot3-7=2+12-7+4\sqrt{6}=7+4\sqrt{6}$

8 $\{(3-\sqrt{5})+\sqrt{8}\}\{(3-\sqrt{5})-\sqrt{8}\}=(3-\sqrt{5})^2-(\sqrt{8})^2$
$=9-6\sqrt{5}+5-8=6-6\sqrt{5}$

9 $\{\sqrt{11}+(\sqrt{6}-\sqrt{3})\}\{\sqrt{11}-\sqrt{6}-\sqrt{3})\}=(\sqrt{11})^2\times(\sqrt{6}-\sqrt{3})^2$
$=11-(6+3-2\sqrt{6}\cdot\sqrt{3})=11-6-3+6\sqrt{2}=2+6\sqrt{2}$

10 $(\sqrt[3]{4}+\sqrt[3]{9}+\sqrt[3]{48})(\sqrt[3]{2}+\sqrt[3]{3})$
$=\left(2^{\frac{2}{3}}+3^{\frac{2}{3}}+2\cdot2^{\frac{1}{3}}\cdot3^{\frac{1}{3}}\right)\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$
$=\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)^2\cdot\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)=\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)^3$
$=2+3+3\cdot2^{\frac{1}{3}}\cdot3^{\frac{1}{3}}\left(2^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$
$=5+3\sqrt[3]{2\cdot2\cdot3}+3\sqrt[3]{2\cdot3\cdot3}=5+3\sqrt[3]{12}+3\sqrt[3]{18}$

11 $(\sqrt{x+a}-\sqrt{x-a})^2=(x+a)+(x-a)-2\sqrt{x^2-a^2}$
$=2x-2\sqrt{x^2-a^2}$

12 $(2\sqrt{8}+5\sqrt{6})^2=4\cdot8+2\cdot2\sqrt{8}\cdot5\sqrt{6}+25\cdot6$
$=32+20\sqrt{8\cdot6}+150=182+20\cdot4\sqrt{3}=182+80\sqrt{3}$

13 $(2\sqrt{5}+3\sqrt{7})^2=4\cdot5+2\cdot2\sqrt{5}\cdot3\sqrt{7}+9\cdot7=20+12\sqrt{35}+63$
$=83+12\sqrt{35}$

14 $(\sqrt{a^2+2b^2}-\sqrt{a^2-2b^2})^2$
$=a^2+2b^2-2\sqrt{a^2+2b^2}\cdot\sqrt{a^2-2b^2}+a^2-2b^2$
$=2a^2-2\sqrt{a^4-4b^4}$

15 $(2\sqrt{x^2+y^2}+5\sqrt{x^2-y^2})^2=4(x^2+y^2)+2\cdot2\cdot5\sqrt{x^4-y^4}+25(x^2-y^2)$
$=29x^2-21y^2+20\sqrt{x^4-y^4}$

Exercise (87).

1 The given expression

$$=\frac{(5\sqrt{3}+\sqrt{7})(4\sqrt{3}-2\sqrt{7})}{(4\sqrt{3}+2\sqrt{7})(4\sqrt{3}-2\sqrt{7})}=\frac{20\cdot3+4\sqrt{21}-10\sqrt{21}-2\cdot7}{16\cdot3-4\cdot7}$$

$$=\frac{60-6\sqrt{21}-14}{48-28}=\frac{46-6\sqrt{21}}{20}=\frac{23-6\sqrt{21}}{10}$$

2 The given expression

$$=\frac{(3\sqrt{2}+2\sqrt{3})^2}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}=\frac{9\cdot 2+4\cdot 3+12\sqrt{6}}{9\cdot 2-4\cdot 3}$$

$$=\frac{18+12+12\sqrt{6}}{18-12}=\frac{30+12\sqrt{6}}{6}=5+2\sqrt{6}$$

3 The given expression

$$=\frac{(4+3\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}=\frac{12+9\sqrt{2}+8\sqrt{2}+6\cdot 2}{9-4\cdot 2}$$

$$=\frac{24+17\sqrt{2}}{9-8}=24+17\sqrt{2}$$

4. The given expression

$$=\frac{(3\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}=\frac{3\cdot 5+\sqrt{15}+3\sqrt{15}+3}{5-3}$$

$$=\frac{15+4\sqrt{15}+3}{2}=\frac{18+4\sqrt{15}}{2}=9+2\sqrt{15}$$

5 The given expression

$$=\frac{(\sqrt{a+x}+\sqrt{a-x})^2}{(\sqrt{a+x}-\sqrt{a+x})(\sqrt{a+x}+\sqrt{a-x})}$$

$$=\frac{(a+x)+(a-x)+2\sqrt{a^2-x^2}}{(a+x)-(a-x)}$$

$$=\frac{2a+2\sqrt{a^2-x^2}}{2x}=\frac{a+\sqrt{a^2-x^2}}{x}$$

6 The given expression

$$=\frac{(\sqrt{x^2+1}-\sqrt{x^2-1})^2}{(\sqrt{x^2+1}+\sqrt{x^2-1})(\sqrt{x^2+1}-\sqrt{x^2-1})}$$

$$=\frac{(x^2+1)+(x^2-1)-2\sqrt{x^4-1}}{(x^2+1)-(x^2-1)}=\frac{2x^2-2\sqrt{x^4-1}}{2}=x^2-\sqrt{x^4-1}$$

7 The given expression

$$=\frac{(1+\sqrt{2})-\sqrt{3}}{\{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\}}=\frac{1+\sqrt{2}-\sqrt{3}}{(1+\sqrt{2})^2-3}$$

$$=\frac{1+\sqrt{2}-\sqrt{3}}{1+2+2\sqrt{2}-3}=\frac{1+\sqrt{2}-\sqrt{3}}{2\sqrt{2}}=\frac{(1+\sqrt{2}-\sqrt{3})\sqrt{2}}{2\sqrt{2}\sqrt{2}}$$

$$=\frac{\sqrt{2}+2-\sqrt{6}}{2\cdot 2}=\frac{2+\sqrt{2}-\sqrt{6}}{4}.$$

8 $\frac{\sqrt{2}+1}{\sqrt{2}-1}=\frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{2+1+2\sqrt{2}}{2-1}=3+2(1\,414)$

$=3+2\,828=5\,828$

9 $\frac{\sqrt{3}}{2-\sqrt{3}}=\frac{\sqrt{3}(2+\sqrt{3})}{4-3}=2\sqrt{3}+3=2(1\,732)+3=3\,464+3$

$=6\,464$

10 $\frac{8-5\sqrt{2}}{3-2\sqrt{2}}=\frac{(8-5\sqrt{2})(3+2\sqrt{2})}{9-4\,2}=\frac{24-15\sqrt{2}+16\sqrt{2}-10\,2}{9-8}$

$=24-20+\sqrt{2}=4+\sqrt{2}=4+1\,414=5\,414$

11 $\frac{3}{\sqrt{5}-\sqrt{2}}=\frac{3(\sqrt{5}+\sqrt{2})}{5-2}=\sqrt{5}+\sqrt{2}=2\,236+1\,414=3\,65$

12 $\frac{3+\sqrt{5}}{3-\sqrt{5}}=\frac{(3+\sqrt{5})^2}{9-5}=\frac{9+5+6\sqrt{5}}{4}=\frac{14+6\sqrt{5}}{4}=\frac{7+3\,(2\,236)}{2}$

$=\frac{7+6\,708}{2}=\frac{13\,708}{2}=6\,854$

13 $\frac{\sqrt{5}+\sqrt{3}}{4+\sqrt{15}}=\frac{(\sqrt{5}+\sqrt{3})(4-\sqrt{15})}{16-15}$

$=4\sqrt{5}+4\sqrt{3}-\sqrt{5}\,\sqrt{5\cdot5}-\sqrt{3}\,\sqrt{5\,3}$

$=4\sqrt{5}+4\sqrt{3}-5\sqrt{3}-3\sqrt{5}=\sqrt{5}-\sqrt{3}=2\,236-1\,732=\,504.$

14 $\frac{1}{x+\sqrt{x^2-1}}+\frac{1}{x-\sqrt{x^2-1}}=\frac{x-\sqrt{x^2-1}+x+\sqrt{x^2-1}}{x^2-(x^2-1)}=2x$

15 The given expression

$=\frac{15}{\sqrt{5\,2}+2\sqrt{5}+2\sqrt{5\,2}-\sqrt{5}-4\sqrt{5}}$

$=\frac{15}{\sqrt{5}(\sqrt{2}+2+2\sqrt{2}-1-4)}=\frac{15}{\sqrt{5}(3\sqrt{2}-3)}=\frac{15}{3\sqrt{5}(\sqrt{2}-1)}$

$=\frac{\sqrt{5}}{\sqrt{2}-1}=\frac{\sqrt{5}(\sqrt{2}+1)}{2-1}=\sqrt{10}+\sqrt{5}$

16 The given expression

$=\frac{\sqrt{2}(2\sqrt{3}+2-3-\sqrt{3})}{(3\sqrt{3}-5)(\sqrt{2}-1)(\sqrt{2}+1)\sqrt{2}}=\frac{(\sqrt{3}-1)}{(3\sqrt{3}-5)(2-1)}=\frac{\sqrt{3}-1}{3\sqrt{3}-5}$

$=\frac{(\sqrt{3}-1)(3\sqrt{3}+5)}{9\,3-25}=\frac{3\,3-3\sqrt{3}+5\sqrt{3}-5}{2}=\frac{9-5+2\sqrt{3}}{2}$

$=\frac{4+2\sqrt{3}}{2}=2+\sqrt{3}$

17. The given expression

$$=\frac{4(\sqrt{3}+\sqrt{5}+\sqrt{2})}{(\sqrt{3}+\sqrt{5})^2-(\sqrt{2})^2}=\frac{4(\sqrt{3}+\sqrt{5}+\sqrt{2})}{3+5+2\sqrt{15}-2}=\frac{4(\sqrt{3}+\sqrt{5}+\sqrt{2})}{6+2\sqrt{15}}$$

$$=\frac{2(\sqrt{3}+\sqrt{5}+\sqrt{2})}{3+\sqrt{15}}=\frac{2(\sqrt{3}+\sqrt{5}+\sqrt{2})(3-\sqrt{15})}{9-15}$$

$$=\frac{2(3\sqrt{3}+3\sqrt{5}+3\sqrt{2}-3\sqrt{5}-5\sqrt{3}-\sqrt{30})}{-6}$$

$$=\tfrac{1}{3}(2\sqrt{3}-3\sqrt{2}+\sqrt{30})$$

18 The given expression

$$=\frac{1}{(3+2\sqrt{2})^3}+\frac{1}{(3-2\sqrt{2})^3}=\frac{(3-2\sqrt{2})^3+(3+2\sqrt{2})^3}{(9-8)^3}$$

$$=27-54\sqrt{2}+72-8+27+54\sqrt{2}+72+8=54+144=198$$

19 The given expression

$$=\frac{(x+\sqrt{x^2-1})^2-(x-\sqrt{x^2-1})^2}{x^2-(x^2-1)}=4x\sqrt{x^2-1}$$

20 The given expression

$$=\frac{(\sqrt{x^2+1}+\sqrt{x^2-1})^2+(\sqrt{x^2+1}-\sqrt{x^2-1})^2}{(x^2+1)-(x^2-1)}$$

$$=\frac{2\{(x^2+1)+(x^2-1)\}}{2}=2x^2$$

21 $$\frac{1}{\sqrt[3]{3}+\sqrt[3]{2}}=\frac{3^{\frac{2}{3}}-3^{\frac{1}{3}}2^{\frac{1}{3}}+2^{\frac{2}{3}}}{(3^{\frac{1}{3}}+2^{\frac{1}{3}})(3^{\frac{2}{3}}-3^{\frac{1}{3}}2^{\frac{1}{3}}+2^{\frac{2}{3}})}$$

$$=\frac{9^{\frac{1}{3}}-6^{\frac{1}{3}}+4^{\frac{1}{3}}}{3+2}=\frac{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}{5}$$

22 $$\frac{1}{\sqrt[3]{4}-\sqrt[3]{3}}=\frac{4^{\frac{2}{3}}+4^{\frac{1}{3}}3^{\frac{1}{3}}+3^{\frac{2}{3}}}{(4^{\frac{1}{3}}-3^{\frac{1}{3}})(4^{\frac{2}{3}}+4^{\frac{1}{3}}3^{\frac{1}{3}}+3^{\frac{2}{3}})}$$

$$=\frac{16^{\frac{1}{3}}+12^{\frac{1}{3}}+9^{\frac{1}{3}}}{4-3}=\sqrt[3]{16}+\sqrt[3]{12}+\sqrt[3]{9}=2\sqrt[3]{2}+\sqrt[3]{12}+\sqrt[3]{9}$$

Exercise (88)

1 Let $\sqrt{4-2\sqrt{3}} = \sqrt{x} - \sqrt{y}$

Then, squaring both sides, $4-2\sqrt{3} = x+y-2\sqrt{xy}$

Hence, $x+y=4$ and $xy=3$

These relations are satisfied by the numbers 3 and 1

Thus the required root $= \sqrt{3}-1$

2 Let $\sqrt{7+4\sqrt{3}} = \sqrt{x} + \sqrt{y}$

Then, $7+4\sqrt{3} = x+y+2\sqrt{xy}$

Hence, $x+y=7$ and $2\sqrt{xy}=4\sqrt{3}$ or $xy=12$

The numbers satisfying these relations are 4 and 3

Thus the required root $= \sqrt{4}+\sqrt{3} = 2+\sqrt{3}$

3. Let $\sqrt{11-6\sqrt{2}} = \sqrt{x} - \sqrt{y}$

Then, $11-6\sqrt{2} = x+y-2\sqrt{xy}$

Hence, $x+y=11$ and $2\sqrt{xy}=6\sqrt{2}$, or $xy=18$

The numbers satisfying these relations are 9 and 2

Thus the required root $= \sqrt{9}-\sqrt{2} = 3-\sqrt{2}$

4 Let $\sqrt{8+2\sqrt{15}} = \sqrt{x} + \sqrt{y}$

Then, $8+2\sqrt{15} = x+y+2\sqrt{xy}$

Hence, $x+y=8$ and $xy=15$

The numbers satisfying these relations are 5 and 3

Thus the required root $= \sqrt{5}+\sqrt{3}$

5 Let $\sqrt{14-6\sqrt{5}} = \sqrt{x} - \sqrt{y}$

Then, $14-6\sqrt{5} = x+y-2\sqrt{xy}$

Hence, $x+y=14$ and $2\sqrt{xy}=6\sqrt{5}$, or $xy=45$.

The numbers satisfying these relations are 9 and 5

Thus the required root $= \sqrt{9}-\sqrt{5} = 3-\sqrt{5}$

6 Let $\sqrt{28+10\sqrt{3}} = \sqrt{x} + \sqrt{y}$

Then $28+10\sqrt{3} = x+y+2\sqrt{xy}$

Hence, $x+y=28$ and $2\sqrt{xy}=10\sqrt{3}$, or $xy=75$

The numbers satisfying these relations are 25 and 3

Thus the required root $= \sqrt{25}+\sqrt{3} = 5+\sqrt{3}$

7 Let $\sqrt{21-8\sqrt{5}}=\sqrt{x}-\sqrt{y}$

Then, $21-8\sqrt{5}=x+y-2\sqrt{xy}$

Hence, $x+y=21$ and $2\sqrt{xy}=8\sqrt{5}$, or $xy=80$

The numbers satisfying these relations are 16 and 5

Thus the required root $=\sqrt{16}-\sqrt{5}=4-\sqrt{5}$

8 Let $\sqrt{17+12\sqrt{2}}=\sqrt{x}+\sqrt{y}$

Then, $17+12\sqrt{2}=x+y+2\sqrt{xy}$

Hence, $x+y=17$ and $2\sqrt{xy}=12\sqrt{2}$, or $xy=72$

The numbers satisfying these relations are 9 and 8

Thus the required root$=\sqrt{9}+\sqrt{8}=3+2\sqrt{2}$

9 Let $\sqrt{41+12\sqrt{5}}=\sqrt{x}+\sqrt{y}$

Then, $41+12\sqrt{5}=x+y+3\sqrt{xy}$

Hence, $x+y=41$ and $\sqrt{xy}=12\sqrt{5}$, or $xy=180$

The numbers satisfying these relations are 36 and 5

Thus the required root$=\sqrt{36}+\sqrt{5}=6+\sqrt{5}$

10 Let $\sqrt{37-20\sqrt{3}}=\sqrt{x}-\sqrt{y}$

Then, $37-20\sqrt{3}=x+y-2\sqrt{xy}$

Hence, $x+y=37$ and $2\sqrt{xy}=20\sqrt{3}$ or $xy=300$

The numbers satisfying these relations are 25 and 12

Thus the required root$=\sqrt{25}-\sqrt{12}=5-2\sqrt{3}$

11 Let $\sqrt{31+4\sqrt{21}}=\sqrt{x}+\sqrt{y}$

Then, $31+4\sqrt{21}=x+y+2\sqrt{xy}$

Hence, $x+y=31$ and $2\sqrt{xy}=4\sqrt{21}$, or $xy=84$

The numbers satisfying these relations are 28 and 3

Thus the required root$=\sqrt{28}+\sqrt{3}=2\sqrt{7}+\sqrt{3}$

12 Let $\sqrt{73-12\sqrt{35}}=\sqrt{x}-\sqrt{y}$

Then, $73-12\sqrt{35}=x+y-2\sqrt{xy}$

Hence, $x+y=73$ and $2\sqrt{xy}=12\sqrt{35}$, or $xy=1260$

The numbers satisfying these relations are 45 and 28

Thus the required root$=\sqrt{45}-\sqrt{28}=3\sqrt{5}-2\sqrt{7}$

13 Let $\sqrt{47+4\sqrt{33}}=\sqrt{x}+\sqrt{y}$

Then, $47+4\sqrt{33}=x+y+2\sqrt{xy}$

Hence, $x+y=47$ and $2\sqrt{xy}=4\sqrt{33}$, or $xy=132$

The numbers satisfying these relations are 44 and 3

Thus the required root $=\sqrt{44}-\sqrt{3}=2\sqrt{11}-\sqrt{3}$

14 Let $\sqrt{4-\sqrt{7}}=\sqrt{x}-\sqrt{y}$

Then, $4-\sqrt{7}=x+y-2\sqrt{xy}$

Hence, $x+y=4$ and $2\sqrt{xy}=\sqrt{7}$, or $xy=\frac{7}{4}$

$x-y=\sqrt{(x+y)^2-4xy}=\sqrt{16-7}=\sqrt{9}=3.$

Thus we have $x+y=4$ and $x-y=3$

Hence, $x=\frac{7}{2}$ and $y=\frac{1}{2}$

The required root is therefore $=\sqrt{\frac{7}{2}}-\sqrt{\frac{1}{2}}$

15 Let $\sqrt{6-\sqrt{35}}=\sqrt{x}-\sqrt{y}$

Then, $6-\sqrt{35}=x+y-2\sqrt{xy}$

Threfore, $x+y=6$ and $2\sqrt{xy}=\sqrt{35}$

Hence, $x-y=\sqrt{(x+y)^2-4xy}=\sqrt{36-35}=1$

Thus we have $x+y=6$ and $x-y=1$

Hence, $x=\frac{7}{2}$ and $y=\frac{5}{2}$,

the required root is $\sqrt{\frac{7}{2}}-\sqrt{\frac{5}{2}}$

16 $\sqrt{18}-\sqrt{16}=\sqrt{2}(\sqrt{9}-\sqrt{8})=\sqrt{2}(3-\sqrt{8})$

Then $\sqrt{\sqrt{18}-\sqrt{16}}=\sqrt[4]{2}\sqrt{3-\sqrt{8}}$

Now let $\sqrt{3-\sqrt{8}}=\sqrt{x}-\sqrt{y}$

Then, $3-\sqrt{8}=x+y-2\sqrt{xy}$

Therefore, $x+y=3$ and $2\sqrt{xy}=\sqrt{8}$

Hence $x-y=\sqrt{(x+y)^2-4xy}=\sqrt{9-8}=1$

Thus we have $x+y=3$ and $x-y=1$

Thus $\sqrt{3-\sqrt{8}}=\sqrt{2}-1$

$\therefore \sqrt{\sqrt{18}-\sqrt{16}}=\sqrt[4]{2}(\sqrt{2}-1)$

17 $\sqrt{32}-\sqrt{24}=\sqrt{8}(\sqrt{4}-\sqrt{3})=\sqrt{8}(2-\sqrt{3})$

Then $\sqrt{\sqrt{32}-\sqrt{24}}=\sqrt[4]{8}\sqrt{2-\sqrt{3}}$

Now let $\sqrt{2-\sqrt{3}}=\sqrt{x}-\sqrt{y}$

Hence, $2-\sqrt{3}=x+y-2\sqrt{xy}$

Therefore, $x+y=2$ and $2\sqrt{xy}=\sqrt{3}$,

$$(x+y)=\sqrt{(x+y)^2-4xy}=\sqrt{4-3}=1$$

Thus we have $x+y=2$ and $x-y=1$, $x=$ and $y=\frac{1}{2}$

Hence, $\sqrt{2-\sqrt{3}}=\sqrt{\frac{3}{2}}-\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}(\sqrt{3}-1)$,

the required root $=\sqrt[4]{8}\ \frac{1}{\sqrt{2}}(\sqrt{3}-1)$

$$=\sqrt{2}\ \sqrt[4]{2}\ \frac{1}{\sqrt{2}}(\sqrt{3}-1)=\sqrt[4]{2}(\sqrt{3}-1)$$

18 $\sqrt{27}+\sqrt{24}=\sqrt{3}(\sqrt{9}+\sqrt{8})=\sqrt{3}(3+\sqrt{8})$

Then $\sqrt{\sqrt{27}+\sqrt{24}}=\sqrt[4]{3}\sqrt{3+\sqrt{8}}$

Now let $\sqrt{3+\sqrt{8}}=\sqrt{x}+\sqrt{y}$,

$$3+\sqrt{8}=x+y+2\sqrt{xy}$$

Hence, $x+y=3$ and $2\sqrt{xy}=\sqrt{8}$,

$$x-y=\sqrt{(x+y)^2-4xy}=\sqrt{9-8}=1$$

Thus we have $x+y=3$ and $x-y=1$

$$x=2 \text{ and } y=1$$

Thus $\sqrt{3+\sqrt{8}}=\sqrt{2}+1$

the required root $=\sqrt[4]{3}(\sqrt{2}+1)$

19 $5\sqrt{5}+\sqrt{120}=\sqrt{5}(5+\sqrt{24})=\sqrt{5}(5+2\sqrt{6})$

Then $\sqrt{5\sqrt{5}+\sqrt{120}}=\sqrt[4]{5}\sqrt{(5+2\sqrt{6})}$

Now let $\sqrt{5+2\sqrt{6}}=\sqrt{x}+\sqrt{y}$

$$5+2\sqrt{6}=x+y+2\sqrt{xy}$$

Hence, $x+y=5$ and $2\sqrt{xy}=2\sqrt{6}$

The nnmbers satisfying these relations are 3 and 2

Thus $$\sqrt{5+2\sqrt{6}}=\sqrt{3}+\sqrt{2}$$

the required root $=\sqrt[4]{5}(\sqrt{3}+\sqrt{2})$

20 We have $2+\sqrt{3}=2+2\sqrt{\frac{3}{4}}$

$$=2+2\sqrt{\tfrac{3}{2}\cdot\tfrac{1}{2}}=\tfrac{3}{2}+\tfrac{1}{2}+\sqrt{\tfrac{3}{2}\cdot\tfrac{1}{2}}=\left(\sqrt{\tfrac{3}{2}}+\sqrt{\tfrac{1}{2}}\right)^2,$$

$$\sqrt{2+\sqrt{3}}=\sqrt{\tfrac{3}{2}}+\sqrt{\tfrac{1}{2}}=\frac{\sqrt{3}+1}{\sqrt{2}}$$

Similarly $$\sqrt{2-\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{2}}$$

the given expression

$$=\frac{\sqrt{2}(2+\sqrt{3})}{2+\sqrt{3}+1}+\frac{\sqrt{2}(2-\sqrt{3})}{2-(\sqrt{3}-1)}=\frac{\sqrt{2}(2+\sqrt{3})}{\sqrt{3}(\sqrt{3}+1)}+\frac{\sqrt{2}(2-\sqrt{3})}{\sqrt{3}(\sqrt{3}-1)}$$

$$=\frac{\sqrt{2}}{\sqrt{3}}\ \frac{2\sqrt{3}-2+3-\sqrt{3}+2\sqrt{3}+2-3+\sqrt{3}}{(3-1)}$$

$$=\frac{\sqrt{2}}{\sqrt{3}}\ \frac{2\sqrt{3}}{2}=\sqrt{2}$$

21 We have $1+x=1+\dfrac{\sqrt{3}}{2}=\dfrac{2+\sqrt{3}}{2}$

and $1-x=1-\dfrac{\sqrt{3}}{2}=\dfrac{2-\sqrt{3}}{2}$

Let $\sqrt{2+\sqrt{3}}=\sqrt{a}+\sqrt{b}$

$$2+\sqrt{3}=a+b+2\sqrt{ab}$$

$$a+b=2 \text{ and } 2\sqrt{ab}=\sqrt{3}$$

$$(a-b)^2=(a+b)^2-4ab=4-3=1,$$

$$a-b=\pm 1$$

$$a=\tfrac{3}{2} \text{ and } b=\tfrac{1}{2}$$

or, $$a=\tfrac{1}{2} \text{ and } b=\tfrac{3}{2}$$

$$\sqrt{2+\sqrt{3}}=\sqrt{\tfrac{3}{2}}+\sqrt{\tfrac{1}{2}}$$

Similarly $\sqrt{2-\sqrt{3}}=\pm\left(\sqrt{\tfrac{3}{2}}-\sqrt{\tfrac{1}{2}}\right)$

Now $\sqrt{1+x}=\sqrt{\dfrac{2+\sqrt{3}}{2}}=\dfrac{\sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}}{\sqrt{2}}=\tfrac{1}{2}(\sqrt{3}+1)$

and $\sqrt{1-x}=\sqrt{\dfrac{2-\sqrt{3}}{2}}=\dfrac{\pm\left(\sqrt{\frac{3}{2}}-\sqrt{\frac{1}{2}}\right)}{\sqrt{2}}=\pm\tfrac{1}{2}(\sqrt{3}-1)$

the given expression

$$=\frac{\frac{2+\sqrt{3}}{2}}{1+\frac{\sqrt{3}+1}{2}}+\frac{\frac{2-\sqrt{3}}{2}}{1\pm\frac{\sqrt{3}-1}{2}}=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{2\pm(\sqrt{3}-1)}$$

$$=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}-1}\text{ or }\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{2-\sqrt{3}+1}$$

$$=\frac{2+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)}+\frac{2-\sqrt{3}}{\sqrt{3}+1}\text{ or }\frac{2+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)}+\frac{2-\sqrt{3}}{\sqrt{3}(\sqrt{3}-1)}$$

$$=\frac{2+\sqrt{3}+2\sqrt{3}-3}{\sqrt{3}(\sqrt{3}+1)}\text{ or }\frac{(2+\sqrt{3})(\sqrt{3}-1)+(2-\sqrt{3})(\sqrt{3}+1)}{\sqrt{3}\,(3-1)}$$

$$=\frac{3\sqrt{3}-1}{\sqrt{3}(\sqrt{3}+1)}\text{ or }\frac{2\sqrt{3}+3-2-\sqrt{3}+2\sqrt{3}-3+2-\sqrt{3}}{2\sqrt{3}}$$

$$=\frac{3\sqrt{3}-1}{\sqrt{3}(\sqrt{3}+1)}\text{ or }\frac{2\sqrt{3}}{2\sqrt{3}}\ (\text{i e }1)$$

22 The given expression

$$=\frac{(\sqrt{a+x}+\sqrt{a-x})^2}{(a+x)-(a-x)}=\frac{(a+x)+(a-x)+2\sqrt{a^2-x^2}}{2x}$$

$$=\frac{2a+2\sqrt{a^2-x^2}}{2x}=\frac{a+\sqrt{a^2-\frac{4a^2b^2}{(b^2+1)^2}}}{\frac{2ab}{b^2+1}}$$

$$=\frac{a(b^2+1)+a\sqrt{(b^2+1)^2-4b^2}}{2ab}=\frac{a(b^2+1)+a\sqrt{(b^2-1)^2}}{2ab}$$

$$=\frac{b^2+1+b^2-1}{2b}=\frac{2b^2}{2b}=b$$

23 Let $\sqrt{a^2+2x\sqrt{a^2-x^2}}=\sqrt{m}+\sqrt{n}$

$$a^2+2x\sqrt{a^2-x^2}=m+n+2\sqrt{mn},$$
$$m+n=a^2 \text{ and } mn=x^2(a^2-x^2),$$
$$m-n=\sqrt{(m+n)^2-4mn}=\sqrt{a^4-4x^2a^2+4x^4}$$
$$=a^2-2x^2,$$
$$m=a^2-x^2 \text{ and } n=x^2$$

Thus the required root $=\sqrt{a^2-x^2}+x$

24 Let $\sqrt{2a+2\sqrt{a^2-b^2}}=\sqrt{m}+\sqrt{n}$

$$2a+2\sqrt{a^2-b^2}=m+n+2\sqrt{mn},$$
$$m+n=2a \text{ and } mn=a^2-b^2,$$
$$m-n=\sqrt{4a^2-4a^2+4b^2}=2b\ ,$$
$$m=a+b \text{ and } n=a-b$$

Thus the required root $=\sqrt{a+b}+\sqrt{a-b}$

25 Let $\sqrt{a+x+\sqrt{2ax+x^2}}=\sqrt{m}+\sqrt{n}$

$$a+x+\sqrt{2ax+x^2}=m+n+2\sqrt{mn},$$
$$m+n=a+x \text{ and } 2\sqrt{mn}=\sqrt{2ax+x^2},$$
$$m-n=\sqrt{(m+n)^2-4mn}=\sqrt{(a+x)^2-2ax-x^2}=a,$$
$$m=\frac{2a+x}{2} \text{ and } n=\frac{x}{2}\ ,$$

Thus the required root $=\sqrt{\dfrac{2a+x}{2}}+\sqrt{\dfrac{x}{2}}=\sqrt{a+\tfrac{1}{2}x}+\sqrt{\tfrac{1}{2}x}$

26 The given expression

$$=2x-1+2\sqrt{(x-3)(x+2)}$$
$$=(x+2)+(x-3)+2\sqrt{(x+2)(x-3)}=\{\sqrt{x+2}+\sqrt{x-3}\}^2$$

the required root $=\sqrt{x+2}+\sqrt{x-3}$

27 The given expression $=(x+y)+z+2\sqrt{(x+y)z}$

$$=(\sqrt{x+y}+\sqrt{z})^2\ ,$$

the required root $=\sqrt{x+y}+\sqrt{z}$

Exercise (89).

1 Squaring both sides, we have $x+7=1+2\sqrt{x}+x$
Hence, $2\sqrt{x}=6$ or, $\sqrt{x}=3$, $\quad x=9$

2 Squaring both sides, we have $3x+16=3x+4\sqrt{3x}+4$
Hence, $4\sqrt{3x}=12$ or, $\sqrt{3x}=3$ or, $3x=9$ $\quad x=3$

3 Squaring both sides, we have $x+9=1+x+2\sqrt{x}$
Hence, $2\sqrt{x}=8$ or, $\sqrt{x}=4$, $\quad x=16$

4 Squaring both sides we have $3x+16-8\sqrt{3x}=3x+4$
Hence, $8\sqrt{3x}=12$ or, $2\sqrt{3x}=3$ or, $12x=9$, $\quad x=\frac{3}{4}$

5 Squaring both sides, we have $5x+10=5x+4\sqrt{5x}+4$
Hence, $4\sqrt{5x}=6$ or, $2\sqrt{5x}=3$ or, $20x=9$, $\quad x=\frac{9}{20}$

6 By transposition, we have $\sqrt{x-16}=8-\sqrt{x}$
Hence, squaring both sides $x-16=64-16\sqrt{x}+x$
or, $16\sqrt{x}=80$ or, $\sqrt{x}=5$, $\quad x=25$

7 By transposition, we have $\sqrt{2x+9}=9-\sqrt{2x}$
Hence squaring both sides, $2x+9=81-18\sqrt{2x}+2x$
or, $18\sqrt{2x}=72$ or, $\sqrt{2x}=4$ or, $2x=16$, $\quad x=8$

8 By transposition, we have $\sqrt{x+11}=1+\sqrt{x}$
Hence, squaring both sides $x+11=1+2\sqrt{x}+x$
or, $2\sqrt{x}=10$ or, $\sqrt{x}=5$, $\quad x=25$

9 By transposition, we have $\sqrt{8x+33}=2\sqrt{2x}+3$
Hence, squaring both sides, $8x+33=8x+12\sqrt{2x}+9$
or, $12\sqrt{2x}=24$ or, $\sqrt{2x}=2$ or, $2x=4$, $\quad x=2$

10 By transposition, we have $\sqrt{2ax+x^2}=a-x$
Hence, squaring both sides, $2ax+x^2=a^2+x^2-2ax$
or, $4ax=a^2$ or, $4x=a$, $\quad x=\dfrac{a}{4}$

11 By transposition, we have $\sqrt{2ax+x^2}=b-a-x$
Hence, squaring both sides,
$$2ax+x^2=b^2+a^2+x^2-2ab-2bx+2ax$$
or, $a^2+b^2-2ab=2bx$, $\quad x=\dfrac{(b-a)^2}{2b}$

12 Squaring both sides, we have

$$x-4+9+6\sqrt{x-4}=x+11$$

or, $6\sqrt{x-4}=6$ or, $x-4=1$, $\quad x=5$

13 Squaring both sides, we have

$$x-5=36-12\sqrt{x+7}+x+7$$

or $12\sqrt{x+7}=48$ or, $\sqrt{x+7}=4$ or $x+7=16$ $\quad x=9$

14 By transposition, we have $\sqrt{x+9}=1+\sqrt{x+2}$

Hence, squaring both sides

$$x+9=1+x+2+2\sqrt{x+2}$$

or, $2\sqrt{x+2}=6$ or, $\sqrt{x+2}=3$ or, $x+2=9$, $\quad x=7$

15 By transposition, we have $\sqrt{3x+1}=2+\sqrt{3x-11}$

Hence, squaring both sides, $3x+1=4+4\sqrt{3x-11}+3x-11$

or, $4\sqrt{3x-11}=8$ or, $\sqrt{3x-11}=2$

or $3x-11=4$ or $3x=15$, $\quad x=5$

16 By transposition, we have $\sqrt{5x+6}=10-\sqrt{5x-14}$

Hence, squaring both sides, $5x+6=100-20\sqrt{5x-14}+5x-14$

or, $20\sqrt{5x-14}=80$ or, $\sqrt{5x-14}=4$

or, $5x-14=16$ or, $5x=30$, $\quad x=6$

17 By transposition, we have

$$\sqrt{7x-12}=8-\sqrt{7x-4}$$

Hence, squaring both sides,

$$7x-12=64-16\sqrt{7x+4}+7x+4$$

or, $16\sqrt{7x+4}=80$ or, $\sqrt{7x+4}=5$

or, $7x+4=25$ or, $7x=21$ $\quad x=3$

18 By transposition, we have

$$\sqrt{x^2-3x+5}=1+\sqrt{x^2-x+1}$$

Hence, squaring both sides,

$$x^2-3x+5=1+x^2-x+1+2\sqrt{x^2-x+1}$$

or, $-2x+3=2\sqrt{x^2-x+1}$

or, $4x^2-12x+9=4x^2-4x+4$

or, $8x=5$, $\quad x=\frac{5}{8}$

19 $$\frac{x-1}{\sqrt{x}+1}=4+\frac{\sqrt{x}-1}{2}$$

or, $$\sqrt{x}-1=4+\frac{\sqrt{x}-1}{2}$$

or, $$\tfrac{1}{2}(\sqrt{x}-1)=4$$

or, $$\sqrt{x}-1=8 \quad \text{or,} \quad \sqrt{x}=9\,, \qquad x=81$$

20 $$\frac{ax-1}{\sqrt{ax}+1}=4+\frac{\sqrt{ax}-1}{2}$$

or, $$\sqrt{ax}-1=4+\tfrac{1}{2}(\sqrt{ax}-1)$$

or, $$\tfrac{1}{2}(\sqrt{ax}-1)=4$$

or, $$\sqrt{ax}-1=8 \quad \text{or,} \quad \sqrt{ax}=9 \quad \text{or,} \quad ax=81\,, \qquad x=\frac{81}{a}$$

21 $$\frac{ax-b^2}{\sqrt{ax}+b}=c+\frac{\sqrt{ax}-b}{c}$$

or, $$\sqrt{ax}-b=c+\frac{1}{c}(\sqrt{ax}-b)$$

or, $$\left(1-\frac{1}{c}\right)(\sqrt{ax}-b)=c$$

or, $$(c-1)(\sqrt{ax}-b)=c^2$$

or, $$\sqrt{ax}-b=\frac{c^2}{c-1}$$

or $$\sqrt{ax}=\frac{c^2}{c-1}+b$$

or, $$ax=\left(\frac{c^2}{c-1}+b\right)^2,$$

$$x=\frac{1}{a}\left(\frac{c^2}{c-1}+b\right)^2$$

22 $$\frac{200+120\sqrt{5x}}{9x-5}=(3\sqrt{x}-\sqrt{5})^2$$

or, $$\frac{40\sqrt{5}(3\sqrt{x}+\sqrt{5})}{(3\sqrt{x})^2-(\sqrt{5})^2}=(3\sqrt{x}-\sqrt{5})^2$$

or, $$\frac{40\sqrt{5}}{3\sqrt{x}-\sqrt{5}}=(3\sqrt{x}-\sqrt{5})^2$$

or, $$40\sqrt{5}=(3\sqrt{x}-\sqrt{5})^3$$

or, $$(2\sqrt{5})^2=(3\sqrt{x}-\sqrt{5})^2$$

or, $$2\sqrt{5}=3\sqrt{x}-\sqrt{5}$$

or, $$3\sqrt{5}=3\sqrt{x}\ , \qquad x=5$$

23 Squaring both sides, we have

$$4a+x+a+x-2\sqrt{(a+x)(4a+x)}=4x-8a$$

or, $$-2\sqrt{(a+x)(4a+x)}=2x-13a$$

or, $$4(4a^2+5ax+x^2)=4x^2-52ax+169a^2$$

or, $$72ax=153a^2$$

or, $$8x=17a\ , \qquad x=\frac{17a}{8}$$

24 $$\sqrt{x}+\sqrt{a+x}=\frac{3a}{\sqrt{a+x}}$$

or, $$\sqrt{ax+x^2}+a+x=3a$$

or, $$\sqrt{ax+x^2}=2a-x$$

or, $$ax+x^2=4a^2-4ax+x^2$$

or, $$5ax=4a^2\ , \qquad x=\frac{4a}{5}$$

25 $$\sqrt{x}+\sqrt{x+13}=\frac{91}{\sqrt{x+13}}$$

or, $$\sqrt{x^2+13x}+x+13=91$$

or, $$\sqrt{x^2+13x}=78-x$$

or, $$x^2+13x=6084+x^2-156x$$

or, $$169x=6084\ , \qquad x=36$$

26 $$\sqrt{x+a}+\sqrt{x-a}=\frac{b}{\sqrt{x+a}}$$

or, $$x+a+\sqrt{x^2-a^2}=b$$

or, $$\sqrt{x^2-a^2}=b-a-x$$

or, $$x^2-a^2=b^2+a^2+x^2-2ab-2bx+2ax$$

or, $$2x(b-a)=2a^2+b^2-2ab\ ,$$

$$x=\frac{2a^2-2ab+b^2}{2(b-a)}$$

27 $$\frac{3\sqrt{x}-4}{\sqrt{x}+2}=\frac{15+3\sqrt{x}}{\sqrt{x}+40}$$

or, $$3-\frac{10}{\sqrt{x}+2}=3-\frac{105}{\sqrt{x}+40}$$

or, $$\frac{2}{\sqrt{x}+2}=\frac{21}{\sqrt{x}+40}$$

or $$2\sqrt{x}+80=21\sqrt{x}+42$$

or, $$19\sqrt{x}=38$$

or, $$\sqrt{x}=2\,, \qquad x=4$$

28 $$\sqrt{x}-\sqrt{x-\sqrt{1-x}}=1$$

or, $$\sqrt{x-\sqrt{1-x}}=1-\sqrt{x}$$

or, $$x-\sqrt{1-x}=1+x-2\sqrt{x}$$

or, $$-\sqrt{1-x}=1-2\sqrt{x}$$

or, $$1-x=1+4x-4\sqrt{x}$$

or, $5x=4\sqrt{x}$ or, $5\sqrt{x}=4$ or, $\sqrt{x}=\frac{4}{5}$, $\therefore$ $x=\frac{16}{25}$

29 By transposition, we have

$$\sqrt{8-\sqrt{x^2+8x}}=2\sqrt{2}-\sqrt{x}$$

Hence, squaring both sides,

$$8-\sqrt{x^2+8x}=8+x-4\sqrt{2x}$$

or, $$-\sqrt{x+8}=\sqrt{x}-4\sqrt{2}$$

or, $$x+8=x+32-8\sqrt{2x}$$

or, $$8\sqrt{2x}=24$$

or $\sqrt{2x}=3$ or, $2x=9$, $x=\frac{9}{2}=4\frac{1}{2}$

30 $$\sqrt{1-x}+\sqrt{1-x+\sqrt{1+x}}=\sqrt{1+x}$$

or, $$\sqrt{1-x+\sqrt{1+x}}=\sqrt{1+x}-\sqrt{1-x}$$

Squaring both sides, we have

$$1-x+\sqrt{1+x}=1+x+1-x-2\sqrt{1-x^2}$$

or, $$\sqrt{1+x}=(1+x)-2\sqrt{1-x^2}$$

or, $$1=\sqrt{1+x}-2\sqrt{1-x}$$

or, $$1=1+x+4-4x-4\sqrt{1-x^2}$$

or, $$3x-4=4\sqrt{1-x^2}$$

or, $$9x^2-24x+16=16-16x^2$$

or, $$25x^2=24x\,, \qquad x=\frac{24}{25}$$

31 $$\frac{\sqrt{a+x}}{a}+\frac{\sqrt{a+x}}{x}=\frac{\sqrt{x}}{b}$$

or, $$\sqrt{a+x}\,(a+x)=\frac{ax\sqrt{x}}{b}$$

Hence, squaring both sides, $(a+x)^3=\dfrac{a^2x^3}{b^2}$

or, $$a+x=x\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}$$

or, $$x\left(\frac{a^{\frac{2}{3}}}{b^{\frac{2}{3}}}-1\right)=a\,, \qquad x=\frac{ab^{\frac{2}{3}}}{a^{\frac{2}{3}}-b^{\frac{2}{3}}}$$

32 $$\sqrt[5]{x+8}=\sqrt[10]{x^2+64x+36}$$

or, $$x+8=\sqrt{x^2+64x+36}$$

or, $$x^2+16x+64=x^2+64x+36$$

or, $$48x=28\,, \qquad x=\frac{28}{48}=\frac{7}{12}$$

33 $$(1+x)^{\frac{1}{3}}+(1-x)^{\frac{1}{3}}=2^{\frac{1}{3}}$$

Cubing both sides, we have

$$(1+x)+(1-x)+3(1-x^2)^{\frac{1}{3}}\,2^{\frac{1}{3}}=2$$

or, $1-x^2=0$ or $x^2=1$, $\qquad x=\pm 1$

34 $$(a+x)^{\frac{2}{3}}+(a-x)^{\frac{2}{3}}=3(a^2-x^2)^{\frac{1}{3}}$$

Cubing both sides, we have

$$(a+x)^2+(a-x)^2+3(a^2-x^2)^{\frac{2}{3}}\,3(a^2-x^2)^{\frac{1}{3}}=27(a^2-x^2)$$

or, $$2(a^2+x^2)+9(a^2-x^2)=27(a^2-x^2)$$

or, $20x^2 = 16a^2$

or, $5x^2 = 4a^2, \qquad x = \dfrac{2a}{\sqrt{5}}$

35 $$\left(\frac{x}{a}+\frac{a}{b}\right)^{\frac{1}{2}}+9\left(\frac{x}{a}-\frac{a}{b}\right)^{\frac{1}{2}}=6\left(\frac{x^2}{a^2}-\frac{a^2}{b^2}\right)^{\frac{1}{4}}$$

or, $$\left\{\left(\left(\frac{x}{a}+\frac{a}{b}\right)\right)^{\frac{1}{4}}\right\}^2+\left\{3\left(\frac{x}{a}-\frac{a}{b}\right)^{\frac{1}{4}}\right\}^2$$

$$-2\left(\frac{x}{a}+\frac{a}{b}\right)^{\frac{1}{4}}3\left(\frac{x}{a}-\frac{a}{b}\right)^{\frac{1}{4}}=0$$

or, $$\left(\frac{x}{a}+\frac{a}{b}\right)^{\frac{1}{4}}-3\left(\frac{x}{a}-\frac{a}{b}\right)^{\frac{1}{4}}=0$$

or, $$\left(\frac{x}{a}+\frac{a}{b}\right)^{\frac{1}{4}}=3\left(\frac{x}{a}-\frac{a}{b}\right)^{\frac{1}{4}}$$

$$\frac{x}{a}+\frac{a}{b}=81\left(\frac{x}{a}-\frac{a}{b}\right)$$

or, $$\frac{80x}{a}=\frac{82a}{b}, \qquad x=\frac{82a^2}{80b}=\frac{41a^2}{40b}$$

36 $$\frac{x-47}{\sqrt{x+2}-7}+\frac{x-19}{\sqrt{x-3}-4}=\frac{4x-124}{\sqrt{4x-5}-11}$$

or, $$\frac{(x+2)-49}{\sqrt{x+2}-7}+\frac{(x-3)-16}{\sqrt{x-3}-4}=\frac{(4x-3)-121}{\sqrt{4x-3}-11}$$

or, $$\sqrt{x+2}+7+\sqrt{x-3}+4=\sqrt{4x-3}+11$$

or, $$\sqrt{x+2}+\sqrt{x-3}=\sqrt{4x-3},$$

$$x+2+x-3+2\sqrt{x^2-x-6}=4x-3$$

or, $$\sqrt{x^2-x-6}=x-1$$

or, $$x^2-x-6=x^2-2x+1, \qquad x=7$$

37 $$\frac{2x-49}{\sqrt{2x+15}-8}+\frac{18x+22}{\sqrt{18x+31}+3}=\frac{8x+191}{2\sqrt{2x+54}-5}$$

or, $$\frac{(2x+15)-64}{\sqrt{2x+15}-8}+\frac{(18x+31)-9}{\sqrt{18x+31}+3}=\frac{4(2x+54)-25}{2\sqrt{2x+54}-5}$$

or, $$\sqrt{2x+15}+8+\sqrt{18x+31}-3=2\sqrt{2x+54}+5$$

or, $$\sqrt{2x+15}+\sqrt{18x+31}=2\sqrt{2x+54}$$

Hence, squaring both sides,

$$2x+15+18x+31+2\sqrt{36x^2+332x+465}=8x+216$$

or, $12x-170=-2\sqrt{36x^2+332x+465}$

or, $6x-85=-\sqrt{36x^2+332x+465}$

$$36x^2-1020x+7225=36x^2+332x+465$$

or, $1352x=6760$, $x=5$

38 By transposition, we have

$$x+a=\sqrt{a^2+x\sqrt{b^2+x^2}},$$

$$x^2+2ax+a^2=a^2+x\sqrt{b^2+x^2}$$

or, $x+2a=\sqrt{b^2+x^2}$

$$x^2+4ax+4a^2=b^2+x^2$$

or, $4ax=b^2-4a^2$, $\frac{b^2-4a^2}{4a}$

39 $$\sqrt{x^2+9}+\sqrt{x^2-9}=4+\sqrt{34}$$

or, $\sqrt{x^2+9}-4=\sqrt{34}-\sqrt{x^2-9}$

$$x^2+9+16-8\sqrt{x^2+9}=34+x^2-9-2\sqrt{34x^2-306}$$

or, $4\sqrt{x^2+9}=\sqrt{34x^2-306}$

$$16x^2+144=34x^2-306$$

or, $18x^2=450$ or, $x^2=25$, $x=5$

40 $$\sqrt{3x^2+16}-\sqrt{3x^2-16}=8-4\sqrt{2}$$

or, $\sqrt{3x^2+16}+4\sqrt{2}=\sqrt{3x^2-16}+8$

$$3x^2+16+32+8\sqrt{6x^2+32}=3x^2-16+64+16\sqrt{3x^2-16}$$

or, $\sqrt{6x^2+32}=2\sqrt{3x^2-16}$

$$6x^2+32=12x^2-64$$

or, $6x^2=96$ or, $x^2=16$, $x=4$

Exercise (90)

Note —The expressions must first be arranged according to either descending or ascending powers of any of the algebracal quantities and then the operation performed

1

$$
\begin{array}{r|l|l}
 & 4x^2z^2+12xyz+9y^2 & 2xz+3y \\
 & 4x^2z^2 & \\ \hline
4xz+3y & 12xyz+9y^2 & \\
 & 12xyz+9y^2 & \\ \hline
\end{array}
$$

Thus the required root $=2xz+3y$

2

$$
\begin{array}{r|l|l}
 & x^4-4x^3+10x^2-12x+9 & x^2-2x+3 \\
 & x^4 & \\ \hline
2x^2-2x & -4x^3+10x^2-12x+9 & \\
 & -4x^3+\ \ 4x^2 & \\ \hline
2x^2-4x+3 & 6x^2-12x+9 & \\
 & 6x^2-12x+9 & \\ \hline
\end{array}
$$

Thus the required root $=x^2-2x+3$

3

$$
\begin{array}{r|l|l}
 & x^6-2x^4+2x^3+x^2-2x+1 & x^3-x+1 \\
 & x^6 & \\ \hline
2x^3-x & -2x^4+2x^3+x^2-2x+1 & \\
 & -2x^4\qquad\ \ +x^2 & \\ \hline
2x^3-2x+1 & 2x^3-2x+1 & \\
 & 2x^3-2x+1 & \\
\end{array}
$$

Thus the required root $=x^3-x+1$

4

$$
\begin{array}{r|l|l}
 & 4x^4-12x^3+25x^2-24x+16 & 2x^2-3x+4 \\
 & 4x^4 & \\ \hline
4x^2-3x & -12x^3+25x^2-24x+16 & \\
 & -12x^3+\ \ 9x^2 & \\ \hline
4x^2-6x+4 & 16x^2-24x+16 & \\
 & 16x^2-24x+16 & \\ \hline
\end{array}
$$

Thus the required root $=2x^2-3x+4$

5

$$
\begin{array}{r|l|l}
 & 4x^4+8ax^3+4a^2x^2+16b^2x^2+16ab^2x+16b^4 & 2x^2+2ax+4b^2 \\
 & 4x^4 & \\ \hline
4x^2+2ax & 8ax^3+4a^2x^2+16b^2x^2+16ab^2x+16b^4 & \\
 & 8ax^3+4a^2x^2 & \\ \hline
4x^2+4ax+4b^2 & 16b^2x^2+16ab^2x+16b^4 & \\
 & 16b^2x^2+16ab^2x+16b^4 & \\ \hline
\end{array}
$$

Thus the required root $=2x^2+2ax+4b^2$

6

$$9x^4-2x^3y+\tfrac{163}{9}x^2y^2-2xy^3+9y^4\ \Big(\ 3x^2-\frac{xy}{3}+3y^2$$
$$9x^4$$
$$6x^2-\frac{xy}{3}\Big)\ -2x^3y+\tfrac{163}{9}x^2y^2-2xy^3+9y^4$$
$$-2x^3y+\tfrac{1}{9}x^2y^2$$
$$6x^2-\frac{2xy}{3}+3y^2\Big)\ 18x^2y^2-2xy^3+9y^4$$
$$18x^2y^2-2xy^3+9y^4$$

Thus the required root $=3x^2-\dfrac{xy}{3}+3y^2$

7

$$x^4-2x^3+\frac{3x^2}{2}-\frac{x}{2}+\frac{1}{16}\ \Big(\ x^2-x+\tfrac{1}{4}$$
$$x^4$$
$$2x^2-x\Big)\ -2x^3+\frac{3x^2}{2}-\frac{x}{2}+\frac{1}{16}$$
$$-2x^3+x^2$$
$$2x^2-2x+\tfrac{1}{4}\Big)\ \frac{x^2}{2}-\frac{x}{2}+\frac{1}{16}$$
$$\frac{x^2}{2}-\frac{x}{2}+\frac{1}{16}$$

Thus the required root $=x^2-x+\frac{1}{4}$

8

$$49x^4-\frac{14x^3}{5}+\frac{1051x^2}{25}-\frac{6x}{5}+9\ \Big(\ 7x^2-\frac{x}{5}+3$$
$$49x^4$$
$$14x^2-\frac{x}{5}\Big)\ -\frac{14x^3}{5}+\frac{1051x^2}{25}-\frac{6x}{5}+9$$
$$-\frac{14x^3}{5}+\frac{x^2}{25}$$
$$14x^2-\frac{2x}{5}+3\Big)\ 42x^2-\frac{6x}{5}+9$$
$$42x^2-\frac{6x}{5}+9$$

Thus the required root $=7x^2-\dfrac{x}{5}+3$

9

$$x^4-x^3+\frac{x^2}{4}+4x-2+\frac{4}{x^2}\Big(x^2-\frac{x}{2}+\frac{2}{x}$$

$$x^4$$

$$2x^2-\frac{x}{2}\Big)\;-x^3+\frac{x^2}{4}+4x-2+\frac{4}{x^2}$$

$$-x^3+\frac{x^2}{4}$$

$$2x^2-x+\frac{2}{x}\Big)\;4x-2+\frac{4}{x^2}$$

$$4x-2+\frac{4}{x^2}$$

Thus the required root $=x^2-\frac{x}{2}+\frac{2}{x}$

10

$$\frac{a^4}{4}+\frac{a^3}{x}+\frac{a^2}{x^2}-ax-2+\frac{x^2}{a^2}\Big(\frac{a^2}{2}+\frac{a}{x}-\frac{x}{a}$$

$$\frac{a^4}{4}$$

$$a^2+\frac{a}{x}\Big)\;\frac{a^3}{x}+\frac{a^2}{x^2}-ax-2+\frac{x^2}{a^2}$$

$$\frac{a^3}{x}+\frac{a^2}{x^2}$$

$$a^2+\frac{2a}{x}-\frac{x}{a}\Big)\;-ax-2+\frac{x^2}{a^2}$$

$$-ax-2+\frac{a^2}{x^2}$$

Thus the required root $=\frac{a^2}{2}+\frac{a}{x}-\frac{x}{a}$

11

$$\frac{a^2}{4b^2}-\frac{a}{b}\quad 1+\frac{4b}{a}+\frac{4b^2}{a^2}\Big(\frac{b}{2b}-1-\frac{2b}{a}$$

$$\frac{a^2}{4b^2}$$

$$\frac{a}{b}-1\Big)\;-\frac{a}{b}-1$$

$$-\frac{a}{b}+1$$

$$\frac{a}{b}-2-\frac{2b}{a}\Big)\ -2+\frac{4b}{a}+\frac{4b^2}{a^2}$$
$$-2+\frac{4b}{a}+\frac{4b^2}{a^2}$$

Thus the required root $=\frac{a}{2b}-1-\frac{2b}{a}$

12

$$\frac{9a^2}{x^2}-\frac{6a}{5x}+\frac{101}{25}-\frac{4x}{15a}+\frac{4x^2}{9a^2}\Big(\frac{3a}{x}-\frac{1}{5}+\frac{2x}{3a}$$
$$\frac{9a^2}{x^2}$$
$$\frac{6a}{x}-\frac{1}{5}\Big)\ -\frac{6a}{5x}+\frac{101}{25}$$
$$-\frac{6a}{5x}+\frac{1}{25}$$
$$\frac{6a}{x}-\frac{2}{5}+\frac{2x}{3a}\Big)\ 4-\frac{4x}{15a}+\frac{4x^2}{9a^2}$$
$$4-\frac{4x}{15a}+\frac{4x^2}{9a^2}$$

Thus the required root $=\frac{3a}{x}-\frac{1}{5}+\frac{2x}{3a}$

13

$$4x^4-8x^3y^2+4xy^6+y^8\Big(2x^2-2xy^2-y^4$$
$$4x^4$$
$$4x^2-2xy^2\Big)\ -8x^3y^2+4xy^6+y^8$$
$$-8x^3y^2+4x^2y^4$$
$$4x^2-4xy^2-y^4\Big)\ -4x^2y^4+4xy^6+y^8$$
$$-4x^2y^4+4xy^6+y^8$$

Thus the required root $=2x^2-2xy^2-y^4$

14

$$\frac{49x^2}{y^2}-\frac{42x}{y}+7+\frac{6y}{7x}+\frac{y^2}{49x^2}\Big(\frac{7x}{y}-3-\frac{y}{7x}$$
$$\frac{49x^2}{y^2}$$
$$\frac{14x}{y}-3\Big)\ -\frac{42x}{y}+7+\frac{6y}{7x}+\frac{y^2}{49x^2}$$
$$-\frac{42x}{y}+9$$

$$\frac{14x}{y} - 6 - \frac{y}{7x} \Big) -2 + \frac{6y}{7x} + \frac{y^2}{49x^2}$$

$$-2 + \frac{6y}{7x} + \frac{y^2}{49x^2}$$

Thus the required root $= \frac{7x}{y} - 3 - \frac{y}{7x}$

15

$$\frac{x^2}{y^2} - \frac{x}{y} - 1\tfrac{1}{4} + \frac{y}{x} + \frac{y^2}{x^2} \Big(\frac{x}{y} - \frac{1}{2} - \frac{y}{x}$$

$$\frac{x^2}{y^2}$$

$$\frac{2x}{y} - \frac{1}{2} \Big) -\frac{x}{y} - 1\tfrac{1}{4} + \frac{y}{x} + \frac{y^2}{x^2}$$

$$-\frac{x}{y} + \frac{1}{4}$$

$$\frac{2x}{y} - 1 - \frac{y}{x} \Big) -2 + \frac{y}{x} + \frac{y^2}{x^2}$$

$$-2 + \frac{y}{x} + \frac{y^2}{x^2}$$

Thus the required root $= \frac{x}{y} - \frac{1}{2} - \frac{y}{x}$

16

$$\frac{4x^2}{49y^2} - \frac{20x}{7y} + 25\tfrac{3}{7} - \frac{15y}{2x} + \frac{9y^2}{16x^2} \Big(\frac{2x}{7y} - 5 + \frac{3y}{4x}$$

$$\frac{4x^2}{49y^2}$$

$$\frac{4x}{7y} - 5 \Big) -\frac{20x}{7y} + 25\tfrac{3}{7} - \frac{15y}{2x} + \frac{9y^2}{16x^2}$$

$$-\frac{20x}{7y} + 25$$

$$\frac{4x}{7y} - 10 + \frac{3y}{4x} \Big) \frac{3}{7} - \frac{15y}{2x} + \frac{9y^2}{16x^2}$$

$$\frac{3}{7} - \frac{15y}{2x} + \frac{9y^2}{16x^2}$$

Thus the required root $= \frac{2x}{7y} - 5 + \frac{3y}{4x}$

17

$$x^2 - 2x^{\frac{3}{2}} + 3x - 2x^{\frac{1}{2}} + 1 \left(x - x^{\frac{1}{2}} + 1 \right.$$
$$x^2$$
$$2x - x^{\frac{1}{2}} \left) \; -2x^{\frac{3}{2}} + 3x - 2x^{\frac{1}{2}} + 1 \right.$$
$$-2x^{\frac{3}{2}} + x$$
$$2x - 2x^{\frac{1}{2}} + 1 \left) \; 2x - 2x^{\frac{1}{2}} + 1 \right.$$
$$2x - 2x^{\frac{1}{2}} + 1$$

Thus the required root $= x - x^{\frac{1}{2}} + 1$

18

$$x^{\frac{5}{3}} - 4x^{\frac{4}{3}} + 2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}} \left(x^{\frac{5}{6}} - 2x^{\frac{1}{2}} - x^{\frac{1}{6}} \right.$$
$$x^{\frac{5}{3}}$$
$$2x^{\frac{5}{6}} - 2x^{\frac{1}{2}} \left) \; -4x^{\frac{4}{3}} + 2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}} \right.$$
$$-4x^{\frac{4}{3}} + 4x$$
$$2x^{\frac{5}{6}} - 4x^{\frac{1}{2}} - x^{\frac{1}{6}} \left) \; -2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}} \right.$$
$$-2x + 4x^{\frac{2}{3}} + x^{\frac{1}{3}}$$

Thus the required root $= x^{\frac{5}{6}} - 2x^{\frac{1}{2}} - x^{\frac{1}{6}}$

19

$$a^2x^{-2} + 2ax^{-1} + 3 + 2a^{-1}x + a^{-2}x^2 \left(ax^{-1} + 1 + a^{-1}x \right.$$
$$a^2x^{-2}$$
$$2ax^{-1} + 1 \left) \; 2ax^{-1} + 3 + 2a^{-1}x + a^{-2}x^2 \right.$$
$$2ax^{-1} + 1$$
$$2ax^{-1} + 2 + a^{-1}x \left) \; 2 + 2a^{-1}x + a^{-2}x^2 \right.$$
$$2 + 2a^{-1}x + a^{-2}x^2$$

Thus the required root $= ax^{-1} + 1 + a^{-1}x$

20

$$\left[\left(x^{\frac{3}{4}} - x^{\frac{1}{2}}y^{-\frac{1}{4}} + y^{\frac{1}{2}} \right.\right.$$
$$x^{\frac{3}{2}} - x^{\frac{5}{4}}y^{-\frac{1}{4}} + xy^{-\frac{1}{2}} + 2x^{\frac{3}{4}}y^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{4}} + y$$
$$x^{\frac{3}{2}}$$
$$2x^{\frac{3}{4}} - x^{\frac{1}{2}}y^{-\frac{1}{4}} \left) \; -2x^{\frac{5}{4}}y^{-\frac{1}{4}} + xy^{-\frac{1}{2}} \right.$$
$$-2x^{\frac{5}{4}}y^{-\frac{1}{4}} + xy^{-\frac{1}{2}}$$
$$2x^{\frac{3}{4}} - 2x^{\frac{1}{2}}y^{-\frac{1}{4}} + y^{\frac{1}{2}} \left) \; 2x^{\frac{3}{4}}y^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{4}} + y \right.$$
$$2x^{\frac{3}{4}}y^{\frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{4}} + y$$

Thus the required root $= x^{\frac{3}{4}} - x^{\frac{1}{2}}y^{-\frac{1}{4}} + y^{\frac{1}{2}}$

$$\left[\left(\frac{3x^{\frac{3}{2}}}{2} - \frac{5}{3}xy^{\frac{1}{2}} + \frac{2}{5}x^{\frac{1}{2}}y\right.\right.$$

21

$$\frac{9x^3}{4} - 5x^{\frac{5}{2}}y^{\frac{1}{2}} + \frac{179x^2y}{45} - \frac{4x^{\frac{3}{2}}y^{\frac{3}{2}}}{3} + \frac{4xy^2}{25}$$

$$\frac{9x^3}{4}$$

$$3x^{\frac{3}{2}} - \frac{5}{3}xy^{\frac{1}{2}}\ \Big)\ -5x^{\frac{5}{2}}y^{\frac{1}{2}} + \frac{179x^2y}{45} - \frac{4x^{\frac{3}{2}}y^{\frac{3}{2}}}{3} + \frac{4xy^2}{25}$$

$$-5x^{\frac{5}{2}}y^{\frac{1}{2}} + \frac{25}{9}x^2y$$

$$3x^{\frac{3}{2}} - \frac{10}{3}xy^{\frac{1}{2}} + \frac{2}{5}x^{\frac{1}{2}}y\ \Big)\ \frac{6}{5}x^2y - \frac{4x^{\frac{3}{2}}y^{\frac{3}{2}}}{3} + \frac{4xy^2}{25}$$

$$\frac{6}{5}x^2y - \frac{4x^{\frac{3}{2}}y^{\frac{3}{2}}}{3} + \frac{4xy^2}{25}$$

Thus the required root $= \frac{3x^{\frac{3}{2}}}{2} - \frac{5}{3}xy^{\frac{1}{2}} + \frac{2}{5}x^{\frac{1}{2}}y$

22

$$a^{2m} - 4a^{m+n} + 4a^{2n}\ \Big(\ a^m - 2a^n$$

$$a^{2m}$$

$$2a^m - 2a^n\ \Big)\ -4a^{m+n} + 4a^{2n}$$

$$-4a^{m+n} + 4a^{2n}$$

Thus the required root $= a^m - 2a^n$

$$\left[\left(\ a^{2m+1} + 3a^m - 5c^{m-2}\right.\right.$$

23

$$a^{4m+2} + 6a^{3m+1} + 9a^{2m} - 10a^{2m+1}c^{m-2} - 30a^mc^{m-2} + 25c^{2m-4}$$

$$a^{4m+2}$$

$$2a^{2m+1} + 3a^m\ \Big)\ 6a^{3m+1} + 9a^{2m} - 10a^{2m+1}c^{m-2} - 30a^mc^{m-2}$$

$$6a^{3m+1} + 9a^{2m}$$

$$2a^{2m+1} + 6a^m - 5c^{m-2}\ \Big)\ -10a^{2m+1}c^{m-2} - 30a^mc^{m-2} + 25c^{2m-4}$$

$$-10a^{2m+1}c^{m-2} - 30a^mc^{m-2} + 25c^{2m-4}$$

Thus the required root $= a^{2m+1} + 3a^m - 5c^{m-2}$

Exercise (91)

1 $25x^2y^2-40xy+16=(5xy)^2-2\cdot 5xy\cdot 4+(4)^2=(5xy-4)^2$,

the required root $=5xy-4$

2 $49a^2x^4-42ab^2x^2+9b^4=(7ax^2)^2-2(7ax^2)(3b^2)+(3b^2)^2$

$=(7ax^2-3b^2)^2$,

the required root $=7ax^2-3b^2$

3 $49a^6b^8+126a^7b^7+81a^8b^6$

$=(7a^3b^4)^2+2(7a^3b^4)(9a^4b^3)+(9a^4b^3)^2=(7a^3b^4+9a^4b^3)^2$,

the required root $=7a^3b^4+9a^4b^3$.

4 $\dfrac{x^8y^4}{4}-\dfrac{x^7y^7}{5}+\dfrac{x^6y^{10}}{25}$

$=\left(\dfrac{x^4y^2}{2}\right)^2-2\left(\dfrac{x^4y^2}{2}\right)\left(\dfrac{x^3y^5}{5}\right)+\left(\dfrac{x^3y^5}{5}\right)^2=\left(\dfrac{x^4y^2}{2}-\dfrac{x^3y^5}{5}\right)^2$,

the required root $=\dfrac{x^4y^2}{2}-\dfrac{x^3y^5}{5}$

5 $\dfrac{25a^2b^2}{4}+\dfrac{c^4}{9}-\dfrac{5abc^2}{3}$

$=\left(\dfrac{5ab}{2}\right)^2-2\left(\dfrac{5ab}{2}\right)\left(\dfrac{c^2}{3}\right)+\left(\dfrac{c^2}{3}\right)^2=\left(\dfrac{5ab}{2}-\dfrac{c^2}{3}\right)^2$,

the required root $=\dfrac{5ab}{2}-\dfrac{c^2}{3}$

6 The given expression

$=(a^2+2ab+b^2)+(2ac+2bc)+c^2=(a+b)^2+2c(a+b)+c^2$

$=\{(a+b)+c\}^2$,

the required root $=a+b+c$

7 The given expression

$=a^2-2ab+b^2+2ac-2bc+c^2=(a-b)^2+2c(a-b)+c^2$

$=\{(a-b)+c\}^2$,

the required root $=a-b+c$

8 The given expression

$=(2a)^2+b^2-2(2a)b+6bc-12ac+9c^2$

$=(2a-b)^2-2(2a-b)(3c)+(3c)^2=\{(2a-b)-3c\}^2$,

the required root $=2a-b-3c$

9 The given expression

$$=a^4+2a^2(2b^2)+(2b^2)^2-6a^2c^2-12b^2c^2+9c^4$$

$$=(a^2+2b^2)^2-2(a^2+2b^2)(3c^2)+(3c^2)^2=(a^2+2b^2-3c^2)^2\ ,$$

the required root $=a^2+2b^2-3c^2$

10 The given expression

$$=(2a^2)^2-2(2a^2)(3b^2)+(3b^2)^2+20a^2c^2-30b^2c^2+(5c^2)^2$$

$$=(2a^2-3b^2)^2+2(2a^2-3b^2)(5c^2)+(5c^2)^2=(2a^2-3b^2+5c^2)^2\ ,$$

the required root $=2a^2-3b^2+5c^2$

11 The given expression

$$=x^2-bx+\left(\frac{b}{2}\right)^2+\frac{2ax}{3}-\frac{ab}{3}+\frac{a^2}{9}$$

$$=\left(x-\frac{b}{2}\right)^2+2\left(x-\frac{b}{2}\right)\frac{a}{3}+\left(\frac{a}{3}\right)^2=\left(x-\frac{b}{2}+\frac{a}{3}\right)^2\ ,$$

the required root $=x-\frac{b}{2}+\frac{a}{3}$

12 The given expression

$$=\left(x-\frac{1}{x}\right)^2+4-4\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)2+(2)^2$$

$$=\left(x-\frac{1}{x}-2\right)^2\ ,$$

the required root $=x-2-\frac{1}{x}$

13 The given expression

$$=x^4+2+\frac{1}{x^4}+2\left(x^2+\frac{1}{x^2}\right)+1=\left(x^2+\frac{1}{x^2}\right)^2+2\left(x^2+\frac{1}{x^2}\right)+1$$

$$=\left\{\left(x^2+\frac{1}{x^2}\right)+1\right\}^2\ ,$$

the required root $=x^2+1+\frac{1}{x^2}$

14 The given expression

$$=\frac{a^2}{b^2}+2+\frac{b^2}{a^2}+\frac{2a}{b}+\frac{2b}{a}+1$$

$$=\left(\frac{a}{b}+\frac{b}{a}\right)^2+2\left(\frac{a}{b}+\frac{b}{a}\right)+1=\left(\frac{a}{b}+\frac{b}{a}+1\right)^2\ ,$$

the required root $=\frac{a}{b}+1+\frac{b}{a}$

15 The given expression

$$=\frac{x^2}{y^2}+2+\frac{y^2}{x^2}-\left(\frac{x}{y}+\frac{y}{x}\right)\sqrt{2}+\frac{1}{2}$$

$$=\left(\frac{x}{y}+\frac{y}{x}\right)^2-2\left(\frac{x}{y}+\frac{y}{x}\right)\frac{1}{\sqrt{2}}+\left(\frac{1}{\sqrt{2}}\right)^2=\left(\frac{x}{y}+\frac{y}{x}-\frac{1}{\sqrt{2}}\right)^2,$$

$$\text{the required root}=\frac{x}{y}-\frac{1}{\sqrt{2}}+\frac{y}{x}$$

16 The given expression

$$=\left(\frac{3x}{a}\right)^2-2\left(\frac{3x}{a}\right)+1-\frac{2a}{3x}+2+\frac{a^2}{9x^2}$$

$$=\left(\frac{3x}{a}-1\right)^2+2\left(\frac{3x}{a}-1\right)\frac{a}{3x}+\left(\frac{a}{3x}\right)^2=\left(\frac{3x}{a}-1+\frac{a}{3x}\right)^2,$$

$$\text{the required root}=\frac{3x}{a}-1+\frac{a}{3x}$$

17 The given expression

$$=x^2+\frac{1}{x^2}+2+4\left(x+\frac{1}{x}\right)+4$$

$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)2+(2)^2=\left(x+\frac{1}{x}+2\right)^2,$$

$$\text{the required root}=x-2+\frac{1}{x}$$

18 The given expression

$$=\left(a\sqrt{2}\right)^2-2\left(a\sqrt{2}\right)\left(a-\sqrt{2}\right)+\left(a-\sqrt{2}\right)^2$$

$$=\left(a\sqrt{2}-a+\sqrt{2}\right)^2,$$

$$\text{the required root}=a\sqrt{2}-a+\sqrt{2}$$

19 The given expression

$$=a^2-2a(b-c+d)+b^2-2b(c-d)+c^2+d^2-2cd$$

$$=a^2-2a(b-c+d)+b^2-2b(c-d)+(c-d)^2$$

$$=a^2-2a(b-c+d)+\{b-(c-d)\}^2$$

$$=a^2-2a(b-c+d)+(b-c+d)^2=\{a-(b-c+d)\}^2$$

$$=(a-b+c-d)^2$$

$$\therefore\ \text{the required root}=a-b+c-d$$

20 The given expression

$$= \{(a-b)^2\}^2 - 2(a^2+b^2)(a-b)^2 + (a^2+b^2)^2 + a^4 + b^4 - 2a^2b^2$$
$$= \{(a-b)^2 - (a^2+b^2)\}^2 + (a^2-b^2)^2$$
$$= (a^2-2ab+b^2-a^2-b^2)^2 + (a^2-b^2)^2 = (a^2-b^2)^2 + (-2ab)^2$$
$$= a^4+b^4-2a^2b^2+4a^2b^2 = a^4+b^4+2a^2b^2 = (a^2+b^2)^2\ ,$$

the required root $= a^2+b^2$

21 The given expression

$$= a^4+b^4-2a^2b^2+c^4+d^4-2c^2d^2+2a^2c^2-2a^2d^2-2b^2(c^2-d^2)$$
$$= (a^2-b^2)^2+(c^2-d^2)^2+2a^2(c^2-d^2)-2b^2(c^2-d^2)$$
$$= (a^2-b^2)^2+(c^2-d^2)^2+2(a^2-b^2)(c^2-d^2) = \{(a^2-b^2)+(c^2-d^2)\}^2$$
$$= (a^2-b^2+c^2-d^2)^2\ ,$$

the required root $= a^2-b^2+c^2-d^2$

22 The given expression

$$= (a^2)^2 + 2(a^2)a + a^2 - (a^2+a) + \tfrac{1}{4}$$
$$= (a^2+a)^2 - 2(a^2+a)\tfrac{1}{2} + (\tfrac{1}{2})^2 = (a^2+a-\tfrac{1}{2})^2\ ,$$

the required root $= a^2+a-\frac{1}{2}$

23 The given expression

$$= 2\{a^2(b+c)^2 + b^2(c+a)^2 + c^2(a+b)^2 + 2abc(a+b+c)\}$$
$$= 2\{a^2(b+c)^2 + a^2(b^2+c^2+2bc) + 2a(b^2c+c^2b)$$
$$+ 2abc(b+c) + 2b^2c^2\}$$
$$= 2\{a^2(b+c)^2 + a^2(b+c)^2 + 2abc(b+c) + 2abc(b+c) + 2b^2c^2\}$$
$$= 4\{a^2(b+c)^2 + 2abc(b+c) + b^2c^2\} = 4\{a(b+c)+bc\}^2\ ,$$

the required root $= 2a(b+c)+2bc$

Exercise (92)

1

$$\begin{array}{r|l}
 & x^3+27x^2+243x+729\ (\ x+9 \\
 & x^3 \\
\hline
3\times(x)^2 = 3x^2 & 27x^2+243x+729 \\
3x\times 9 = 27x & \\
(9)^2 = 81 & 27x^2+243x+729 \\
\hline
3x^2+27x+81 & \\
\end{array}$$

Thus the required root $= x+9$

2

$$27x^3 - 216x^2 + 567x - 512 \,(\, 3x - 8$$
$$27x^3$$

$$\begin{array}{r|l}
3\times(3x)^2 = 27x^2 & -216x^2 + 567x - 512 \\
3\times 3x\times(-8) = -72x & \\
(-8)^2 = 64 & \\
\hline
27x^2 - 72x + 64 & -216x^2 + 576x - 512
\end{array}$$

Thus the required root $= 3x - 8$

3

$$64a^3 - 144a^2b + 108ab^2 - 27b^3 \,(\, 4a - 3b$$
$$64b^3$$

$$\begin{array}{r|l}
3\times(4a)^2 = 48a^2 & -144a^2b + 108ab^2 - 27b^3 \\
3\times 4a(-3b) = -36ab & \\
(-3b)^2 = 9b^2 & \\
\hline
48a^2 - 36ab + 9b^2 & -144a^2b + 108ab^2 - 27b^3
\end{array}$$

Thus the required root $= 4a - 3b$

4

$$x^6 - 9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8 \,(\, x^2 - 3x + 2$$
$$x^6$$

$$\begin{array}{r|l}
3\times(x^2) = 3x^4 & -9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8 \\
3\times x^2\times(-3x) = -9x^3 & \\
(-3x)^2 = 9x^2 & \\
\hline
3x^4 - 9x^3 + 9x^2 & -9x^5 + 27x^4 - 27x^3 \\
\hline
& 6x^4 - 36x^3 + 66x^2 - 36x + 8 \\
3\times(x^2 - 3x)^2 = 3x^4 - 18x^3 + 27x^2 & \\
3\times(x^2 - 3x)\times 2 = 6x^2 - 18x & \\
(2)^2 = 4 & \\
\hline
3x^4 - 18x^3 + 33x^2 - 18x + 4 & 6x^4 - 36x^3 + 66x^2 - 36x + 8
\end{array}$$

Thus the required root $= x^2 - 3x + 2$

5

$$8x^6 + 12x^5 - 30x^4 - 35x^3 + 45x^2 + 27x - 27 \,(\, 2x^2 + x - 3$$
$$8x^6$$

$$\begin{array}{r|l}
3\times(2x^2)^2 = 12x^4 & 12x^5 - 30x^4 - 35x^3 + 45x^2 + 27x - 27 \\
3\times 2x^2\times x = 6x^3 & \\
(x)^2 = x^2 & \\
\hline
12x^4 + 6x^3 + x^2 & 12x^5 + 6x^4 + x^3 \\
\hline
& -36x^4 - 36x^3 + 45x^2 + 27x - 27 \\
3\times(2x^2 + x)^2 = 12x^4 + 12x^3 + 3x^2 & \\
3\times(2x^2 + x)\times(-3) = \quad -18x^2 - 9x & \\
(-3)^2 = \quad 9 & \\
\hline
12x^4 + 12x^3 - 15x^2 - 9x + 9 & -36x^4 - 36x^3 + 45x^2 + 27x - 27
\end{array}$$

Thus the required root $= 2x + x - 3$

6.

$$1-9x^2+33x^4-63x^6+66x^8-36x^{10}+8x^{12}\,\big(\,1-3x^2+2x^4$$

$$\begin{array}{l} 1 \\ \hline -9x^2+33x^4-63x^6+66x^8-36x^{10}+8x^{12} \\ -9x^2+27x^4-27x^6 \\ \hline 6x^4-36x^6+66x^8-36x^{10}+8x^{12} \\ 6x^4-36x^6+66x^8-36x^{10}+8x^{12} \\ \hline \end{array}$$

$$\begin{array}{rr} 3\times 1^2 = & 3 \\ 3\times 1\times(-3x^2) = & -9x^2 \\ (-3x^2)^2 = & 9x^4 \\ \hline & 3-9x^2+9x^4 \end{array}$$

$$\begin{array}{rr} 3(1-3x^2)^2 = & 3-18x^2+27x^4 \\ 3(1-3x^2)\times 2x^4 = & 6x^4-18x^6 \\ (2x^4)^2 = & 4x^8 \\ \hline & 3-18x^2+33x^4-18x^6 \\ & +4x^8 \end{array}$$

Thus the required root $=1-3x^2+2x^4$

7

$$8x^6-36cx^5+66c^2x^4-63c^3x^3+33c^4x^2-9c^5x+c^6\,\big(\,2x^2-3cx+c^2$$

$$\begin{array}{l} 8x^6 \\ \hline -36cx^5+66c^2x^4-63c^3x^3+33c^4x^2-9c^5x+c^6 \\ -36cx^5+54c^2x^4-27c^3x^3 \\ \hline 12c^2x^4-36c^3x^3+33c^4x^2-9c^5x+c^6 \\ 12c^2x^4-36c^3x^3+33c^4x^2-9c^5x+c^6 \\ \hline \end{array}$$

$$\begin{array}{rr} 3\times(2x^2)^2 = & 12x^4 \\ 3\times(2x^2)\times(-3cx) = & -18cx^3 \\ (-3cx)^2 = & 9c^2x^2 \\ \hline & 12x^4-18cx^3+9c^2x^2 \end{array}$$

$$\begin{array}{rr} 3(2x^2-3cx)^2 = & 12x^4-36cx^3+27c^2x^2 \\ 3(2x^2-3cx)\times c^2 = & 6c^2x^2-9c^3x \\ (c^2)^2 = & c^4 \\ \hline & 12x^4-36cx^3+33c^2x^2 \\ & -9c^3x+c^4 \end{array}$$

Thus the required root $=2x^2-3cx+c^2$

Exercise (93).

1 We have $4 \quad 5=32 \quad 40$ and $7 \quad 8=35 \quad 40$,

$$7 \quad 8>4 \quad 5$$

2 We have $7 \quad 10=49 \quad 70$ and $11 \quad 14=55 \quad 70$,

$$11 . 14>7 \quad 10$$

3 We have $9 \quad 5=72 \quad 40$ and $13 \quad 8=65 \quad 40$,

$$9 \quad 5>13 \quad 8$$

4 We have $22 \quad 27=110 \quad 135$ and $32 \quad 45=96 \quad 135$,

$$22 \quad 27>32 \quad 45$$

5 We have $28 \quad 39=140 \quad 195$, and $49 \quad 65=147 \quad 195$,

$$49 \quad 65>28 \quad 39$$

6 The required ratio$=\frac{a}{b}\times\frac{b}{c}\times\frac{c}{d}=\frac{a}{d}=a \quad d$

7 The required ratio$=\frac{3}{5}\times\frac{7}{9}\times\frac{15}{28}=\frac{1}{4}=1 \quad 4$

8 The required ratio$=\frac{a+x}{a-x}\times\frac{a^2+x^2}{(a+x)^2}\times\frac{(a^2-x^2)^2}{a^4-x^4}$

$$=\frac{(a^2+x^2)(a^2-x^2)}{(a-x)(a+x)(a^2+x^2)}=1=1 \quad 1$$

9 The required ratio$=\frac{16}{5}\times\left(\frac{5}{4}\right)^3\times\left(\frac{9}{4}\right)^{\frac{1}{2}}$

$$=\frac{16}{5}\times\frac{5\times5\times5}{4\times4\times4}\times\frac{3}{2}=\frac{75}{8}=75 \quad 8$$

10 The required ratio$=\frac{25}{18}\times\left(\frac{81}{49}\right)^{\frac{1}{2}}\times\left(\frac{2}{3}\right)^3\times\left(\frac{7}{5}\right)^2$

$$=\frac{25}{18}\times\frac{9}{7}\times\frac{8}{27}\times\frac{49}{25}=\frac{28}{27}=28 \quad 27$$

11 We have $\frac{2x+5y}{3x+5y}=\frac{9}{10}$, $\quad 20x+50y=27x+45y$

or, $\quad 7x=5y \quad$ or, $\frac{x}{y}=\frac{5}{7}$, $\quad x \quad y=5 \quad 7$

12 We have $\frac{x}{y}=\frac{3}{4} \quad x=\frac{3}{4}y$

Hence, $\frac{5x+9y}{16x+5y}=\frac{\frac{15}{4}y+9y}{12y+5y}=\frac{15+36}{4\times17}=\frac{51}{4\times17}=\frac{3}{4}=3 \quad 4$

13 Let x and y be the required numbers

we get $\frac{x}{y}=\frac{7}{8}$ (1)

and $x+y=135$ (2)

From (1), $x=\frac{7}{8}y$

From (2), $y(\frac{7}{8}+1)=135$ or, $\frac{15}{8}y=135$, $\therefore$ $y=72$

Hence $x=135-y=135-72=63$

Thus the numbers are 72 and 63

14 Let x and y be the required numbers of which x is the greater

we get $\frac{x}{y}=\frac{5}{3}$ (1)

and $x-y=34$ (2)

From (1), $x=\frac{5}{3}y$

From (2) $y(\frac{5}{3}-1)=34$ or, $\frac{2}{3}y=34$, $\therefore$ $y=51$

Hence, from (2), $x=85$

Thus the numbers are 85 and 51

15 Let x and y be the required numbers, of which y is the greater

we get $\frac{x}{y}=\frac{4}{5}$ (1)

and $\frac{x+7}{y+7}=\frac{5}{6}$ (2)

From (1) we get $x=\frac{4}{5}y$

From (2) we get $6x+42=5y+35$ or, $\frac{24}{5}y+7=5y$

or, $25y-24y=35$, $y=35$ and $\therefore$ $x=28$

Thus the required numbers are 28 and 35

16 Let x and y be the required numbers, of which y is the greater

$\therefore$ we get $\frac{x}{y}=\frac{7}{9}$ (1)

and $\frac{x-10}{y-10}=\frac{8}{11}$ (2)

From (1) we get $9x-7y=0$ (3)

From (2) we get $11x-110=8y-80$

or, $11x-8y-33=0$ (4)

From (3) and (4), we get by cross multiplication

$$\frac{x}{210}=\frac{y}{270}=\frac{1}{-72+77}=\frac{1}{5}$$

$x=42$ and $y=54$

Thus the required numbers are 42 and 54

17 We have $\dfrac{23+x}{19+x}=2$

or, $23+x=38+2x$, $x=-15$

18 Let x be the required number

Then we have $\dfrac{25+x}{37+x}=\dfrac{5}{6}$

or, $150+6x=185+5x$, $x=35$

19 Let x be the required number

Then we have $\dfrac{29+x}{38+x}=\dfrac{4}{7}$

or, $203+7x=152+4x$ or, $3x=-51$, $x=-71$

20 Let x be the required quantity

Then we have $\dfrac{a+x}{b+x}=\dfrac{c}{d}$

or, $ad+dx=bc+cx$ or, $cx-dx=ad-bc$,

$$x=\frac{ad-bc}{c-d}$$

21 We have $\dfrac{a^2-x^2}{a^2+x^2}-\dfrac{a-x}{a+x}$

$$=(a-x)\left\{\frac{(a+x)^2-(a^2+x^2)}{(a^2+x^2)(a+x)}\right\}=\frac{(a-x)\,2ax}{(a^2+x^2)(a+x)}$$

Now $(a-x)$ is positive, a being greater than x,

the difference of the ratios is positive and hence the former is greater than the latter

22 We have $\dfrac{a^2+b^2}{a+b}-\dfrac{a^2-b^2}{a-b}$

$$=\frac{a^2+b^2}{a+b}-(a+b)=\frac{a^2+b^2-a^2-b^2-2ab}{a+b}=\frac{-2ab}{a+b},$$

which is negative, a and b both being positive

Hence $a^2+b^2 \quad a+b < a^2-b^2 \quad a-b$

23 $(226)^2 \quad (225)^2 = \left(\frac{226}{225}\right)^2 = \left\{1 + \frac{1}{225}\right\}^3 = 1 + \frac{3}{225} +$ etc

$= 1 + \frac{1}{75}$ (neglecting the other terms which are very small)

$= \frac{76}{75}$

24 $\sqrt{3546} \quad \sqrt{3542} = \left(\frac{3546}{3542}\right)^{\frac{1}{2}} = \left(\frac{1773}{1771}\right)^{\frac{1}{2}} = \frac{(1773 \times 1771)^{\frac{1}{2}}}{1771}$

$= \frac{\{(1772+1)(1772-1)\}^{\frac{1}{2}}}{1771} = \frac{\{(1772)^2 - 1\}^{\frac{1}{2}}}{1771} = \frac{1772}{1771}$ (approximately)

25 The ratio of A's expenditure to his income $= \frac{8\frac{3}{4}}{15}$ (1)

B's $= \frac{11\frac{1}{4}}{20}$ (2)

C's $= \frac{15\frac{5}{8}}{25}$ (3)

The ratios are $\frac{35}{4 \times 15}$, $\frac{45}{4 \times 20}$ and $\frac{125}{8 \times 25}$

or $\frac{7}{12}$, $\frac{9}{16}$ and $\frac{5}{8}$ respectively

These ratios are again equivalent to

$\frac{28}{48}$, $\frac{27}{48}$ and $\frac{30}{48}$ respectively

The second ratio being the least B is the most frugal

Exercise (94)

1 Let x be the required quantity

We get $9 \quad 6 = 6 \quad x$ or, $9x = 6^2 = 36$, $x = 4$

Thus the required quantity is 4

2 Let x be the required quantity

Then we have $8 \quad 12 = 12 \quad x$ or, $8x = 144$, $x = 18$

Thus the required quantity is 18

3 Let x be the required quantity

Then we have $6 \quad 15 = 15 \quad x$

or, $6x = (15)^2 = 225$, $x = 37\frac{1}{2}$

Thus the required quantity is $37\frac{1}{2}$

4 Let x be the required quantity
Then we have $16 \quad 24=24 \quad x$
or, $16x=(24)^2=576$, $\therefore x=36$
Thus 36 is the required quantity

5 Let x be the required quantity
Then we have $6 \quad 8=15 \quad x$ or, $6x=120$, $x=20$
Thus 20 is the required quantity

6 Let x be required quantity
Then we have $14 \quad 24=35 \quad x$ or, $14x=24\times 35$, $x=60$
Thus 60 is the required quantity

7 Let x be the required quantity
Then we have $0014 \quad 1\ 4=02 \quad x$
or, $0014x=1\ 4\times 02$ or, $14x=14\times 20$, $x=20$
Thus 20 is the required quantity

8 Let x be the required quantity
Then we have $4 \quad x=x \quad 9$ or, $x^2=36$, $x=6$
Thus 6 is the required quantity

9 Let x be the required number
Then we have $7 \quad x=x \quad 28$ or, $x^2=7\times 28$ $\therefore$ $x=14$
Thus 14 is the required number

10 Let x be the required number
Then we have $6 \quad x=x \quad 54$ or, $x^2=6\times 54$, $x=18$
Thus 18 is the required number

Exercise (95)

1 From (1), by componendo and dividendo,
we get $\frac{x}{y}=\frac{6}{4}=\frac{3}{2}$ or, $2x=3y$ (3)
From (2), we get $4x=36$ $x=9$,
from (3) $3y=18$ $y=6$
Thus $x=9$ and $y=6$

2 From (1), by Componendo and dividendo,
we have $\frac{3x}{-5y}=\frac{5}{-3}$ or, $9x=25y$. (3)
$36x-100y=0$,

and from (2) $36x - 81y = 171$

Hence, $-19y = -171$, $y = 9$,

Hence, from (3), $x = 25$

Thus $x = 25$ and $y = 9$

3 From (1) by componendo and dividendo,

we have $\frac{5x}{-7y} = \frac{8}{-6} = \frac{4}{-3}$ or, $15x = 28y$ (3)

$$15x - 28y = 0$$

and from (2), $15x - 25y = 90$

$$-3y = -90, \qquad y = 30$$

$\therefore$ from (3), $15x = 28 \times 30$, $x = 56$

Thus $x = 56$ and $y = 30$

4 $$16\left(\frac{a-x}{a+x}\right)^3 = \frac{a+x}{a-x}$$

or, $$16\left(\frac{a-x}{a+x}\right)^4 = 1$$

or, $$\left(\frac{a-x}{a+x}\right)^4 = \frac{1}{16} = \left(\frac{1}{2}\right)^4$$

or, $\frac{a-x}{a+x} = \frac{1}{2}$, or, $2a - 2x = a + x$

or, $a = 3x$, $x = \frac{a}{3}$

5 $$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

By componendo and dividendo, we get

$\frac{2x}{\sqrt{4x^2 - 1}} = \frac{5}{3}$, or, $\frac{4x^2}{4x^2 - 1} = \frac{25}{9}$

or, $36x^2 = 100x^2 - 25$, or, $64x^2 = 25$

or, $8x = 5$, $x = \frac{5}{8}$

6 By componendo and dividendo $\frac{1}{-\sqrt{1-x}} = \frac{4}{-2} = -2$

or, $1 = 2\sqrt{1-x}$ or, $1 = 4(1-x)$ or, $4x = 3$,

$x = \frac{3}{4}$

7 By componendo and dividendo, $\frac{\sqrt{36x+1}}{\sqrt{36x}} = \frac{10}{8} = \frac{5}{4}$

Hence, squaring both sides, $$\frac{36x+1}{36x}=\frac{25}{16}$$

or, $$16 \cdot 36x+16=25 \cdot 36x$$

or, $9 \cdot 36x=16$, $$x=\frac{16}{9 \cdot 36}=\frac{4}{81}$$

8 By componendo and dividendo, $$\frac{1+x^2}{x}=\frac{125-x}{125x-1}$$

or, $$125x+125x^3-1-x^3=125x-x^3$$

or, $125x^3-1=0$, or, $(5x)^3=1$, or, $5x=1$, $x=\frac{1}{5}$

9 By componendo and dividendo, $$\frac{\sqrt{5}}{\sqrt{5-x}}=\frac{6}{4}=\frac{3}{2}$$

Hence, squaring both sides,

$$\frac{5}{5-x}=\frac{9}{4}\ , \quad \text{or,} \quad 20=45-9x\ , \quad \text{or,} \quad 9x=25\ ,$$

$$x=\frac{25}{9}=2\tfrac{7}{9}$$

10 By componendo and dividendo, $$\frac{a+x}{\sqrt{a^2-x^2}}=\frac{b+x}{b-x}$$

Hence, squaring both sides,

$$\frac{(a+x)^2}{a^2-x^2}=\frac{(b+x)^2}{(b-x)^2}$$

or, $$\frac{a+x}{a-x}=\frac{(b+x)^2}{(b-x)^2}$$

Again, by componendo and dividendo

$$\frac{a}{x}=\frac{(b+x)^2+(b-x)^2}{(b+x)^2-(b-x)^2}=\frac{b^2+x^2}{2bx}$$

or, $2ab=b^2+x^2$, or, $x^2=2ab-b^2$

$$x=\sqrt{2ab-b^2}$$

11 By componendo and dividendo,

$$\frac{a^{\frac{1}{2}}}{\{a-(a^2-ax)^{\frac{1}{2}}\}^{\frac{1}{2}}}=\frac{b+1}{b-1}$$

Hence, squaring both sides,

$$\frac{a}{a-(a^2-ax)^{\frac{1}{2}}}=\frac{(b+1)^2}{(b-1)^2}$$

or, $$a(b-1)^2=a(b+1)^2-(b+1)^2(a^2-ax)^{\frac{1}{2}}$$

or, $$a\{(b+1)^2-(b-1)^2\}=(b+1)^2(a^2-ax)^{\frac{1}{2}}$$

or, $$(b+1)^2(a^2-ax)^{\frac{1}{2}}=4ab$$

or, $$(b+1)^4(a^2-ax)=16a^2b^2$$

or, $$a-x=\frac{16ab^2}{(b+1)^4}$$

$$x=a-\frac{16ab^2}{(b+1)^4}=a\left\{1-\frac{16b^2}{(b+1)^4}\right\}$$

12 We have

$$\frac{a+3b+2c+6d}{a+3b-2c-6d}=\frac{a-3b+2c-6d}{a-3b+2c-6d}$$

Hence, by componendo and dividendo,

$$\frac{a+3b}{2c+6d}=\frac{a-3b}{2c-6d}$$

or, $$\frac{a+3b}{a-3b}=\frac{2c+6d}{2c-6d}=\frac{c+3d}{c-3d}$$ (by alternendo)

Again, by componendo and dividendo,

$$\frac{a}{3b}=\frac{c}{3d}, \qquad \frac{a}{b}=\frac{c}{d} \quad \text{or,} \quad a \quad b \qquad c \quad d$$

13 We have

$$\frac{2a+b+4c+2d}{2a+b-4c-2d}=\frac{2a-b+4c-2d}{2a-b-4c+2d}$$

By componendo and dividendo,

$$\frac{2a+b}{4c+2d}=\frac{2a-b}{4c-2d}$$

or, $$\frac{2a+b}{2a-b}=\frac{4c+2d}{4c-2d}=\frac{2c+d}{2c-d}$$ (alternendo)

Again, by componendo and dividendo,

$$\frac{2a}{b}=\frac{2c}{d} \quad \text{or,} \quad a \quad b \qquad c \quad d$$

14 We have

$$x=\frac{\sqrt{2a+3b}-\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}.$$

By componendo and dividendo,

$$\frac{x+1}{x-1}=\frac{\sqrt{2a+3b}}{\sqrt{2a-3b}}, \qquad \frac{(x+1)^2}{(x-1)^2}=\frac{2a+3b}{2a-3b}$$

Again, by componendo and dividendo,

$$\frac{(x-1)^2-(x-1)^2}{(x+1)^2-(x-1)^2}=\frac{2a}{3b}$$

or, $\frac{2x^2-2}{2x}=\frac{2a}{3b}$ or $\frac{x^2-1}{x}=\frac{2a}{3b}$

or $3bx^2-3b=2ax$ $\therefore$ $3bx^2-2ax-3b=0$

15 We have $x=\frac{2\sqrt{24}}{\sqrt2-\sqrt3}=\frac{2\sqrt3\sqrt8}{\sqrt2-\sqrt3}$. $\frac{x}{\sqrt8}=\frac{2\sqrt3}{\sqrt2-\sqrt3}$

Hence, by componendo and dividendo

$$\frac{x-\sqrt8}{x-\sqrt8}=\frac{3\sqrt3-\sqrt2}{\sqrt3-\sqrt2}$$

Again $x=\frac{2\sqrt2\sqrt{12}}{\sqrt2-\sqrt3}$ or $\frac{x}{\sqrt{12}}=\frac{2\sqrt2}{\sqrt2-\sqrt3}$

Hence, by componendo and dividendo $\frac{x-\sqrt{12}}{x-\sqrt{12}}=\frac{3\sqrt2-\sqrt3}{\sqrt2-\sqrt3}$;

the given expression

5 We have $a^2+c^2+e^2=x^2(b^2+d^2+f^2)$,

the given expression $=(x^2)^{\frac{1}{2}}=x=$ each of the given ratios

6 We have $a^3-2c^3+3e^3=x^3(b^3-2d^3+3f^3)$,

the given expression $=(x^3)^{\frac{1}{3}}=x=$ each of the given ratios

7 We have $\sqrt[3]{a^3+c^3+e^3}=\sqrt[3]{x^3(b^3+d^3+f^3)}=x\sqrt[3]{b^3+d^3+f^3}$,

the given expression $=x=$ each of the given expressions

In each of the following 3 examples, let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=k$,

$a=bk$, $c=dk$, $e=fk$ and $g=hk$

8 We have $a^{-1}+c^{-1}+e^{-1}+g^{-1}=k^{-1}(b^{-1}+d^{-1}+f^{-1}+h^{-1})$,

the given expression $=(k^{-1})^{-1}=k=$ each of the given ratios

9 We have $a^4-2c^4+3e^4-4g^4=k^4(b^4-2d^4+3f^4-4h^4)$,

the given expression $=\sqrt[4]{k^4}=k=$ each of the given ratios

10 We have

$$3a^{-2}-7c^{-2}-8e^{-2}+15g^{-2}$$
$$=k^{-2}(3b^{-2}-7d^{-2}-8f^{-2}+15h^{-2})$$

the given expression $=\sqrt{(k^{-2})^{-1}}=\sqrt{k^2}=k$

$=$ each of the given ratios

Exercise (97)

1 We have
$$\frac{a}{b}=\frac{c}{d} \qquad (1)$$
$$\frac{a}{c}=\frac{b}{d} \qquad (2) \quad \text{(by alternendo)}$$

Both $\frac{a}{b}$ and $\frac{a}{c}$ are ratios of greater inequality for a is greater than both b and c Hence $\frac{c}{d}$ and $\frac{b}{d}$ also are ratios of greater inequality , b and c are each $> d$

2 Because
$$\frac{a}{b}=\frac{c}{d}$$
$$\frac{a-b}{b}=\frac{c-d}{d}$$
or,
$$\frac{a-b}{c-d}=\frac{b}{d}$$

or, $$\frac{a+c}{b+d}=\frac{c}{d}=\frac{a}{b}$$

i e, $$a \quad b=a+c \quad b+d$$

7 $$\frac{a}{b}=\frac{c}{d}$$

or, $$\frac{a}{c}=\frac{b}{d}$$

or, $$\frac{a^2}{c^2}=\frac{b^2}{d^2}$$

or, $$\frac{a^2+c^2}{c^2}=\frac{b^2+d^2}{d^2}$$

or, $$\frac{a^2+c^2}{b^2+d^2}=\frac{c^2}{d^2}=\frac{a^2}{b^2}$$

i e, $$a^2 \quad b^2=a^2+c^2 \quad b^2+d^2$$

8 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose) },$$

$$a=bk \text{ and } c=dk ,$$

$$\frac{a^2+c^2}{b^2+d^2}=\frac{b^2k^2+d^2k^2}{b^2+d^2}=k^2$$

$$=k \quad k=\frac{a}{b} \quad \frac{c}{d}=\frac{ac}{bd}$$

9 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{(a-c)^2}{(b-d)^2}=\frac{(bk-dk)^2}{(b-d)^2}=k^2=\left(\frac{a}{b}\right)^2=\frac{a^2}{b^2}$$

10 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk$$

$$(a+c)^3=k^3(b+d)^3$$

$$\frac{(a+c)^3}{(b+d)^3}=k^3 \qquad (1)$$

Also, we have $a(a-c)^2=bk(bk-dk)^2=k^3\{b(b-d)^2\}$

$$\frac{a(a-c)^2}{b(b-d)^2}=k^3 \qquad (2)$$

Hence, from (1) and (2),

$$(a+c)^3 \quad (b+d)^3=a(a-c)^2 \quad b(b-d)^2$$

11

$$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{a^2+b^2}{a^2-b^2}=\frac{b^2k^2+b^2}{b^2k^2-b^2}=\frac{k^2+1}{k^2-1} \qquad (1)$$

Also, we have

$$\frac{ac+bd}{ac-bd}=\frac{k^2bd+bd}{k^2bd-bd}=\frac{k^2+1}{k^2-1} \qquad (2)$$

Hence, from (1) and (2),

$$a^2+b^2 \quad a^2-b^2=ac+bd \quad ac-bd$$

12

$$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$\therefore \quad a=bk \text{ and } c=dk,$$

$$\frac{a(a+c)}{c^2}=\frac{bk(bk+dk)}{d^2k^2}=\frac{b(b+d)}{d^2}$$

13

$$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{c}{d}=\frac{dk}{d}=k \qquad (1)$$

Also, we have $$\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}=\frac{\sqrt{k^2b^2+k^2d^2}}{\sqrt{b^2+d^2}}=k \qquad (2)$$

Hence, from (1) and (2),

$$c \quad d=\sqrt{a^2+c^2} \quad \sqrt{b^2+d^2}$$

14

$$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{a+b}{c+d}=\frac{b(k+1)}{d(k+1)}=\frac{b}{d} \qquad (1)$$

Also, $$\frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}=\frac{\sqrt{b^2(k^2+1)}}{\sqrt{d^2(k^2+1)}}=\sqrt{\frac{b^2}{d^2}}=\frac{b}{d} \qquad (2)$$

Hence, from (1) and (2),

$$a+b \quad c+d=\sqrt{a^2+b^2} \quad \sqrt{c^2+d^2}$$

15 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{a+b}{c+d}=\frac{bk+b}{dk+d}=\frac{b}{d} \qquad (1)$$

Also, $$\frac{\sqrt{3a^2+5b^2}}{\sqrt{3c^2+5d^2}}=\frac{\sqrt{3b^2k^2+5b^2}}{\sqrt{3d^2k^2+5d^2}}=\frac{b\sqrt{3k^2+5}}{d\sqrt{3k^2+5}}=\frac{b}{d} \qquad (2)$$

Hence, from (1) and (2),

$$a+b : c+d=\sqrt{3a^2+5b^2} : \sqrt{3c^2+5d^2}$$

16 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{b^2k^2+kb^2+b^2}{b^2k^2-kb^2+b^2}=\frac{k^2+k+1}{k^2-k+1} \qquad (1)$$

Also, $$\frac{c^2+cd+d^2}{c^2-cd+d^2}=\frac{d^2k^2+kd^2+d^2}{d^2k^2-kd^2+d^2}=\frac{k^2+k+1}{k^2-k+1} \qquad (2)$$

Hence, from (1) and (2),

$$a^2+ab+b^2 : a^2-ab+b^2=c^2+cd+d^2 : c^2-cd+d^2$$

17 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{a^2+ac+c^2}{a^2-ac+c^2}=\frac{b^2k^2+bdk^2+d^2k^2}{b^2k^2-bdk^2+d^2k^2}=\frac{b^2+bd+d^2}{b^2-bd+d^2}$$

18 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$$a=bk \text{ and } c=dk,$$

$$\frac{ma+nb}{mc+nd}=\frac{mkb+nb}{mkd+nd}=\frac{b(mk+n)}{d(mk+n)}=\frac{b}{d} \qquad (1)$$

Also, $$\frac{b^2c}{d^2a}=\frac{b^2dk}{d^2bk}=\frac{b}{d} \qquad (2)$$

Hence, from (1) and (2),

$$ma+nb : mc+nd=b^2c : d^2a$$

19 $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k,$ $\qquad a=bk,\ c=dk$ and $e=fk,$

$$\frac{ac}{bd}=\frac{bdk^2}{bd}=k^2 \qquad (1)$$

Also, $\dfrac{2a^2-3c^2-5e^2}{2b^2-3d^2-5f^2}=\dfrac{2b^2k^2-3d^2k^2-5f^2k^2}{2b^2-3d^2-5f^2}=k^2.$ (2)

Hence, from (1) and (2)

$ac : bd=2a^2-3c^2-5e^2 : 2b^2-3d^2-5f^2$

20 $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ (suppose).

$a=bk$, $c=dk$ and $e=fk$,

$\dfrac{a^2+c^2+e^2}{b^2+d^2+f^2}=\dfrac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}=k^2$ (1)

Also, $\dfrac{ce}{df}=\dfrac{k^2df}{df}=k^2$ . .. (2)

Hence, from (1) and (2),

$a^2+c^2+e^2 : b^2+d^2+f^2=ce : df$

21 $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ (suppose)

.. $a=bk$, $c=dk$ and $e=fk$.

$\dfrac{pa+qc+re}{pb+qd+rf}=\dfrac{pkb+qkd+rkf}{pb+qd+rf}=k$. (1)

Also, $\dfrac{\sqrt[3]{ace}}{\sqrt[3]{bdf}}=\dfrac{\sqrt[3]{k^3bdf}}{\sqrt[3]{bdf}}=k$ . .. (2)

Hence, from (1) and (2),

$pa+qc+re : pb+qd+rf=\sqrt[3]{ace} : \sqrt[3]{bdf}$

22 $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ (suppose),

$a=bk$, $c=dk$ and $e=fk$

. $\dfrac{a^2}{b^2}=\dfrac{b^2k^2}{b^2}=k^2$. (1)

Also $\dfrac{ac+ce-ae}{bd+df-bf}=\dfrac{k^2bd+k^2df-k^2bf}{bd+df-bf}=k^2$. (2)

Hence, from (1) and (2),

$a^2 : b^2=ac+ce-ae : bd+df-bf.$

23 $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ (suppose),

$a=bk$, $c=dk$ and $e=fk$,

. $\dfrac{a^3+c^3+e^3}{b^3+d^3+f^3}=\dfrac{k^3b^3+k^3d^3+k^3f^3}{b^3+d^3+f^3}=k^3$ (1)

Also, $\frac{ace}{bdf} = \frac{kb\,kd\,kf}{bdf} = k^3$ (2)

Hence, from (1) and (2),

$$a^3+c^3+e^3 : b^3+d^3+f^3 = ace : bdf$$

24 $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ (suppose),

$a = bk$, $c = dk$ and $e = fk$,

$$\frac{\sqrt{a^2c^2+c^2e^2+a^2e^2}}{\sqrt{b^2d^2+d^2f^2+b^2f^2}} = \frac{\sqrt{k^2b^2\,k^2d^2+k^2d^2\,k^2f^2+k^2b^2\,k^2f^2}}{\sqrt{b^2d^2+d^2f^2+b^2f^2}},$$

$$= \sqrt{k^4} = k^2 \quad (1)$$

Also, $\frac{ace}{bdf} = \frac{kb\,kd\,kf}{bdf} = k^3$ (2)

Hence, from (1) and (2),

$$\sqrt{a^2c^2+c^2e^2+a^2e^2} : \sqrt{b^2d^2+d^2f^2+b^2f^2} = ace : bdf$$

25 $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = k$ (suppose),

$\frac{a}{e} = \frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{d}\cdot\frac{d}{e} = k^4$ (1)

Also, $\frac{a^4}{b^4} = k^4$ (2)

Hence from (1) and (2)

$$a : e = a^4 : b^4$$

26 $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = k$ (suppose),

$x = k(b+c-a)$, $y = k(c+a-b)$ and $z = k(a+b-c)$,

$x(b-c)+y(c-a)+z(a-b)$

$= k(b-c)(b+c-a)+k(c-a)(c+a-b)+k(a-b)(a+b-c)$

$= k\{b^2-c^2-a(b-c)+c^2-a^2-b(c-a)+a^2-b^2-c(a-b)\}$

$= -k\{a(b-c)+b(c-a)+c(a-b)\}$

$= 0$

27 $\frac{a}{b} = \frac{c}{d} = k$ (suppose) $\quad$ $a = bk$ and $c = dk$,

$$\frac{a^2+c^2}{b^2+d^2} = \frac{b^2k^2+d^2k^2}{b^2+d^2} = k^2 \quad (1)$$

Also, $\dfrac{\sqrt{a^4+c^4}}{\sqrt{b^4+d^4}}=\dfrac{\sqrt{b^4k^4+d^4k^4}}{\sqrt{b^4+d^4}}=k^2$ (2)

Hence, from (1) and (2),

$$a^2+c^2 \quad b^2+d^2=\sqrt{a^4+c^4} \quad \sqrt{b^4+d^4}$$

28 $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ (suppose), $a=bk$, $c=dk$ and $e=fk$,

$$27(a+b)(c+d)(e+f)$$
$$=27\,(bk+b)(dk+d)(fk+f)=27bdf\,(k+1)^3 \qquad (1)$$

Also, $bdf\left(\dfrac{a+b}{b}+\dfrac{c+d}{d}+\dfrac{e+f}{f}\right)^3$

$$=bdf\left(\frac{a}{b}+1+\frac{c}{d}+1+\frac{e}{f}+1\right)^3$$
$$=bdf\,(3k+3)^3=27\,bdf\,(k+1)^3 \qquad (2)$$

Hence, from (1) and (2),

$$27(a+b)(c+d)(e+f)=bdf\left(\frac{a+b}{b}+\frac{c+d}{d}+\frac{e+f}{f}\right)^3$$

29 $\dfrac{a}{b}=\dfrac{c}{d}=k$ (suppose), $a=bk$ and $c=dk$,

$$\frac{ad+bc}{2bd}=\frac{kbd+kbd}{2bd}=k \qquad (1)$$

Also, $\dfrac{a^2+c^2}{ab+cd}=\dfrac{k^2b^2+k^2d^2}{kb^2+kd^2}=k$ (2)

Hence, from (1) and (2),

$$ad+bc \quad 2bd=a^2+c^2 \quad ab+cd$$

30 $\dfrac{a}{b}=\dfrac{c}{d}=k$ (suppose), $a=bk$ and $c=dk$,

$$\frac{a^2+b^2}{ab+ad-bc}=\frac{b^2k^2+b^2}{kb^2+kbd-kbd}=\frac{b^2(k^2+1)}{b^2k}$$
$$=\frac{k^2+1}{k} \qquad (1)$$

Also, $\dfrac{c^2+d^2}{cd-ad+bc}=\dfrac{k^2d^2+d^2}{kd^2-kbd+kbd}=\dfrac{d^2(k^2+1)}{d^2k}=\dfrac{k^2+1}{k}$ (2)

Hence, from (1) and (2),

$$a^2+b^2 \quad ab+ad-bc=c^2+d^2 \quad cd-ad+bc$$

31 $\frac{a}{b}=\frac{b}{c}=k$ (suppose) $\therefore a=bk$ and $b=kc$, $\therefore a=ck^2$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{k^4c^2+k^3c^2+k^2c^2}{k^2c^2+kc^2+c^2}=\frac{k^4+k^3+k^2}{k^2+k+1}=\frac{k^2(k^2+k+1)}{k^2+k+1}$$
$$=k^2 \quad (1)$$

Also, $\frac{a}{c}=\frac{ck^2}{c}=k^2$ (2)

Hence, from (1) and (2), $a^2+ab+b^2 : b^2+bc+c^2=a : c$

32 $\frac{a}{b}=\frac{b}{c}=k$ (suppose) $\therefore a=bk$ and $b=ck$, $\therefore a=k^2c$

$$a-2b+c=k^2c-2ck+c=c(k^2-2k+1)=c(k-1)^2 \quad (1)$$

Also $\frac{(a-b)^2}{a}=\frac{(k^2c-ck)^2}{k^2c}=\frac{k^2c^2(k-1)^2}{k^2c}=c(k-1)^2$ (2)

Also $\frac{(b-c)^2}{c}=\frac{(ck-c)^2}{c}=\frac{c^2(k-1)^2}{c}=c(k-1)^2$ (3)

Hence from (1), (2), and (3)

$$a-2b+c=\frac{(a-b)^2}{a}=\frac{(b-c)^2}{c}$$

33 $\frac{a}{b}=\frac{b}{c}=k$ (suppose) $\therefore a=bk$ and $b=ck$

$\therefore a=k^2c$

$$a^2b^2c^2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=\frac{b^2c^2}{a}+\frac{a^2c^2}{b}+\frac{a^2b^2}{c}$$

$$\frac{k^2c^4}{k^4c}+\frac{k^4c^4}{kc}+\frac{k^4c^2k^2c^2}{c}=c^2+k^2c^2+k^4c^2$$

$$\therefore c^2+(kc)^2+(k^2c)^2=a^2+b^2+c^2$$

34 $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=k$ (suppose), $\therefore a=bk$, $b=ck$ and $c=dk$

$\therefore a=k^2c=k^3d$ and $b=k^2d$

$(b+c)(b+d)=(k^2d+kd)(k^2d+d)$

$=kd^2(k+1)(k^2+1)$, (1)

Also, $(c+a)(c+d)=(dk+k^3d)(dk+d)$

$=kd^2(k^2+1)(k+1)$ (2)

Hence, from (1) and (2), $(b+c)(b+d)=(c+a)(c+d)$

35. $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=k$ (suppose)

$a=bk,\ b=ck$ and $c=dk$

$\therefore\ a=k^2c=k^3d$ and $b=k^2d$,

$\therefore\ (a+d)(b+c)-(a+c)(b+d)$

$=(k^3d+d)(k^2d+kd)-(k^3d+kd)(k^2d+d)$

$=kd^2(k^3+1)(k+1)-kd^2(k^2+1)^2$

$=kd^2(k^4+k^3+k+1-k^4-2k^2-1)$

$=kd^2(k^3+k-2k^2)$

$=k^2d^2(k^2-2k+1)$

$=c^2(k-1)^2=(ck-c)^2=(b-c)^2$

36. As in the last sum $c=kd$, $b=k^2d$ and $a=k^3d$.

$\therefore\ \left(\frac{a-b}{c}-\frac{a-c}{b}\right)^2-\left(\frac{d-c}{c}-\frac{d-c}{b}\right)^2$

$=\left(\frac{k^3d-k^2d}{kd}-\frac{k^3d-kd}{k^2d}\right)^2-\left(\frac{c-k^2c}{kc}-\frac{c-kc}{k^2c}\right)^2$

$=\left(k^2-k-\frac{k^2-1}{k}\right)^2-\left(\frac{1-k^2}{k}-\frac{1-k}{k^2}\right)^2$

$=\left(\frac{k^3-k^2-k^2-1}{k}\right)^2-\left(\frac{k-k^3-1-k}{k^2}\right)^2$

$=\left(\frac{k^3-1}{k}\right)^2-\left(\frac{k^3-1}{k^2}\right)^2$

$=(k^3-1)^2\left(\frac{1}{k^2}-\frac{1}{k^4}\right)$ ... (1)

Also $(a-d)^2\left(\frac{1}{c^2}-\frac{1}{b^2}\right)=(k^3d-d)^2\left(\frac{1}{d^2k^2}-\frac{1}{d^2k^4}\right)$

$=d^2(k^3-1)^2\left(\frac{1}{k^2}-\frac{1}{k^4}\right)\times\frac{1}{d^2}=(k^3-1)^2\left(\frac{1}{k^2}-\frac{1}{k^4}\right)$ (2)

Hence, from (1) and (2), the expressions are equal.

37. We have as in the last sum $a=k^3d$, $b=k^2d$ and $c=kd$

$\therefore\ \frac{a}{b}=\frac{k^3d}{k^2d}=k$ ... (1)

also, $\dfrac{\frac{1}{k^2d}-\frac{1}{kd}-\frac{1}{d}}{\frac{1}{k^3d}-\frac{1}{k^2d}-\frac{1}{kd}}=\dfrac{\frac{1}{k^2}-\frac{1}{k}-1}{\frac{1}{k}\left(\frac{1}{k^2}-\frac{1}{k}-1\right)}=k$. (2)

Hence, from (1) and (2), the expressions are equal

38 We have $a=k^3d$, $b=k^2d$ and $c=kd$

$$\frac{a}{d}=\frac{k^3d}{d}=k^3 \qquad (1)$$

also $$\frac{a^3+b^3+c^3}{b^3+c^3+d^3}=\frac{k^9d^3+k^6d^3+k^3d^3}{k^6d^3+k^3d^3+d^3}=\frac{k^3(k^6+k^3+1)}{k^6+k^3+1}=k^3 \qquad (2)$$

Hence, from (1) and (2) the expressions are equal

39 $\frac{a}{b}=\frac{c}{d}=k$ (suppose), $\quad a=bk$ and $c=dk$

$$\frac{a^2+ab}{c^2+cd}=\frac{b^2k^2+b^2k}{d^2k^2+d^2k}=\frac{b^2(k^2+k)}{d^2(k^2+k)}=\frac{b^2}{d^2} \qquad (1)$$

Also, $$\frac{b^2-2ab}{d^2-2cd}=\frac{b^2-2b^2k}{d^2-2d^2k}=\frac{b^2(1-2k)}{d^2(1-2k)}=\frac{b^2}{d^2} \qquad (2)$$

Hence from (1) and (2), $a^2+ab : c^2+cd=b^2-2ab : d^2-2cd$

40 $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ (suppose) $\quad a=bk$, $c=dk$ and $e=fk$,

$$\therefore (a^2+b^2)(ce+df)^2=(b^2k^2+b^2)(dfk^2+df)^2$$

$$=b^2(k^2+1)(k^2+1)^2d^2f^2=d^2f^2b^2(k^2+1)^3 \qquad (1)$$

$$(c^2+d^2)(ae+bf)^2=(d^2k^2+d^2)(bfk^2+bf)^2=b^2f^2d^2(k^2+1)^3 \qquad (2)$$

and $$(e^2+f^2)(ac+bd)^2=(f^2k^2+f^2)(bdk^2+bd)^2=b^2f^2d^2(k^2+1)^3 \qquad (3)$$

Hence, from (1), (2) and (3), the expressions are equal

Exercise (98)

1 From the 1st equation we have $a^2x^2=b^2$
and from the second equation, $c^2x^2=d^2$,

$$\therefore \frac{a^2}{c^2}=\frac{b^2}{d^2}$$

Thus $a^2d^2=b^2c^2$ is the required Eliminant

2 From the 1st equation we have

$$ax^2=b \quad \text{or,} \quad x^2=\frac{b}{a}, \quad \text{or,} \quad x^6=\frac{b^3}{a^3},$$

and from the 2nd equation,

$$cx^3=d, \quad \text{or,} \quad x^3=\frac{d}{c}, \quad \text{or,} \quad x^6=\frac{d^2}{c^2},$$

$$\therefore \frac{b^3}{a^3}=\frac{d^2}{c^2}$$

Thus $b^3c^2=a^3d^2$ is the required Eliminant

3 From the 1st equation, we have

$$mx^3=n \quad \text{or,} \quad x^3=\frac{n}{m} \quad \text{or,} \quad x^{12}=\frac{n^4}{m^4},$$

and from the 2nd equation,

$$px^4=q \quad \text{or,} \quad x^4=\frac{q}{p} \quad \text{or,} \quad x^{12}=\frac{q^3}{p^3}, \qquad \frac{n^4}{m^4}=\frac{q^3}{p^3}$$

Thus $n^4p^3=m^4q^3$ is the required Eliminant

4 We have by cross multiplication,

$$\frac{x^2}{bd-c}=\frac{x}{-ad}=\frac{1}{a}, \qquad \frac{x^2}{a(bd-c)}=\frac{x^2}{a^2d^2}, \qquad bd-c=ad^2$$

Thus $ad^2-bd+c=0$ is the required Eliminant

5 By cross multiplication, we have

$$\frac{x^2}{mb-an}=\frac{x}{-lb}=\frac{1}{al} \qquad \frac{x^2}{al(mb-an)}=\frac{x^2}{l^2b^2},$$

$$a(mb-an)=lb^2$$

Thus $lb^2-amb+a^2n=0$ is the required Eliminant

6 By cross multiplication, we have

$$\frac{x^2}{bn-cm}=\frac{x}{cl-an}=\frac{1}{am-bl}$$

$$\frac{x^2}{(bn-cm)(am-bl)}=\frac{x^2}{(cl-an)^2}$$

$(bn-cm)(am-bl)=(cl-an)^2$ is the reqd Eliminant

7 By adding the two equations, we have

$2x=2a$ or, $x=a$ and by subtracting $\frac{2}{x}=2b$ or, $\frac{1}{x}=b$

$$x\frac{1}{x}=ab$$

Thus $ab=1$ is the required Eliminant

8 By addition and subtraction of the two equations, we have respectively

$$4x=10p \quad \text{or,} \quad x=\frac{5p}{2}, \text{ and } \frac{6}{x}=14q \quad \text{or,} \quad \frac{1}{x}=\frac{7q}{3}$$

$$x\frac{1}{x}=\frac{35pq}{6}$$

Thus $35pq=6$ is the required Eliminant.

9 By cross multiplication, we have

$$\frac{x^3}{b_1c_2-c_1b_2}=\frac{x}{c_1a_2-a_1c_2}=\frac{1}{a_1b_2-b_1a_2},$$

$$\left(\frac{x^3}{b_1c_2-c_1b_2}\right)\left(\frac{1}{a_1b_2-b_1a_2}\right)^2=\left(\frac{x}{c_1a_2-a_1c_2}\right)^3$$

Thus $(b_1c_2-c_1b_2)(a_1b_2-b_1a_2)^2=(c_1a_2-a_1c_2)^3$ is the reqd Eliminant

10 By cross multiplication we have

$$\frac{x^3}{b_1c_2-b_2c_1}=\frac{x^2}{c_1a_2-a_1c_2}=\frac{1}{a_1b_2-b_1a_2}$$

$$\left(\frac{x^3}{b_1c_2-b_2c_1}\right)^2\left(\frac{1}{a_1b_2-b_1a_2}\right)=\left(\frac{x^2}{c_1a_2-a_1c_2}\right)^3$$

Thus $(b_1c_2-b_2c_1)^2(a_1b_2-b_1a_2)=(c_1a_2-a_1c_2)^3$ is the reqd Eliminant

11 By cross multiplication, we have

$$\frac{x^4}{b_1c_2-c_1b_2}=\frac{x^3}{c_1a_2-a_1c_2}=\frac{1}{a_1b_2-b_1a_2}$$

$$\left(\frac{x^4}{b_1c_2-c_1b_2}\right)^3\left(\frac{1}{a_1b_2-b_1a_2}\right)=\left(\frac{x^3}{c_1a_2-a_1c_2}\right)^4$$

Thus $(b_1c_2-c_1b_2)^3(a_1b_2-b_1a_2)=(c_1a_2-a_1c_2)^4$ is the reqd Eliminant

12 Multiplying (2) by ax, we have $ax^3+amx^2+anx=0$ (3)

From (1) and (3) by subtraction, we get

$$amx^2+(an-b)x-c=0 \qquad (4)$$

From (2) and (4) by cross multiplication, we have

$$\frac{x^2}{-mc-an^2+nb}=\frac{x}{amn+c}=\frac{1}{an-b-am^2}$$

$$\frac{x^2}{an^2+mc-nb}=\frac{1}{am^2-an+b}=\frac{x^2}{(amn+c)^2}$$

Thus $(an^2+mc-nb)(am^2-an+b)=(amn+c)^2$ is the reqd Eliminant

13 Multiplying the 1st equation by x, we have

$ax^3+bx^2+cx=0$ and the 2nd equation by a,

$ax^3+2ax^2+3a=0$

by subtraction, we have $x^2(2a-b)-cx+3a=0$ (3)

From (1) and (3), by cross multiplication, we have

$$\frac{x^2}{3ab+c^2}=\frac{x}{c(2a-b)-3a^2}=\frac{1}{-ac-b\ 2a-b)},$$

$$\frac{x^2}{3ab+c^2}\ \frac{1}{(b^2-2ab-ac)}=\frac{x^2}{(3a^2-2ac+bc)^2}$$

Thus $(3ab+c^2)(b^2-2ab-ac)=(3a^2-2ac+bc)^2$ is the reqd. Eliminant

14 From (1), we get $ax+by-m=0$

From (2), we get $bx-ay-n=0$

by cross multiplication, $\frac{x}{-bn-am}=\frac{y}{-mb+an}=\frac{1}{-a^2-b^2}$

$x=\frac{bn+am}{a^2+b^2}$ and $y=\frac{bm-an}{a^2+b^2}$,

from (3), we have $\left(\frac{bn+am}{a^2+b^2}\right)^2+\left(\frac{bm-an}{a^2+b^2}\right)^2=1$

or, $(bn+am)^2+(bm-an)^2=(a^2+b^2)^2$

or, $b^2n^2+a^2m^2+2abmn+b^2m^2+a^2n^2-2abmn=(a^2+b^2)^2$

or, $a^2(m^2+n^2)+b^2(m^2+n^2)=(a^2+b^2)^2$

or, $(a^2+b^2)(m^2+n^2)-(a^2+b^2)^2$

Thus $a^2+b^2=m^2+n^2$ is the reqd Eliminant

15 From (1), we get $ax-cy+b=0$

From (2), we get $c_1x-a_1y-b_1=0$

by cross multiplication, $\frac{x}{cb_1+ba_1}=\frac{y}{bc_1+ab_1}=\frac{1}{-aa_1+c}$,

$x=\frac{cb_1+ba_1}{-aa_1+cc_1}$ and $y=\frac{bc_1+ab_1}{-aa_1+cc_1}$,

from (3), we have $\left(\frac{cb_1+ba_1}{-aa_1+cc_1}\right)^2\times\left(\frac{bc_1+ab_1}{-aa_1+cc_1}\right)^2=1$,

$(cb_1+ba_1)^2+(bc_1+ab_1)^2=(cc_1-aa_1)^2$ is the reqd Eliminant

16 Multiplying (1) by x, we have $ax^2+bxy=0$

and $lx^2+mxy+ny^2=0$ (2)

by cross multiplication, $\frac{x^2}{bn}=\frac{xy}{-an}=\frac{y^2}{am-bl}$,

$$\frac{x^2y^2}{bm(am-bl)}=\frac{x^2y^2}{a^2n^2}, \qquad abmn-b^2ln=a^2n^2$$

Thus $a^2n+b^2l=abm$ is the reqd Eliminant

17 From (1) we get $x=ay+az$ or, $x-ay-az=0$ ... (4)

From (2) we get $y=bz+bx$ or, $bx-y+bz=0$ ... (5)

from (4) and (5), we get

$$\frac{x}{-ab-a}=\frac{y}{-ab-b}=\frac{z}{-1+ab}=k \text{ (suppose)},$$

$x=-k(ab+a)$, $y=-k(ab+b)$ and $z=k(ab-1)$,

from (3), we get $\dfrac{k(ab-1)}{-k(ab+a)-k(ab+b)}=c$,

or, $\dfrac{ab-1}{-2ab-a-b}=c$, or, $ab-1=-2abc-ac-bc$

Thus $ab+bc+ca+2abc=1$ is the reqd Eliminant

18 From (1), $\dfrac{y}{-z}=\dfrac{a+1}{a-1}$ (4)

From (2), $\dfrac{z}{-x}=\dfrac{b+1}{b-1}$ (5)

From (3), $\dfrac{x}{-y}=\dfrac{c+1}{c-1}$ (6)

Multiplying (4) and (5), we have $\dfrac{y}{x}=\dfrac{(a+1)(b+1)}{(a-1)(b-1)}$

Also from (3) $\dfrac{y}{x}=\dfrac{1-c}{1+c}$,

$$\frac{(a+1)(b+1)}{(a-1)(b-1)}=\frac{1-c}{1+c} \text{ or, } \frac{ab+a+b+1}{ab-a-b+1}=\frac{1-c}{1+c}$$

or, $\dfrac{ab+1}{-(a+b)}=\dfrac{1}{c}$ (by componendo and dividendo)

or, $abc+c=-a-b$

Thus $a+b+c+abc=0$ is the reqd Eliminant

19 We have, from the given equations,

$$\left(\frac{y}{z}+\frac{z}{y}\right)^2=a^2, \quad \left(\frac{z}{x}+\frac{x}{z}\right)^2=b^2 \quad \text{and} \quad \left(\frac{x}{y}+\frac{y}{x}\right)^2=c^2$$

$$\left(\frac{y}{z}+\frac{z}{y}\right)^2+\left(\frac{z}{x}+\frac{x}{z}\right)^2+\left(\frac{x}{y}+\frac{y}{x}\right)^2=a^2+b^2+c^2$$

(from Ex 6 Page 184,)

$$4+\left(\frac{y}{z}+\frac{z}{y}\right)\left(\frac{z}{x}+\frac{x}{z}\right)\left(\frac{x}{y}+\frac{y}{x}\right)=a^2+b^2+c^2$$

or, $4+abc=a^2+b^2+c^2$

Thus $a^2+b^2+c^2-abc=4$ is the reqd Eliminant

20 By adding the first three equations,

We have $x^2(y-z)+y^2(z-x)+z^2(x-y)=a+b+c$

or, $-(x-y)(y-z)(z-x)=a+b+c$

$-x^2y^2z^2(x-y)(y-z)(z-x)=x^2y^2z^2(a+b+c)$

or, $-abc=d^2(a+b+c)$

Thus $d^2(a+b+c)+abc=0$ is the reqd Eliminant

21 From (1), we have $a-bz-cy=0$

From (2), we have $az-b+cx=0$

by cross multiplication,

$$\frac{a}{-xz-y}=\frac{b}{-yz-x}=\frac{c}{-1+z^2}=k \text{ (suppose)},$$

$a=-k(xz+y)$, $b=-k(yz+x)$, and $c=k(z^2-1)$

from (3), we have $-ky(xz+y)-kx(yz+x)=k(z^2-1)$

or, $-xyz-y^2-xyz-x^2=z^2-1$

Thus $x^2+y^2+z^2+2xyz=1$ is the reqd Eliminant

Miscellaneous Exercises (4)

I

1 The given expression

$=\sqrt{(1-27+1)(-1+3-3)}-\sqrt[3]{(-1)(-27)(1)}$

$=\sqrt{(-25)(-1)}-\sqrt[3]{27}$

$=\sqrt{25}-\sqrt[3]{27}=5-3=1\frac{2}{3}$

2 The given expression

$=3a-2b+2c-2(a-b)+3(c+a)-9c+4(c-a)$

$=3a-2b+2c-2a+2b+3c+3a-9c+4c-4a$

$=0$

3
$$3(a+b)^2-2(a^2-b^2)-a(a+b)$$
$$=(a+b)\{3(a+b)-2(a-b)-a\}$$
$$=(a+b)(3a+3b-2a+2b-a)=5b(a+b)$$

4
$$\text{Quotient}=2x^2-4xy+5y^2$$
$$\begin{array}{r|l} x^2-3xy+4y^2 & 2x^4-10x^3y+25x^2y^2-31xy^3+20y^4 \\ & \underline{2x^4-6x^3y+8x^2y^2} \\ & \quad -4x^3y+17x^2y^2-31xy^3+20y^4 \\ & \quad \underline{-4x^3y+12x^2y^2-16xy^3} \\ & \qquad 5x^2y^2-15xy^3+20y^4 \\ & \qquad \underline{5x^2y^2-15xy^3+20y^4} \end{array}$$

5 The given expression
$$=\frac{b}{a+b}\left\{1-\frac{a}{(a+b)}-\frac{ab}{(a+b)^2}\right\}=\frac{b}{a+b}\left\{\frac{(a+b)^2-a(a+b)-ab}{(a+b)^2}\right\}$$
$$=\frac{b}{a+b}\left\{\frac{a^2+2ab+b^2-a^2-ab-ab}{(a+b)^2}\right\}=\frac{b}{a+b}\cdot\frac{b^2}{(a+b)^2}=\frac{b^3}{(a+b)^3}$$

6
$$\frac{x+a}{x-a}-\frac{x-b}{x+b}=\frac{2(a+b)}{x}$$

or, $$1+\frac{2a}{x-a}-1+\frac{2b}{x+b}=\frac{2(a+b)}{x}$$

or, $$\frac{a}{x-a}+\frac{b}{x+b}=\frac{a+b}{x}$$

or, $$a\left(\frac{1}{x-a}-\frac{1}{x}\right)=b\left(\frac{1}{x}-\frac{1}{x+b}\right)$$

or $$a\left\{\frac{x-x+a}{x(x-a)}\right\}=b\left\{\frac{x+b-x}{x(x+b)}\right\}$$

or, $$\frac{a^2}{x-a}=\frac{b^2}{x+b}$$

or, $$a^2x+a^2b=b^2x-ab^2$$

or, $$x(a^2-b^2)=-ab(a+b), \qquad x=\frac{ab}{b-a}$$

7
$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)$$
$$=(\sqrt{3})^3-3\sqrt{3}=3\sqrt{3}-3\sqrt{3}=0$$

8 $$\frac{3\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{(3\sqrt{3}-2\sqrt{2})(\sqrt{3}+\sqrt{2})}{3-2}$$
$$=9-2\sqrt{6}+3\sqrt{6}-4=5+\sqrt{6}$$

II.

1 The given expression
$$=(2+2)(1-2)+(4-3)(2+3)+(-6+1)(-3-1)$$
$$=4(-1)+(1)(5)+(-5)(-4)$$
$$=-4+5+20=25-4=21$$

2
$$\begin{array}{r|l|l}
2x^2+3x-1 & 8x^4-24x^2+3x+1 & 4x^2-6x-1 \\
 & 8x^4+12x^3 \quad 4x^2 & \\
\hline
 & -12x^3-20x^2+3x+1 & \\
 & -12x^3-18x^2+6x & \\
\hline
 & -2x^2-3x+1 & \\
 & -2x^2-3x+1 & \\
\hline
\end{array}$$

3
$$\begin{array}{r|l|l}
x+4 & x^2+7x+c & x+3 \\
 & x^2+4x & \\
\hline
 & 3x+c & \\
 & 3x+12 & \\
\hline
 & c-12 & \\
\end{array}$$

But in order that x^2+7x+c may be exactly divisible by $x+4$ there should be no remainder, that is, $c-12$ must be equal to o, $c=12$

4 $$\frac{1}{2\sqrt{7}-3\sqrt{2}}-\frac{1}{2\sqrt{7}+3\sqrt{2}}=\frac{2\sqrt{7}+3\sqrt{2}-2\sqrt{7}+3\sqrt{2}}{(2\sqrt{7})^2-(3\sqrt{2})^2}$$
$$=\frac{6\sqrt{2}}{28-18}=\frac{6\sqrt{2}}{10}=\frac{3\sqrt{2}}{5}$$

5
$$\begin{array}{r|l|l}
x^3-7x+6 & x^4-3x^3-2x^2+12x-8 & x-3 \\
 & x^4 \quad -7x^2+6x & \\
\hline
 & -3x^3+5x^2+6x-8 & \\
 & -3x^3+ \quad +21x-18 & \\
\hline
 & 5\,\underline{|\,5x^2-15x+10} & \\
 & x^2-3x+2 & \\
\end{array}$$

$$\begin{array}{r|l|l}
x^2-3x+2 & x^3 \quad -7x+6 & x+3 \\
 & x^3-3x^2+2x & \\
\hline
 & 3x^2-9x+6 & \\
 & 3x^2-9x+6 & \\
\hline
\end{array}$$

Thus the reqd H C F is x^2-3x+2

6 The given expression

$$=5\left(\frac{1}{5}+\frac{7}{x-7}-\frac{3}{x-3}\right)\left(\frac{1}{5}-\frac{7}{x+7}+\frac{3}{x+3}\right)$$

$$=5\left\{\frac{x^2-10x+21+35x-105-15x+105}{5(x-7)(x-3)}\right\}\times$$

$$\left\{\frac{x^2+10x+21-35x-105+15x+105}{5(x+7)(x+3)}\right\}$$

$$=5\left\{\frac{x^2+10x+21}{5(x^2-10x+21)}\right\}\left\{\frac{x^2-10x+21}{5(x^2+10x+21)}\right\}=\frac{1}{5}$$

7, $$\frac{x+1}{6}+\frac{3x-1}{8}-\frac{5x-7}{12}+1=\frac{7x-5}{24}$$

or, $4x+4+9x-3-10x+14+24=7x-5$

or, $-4x=-44$, $x=11$

8 $$x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\;x\,\frac{1}{x}\left(x-\frac{1}{x}\right)=1+3=4$$

III

1 The given expression

$$=\frac{9(-8-64)+4(64-27)+16(27+8)}{-8+12-6}$$

$$=\frac{9(-72)+4(37)+16(35)}{-2}=\frac{-648+148+560}{-2}=\frac{60}{-2}=-30$$

2 $$\frac{1+x}{1-x}+\frac{1-x}{1+x}-\frac{1+x^2}{1-x^2}-\frac{1-x^2}{1+x^2}$$

$$=\frac{(1+x)^2+(1-x)^2-(1+x^2)}{1-x^2}-\frac{1-x^2}{1+x^2}$$

$$=\frac{2(1+x^2)-(1+x^2)}{1-x^2}-\frac{1-x^2}{1+x^2}=\frac{1+x^2}{1-x^2}-\frac{1-x^2}{1+x^2}$$

$$=\frac{(1+x^2)^2-(1-x^2)^2}{1-x^4}=\frac{4x^2}{1-x^4}$$

3 $$a^2-b^2+6bc-9c^2=a^2-(b^2-6bc+9c^2)=a-(b-3c)^2$$

$$=\{a+(b-3c)\}\{a-(b-3c)\}=(a+b-3c)(a-b+3c)$$

4 $$x^3+5ax^2-5a^2x-a^3=(x^3-a^3)+(5ax^2-5a^2x)$$

$$=(x-a)(x^2+ax+a^2+5ax)=(x-a)(x^2+6ax+a^2)$$

and $5x^3-3ax^2-5a^2x+3a^3$

$$=x^2(5x-3a)-a^2(5x-3a)=(x+a)(x-a)(5x-3a)\text{ ,}$$

the reqd H C. F $=(x-a)$

5 $x^2-5x+6=(x-3)(x-2)$

$x^2-4x+3=(x-3)(x-1)$

$x^2-3x+2=(x-2)(x-1)$

the reqd L C M $=(x-1)(x-2)(x-3)$

6 The numerator

$=x^2(x^3+5x^2+8x+4)=x^2(x^3+x^2+4x^2+4x+4x+4)$

$=x^2\{x^2(x+1)+4x(x+1)+4(x+1)\}$

$=x^2(x+1)(x^2+4x+4)=x^2(x+1)(x+2)^2$,

and the denominator

$=x(x^4+x^3+8x+8)=x\{x^3(x+1)+8(x+1)\}$

$=x(x+1)(x^3+8)=x(x+1)(x+2)(x^2-2x+4)$,

the reqd result $=\dfrac{x(x+2)}{x^2-2x+4}$

7 $\dfrac{1}{x+3}+\dfrac{1}{x-2}=\dfrac{2}{x-7}$ or, $\dfrac{1}{x+3}-\dfrac{1}{x-7}=\dfrac{1}{x-7}-\dfrac{1}{x-2}$

or, $\dfrac{x-7-x-3}{(x+3)(x-7)}=\dfrac{x-2-x+7}{(x-7)(x-2)}$

or, $\dfrac{-10}{(x+3)(x-7)}=\dfrac{5}{(x-7)(x-2)}$ or, $\dfrac{-2}{x+3}=\dfrac{1}{x-2}$

or, $-2x+4=x+3$ or, $-3x=-1$, $x=\frac{1}{3}$

8 $\dfrac{a}{b}=\dfrac{x}{y}=k$ (suppose), $a=bk$ and $x=yk$

$\dfrac{ab}{xy}=\dfrac{kb^2}{ky^2}=\dfrac{b^2}{y^2}$,

also, $\dfrac{a^2+b^2}{x^2+y^2}=\dfrac{k^2b^2+b^2}{k^2y^2+y^2}=\dfrac{b^2(k^2+1)}{y^2(k^2+1)}=\dfrac{b^2}{y^2}$

Hence, $ab \quad xy \quad a^2+b^2 \quad x^2+y^2$

IV

1 The given expression

$$=\frac{x+\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}+y}\times\frac{x-\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}-y}=\frac{x^2-(x^2-y^2)}{(x^2+y^2)-y^2}=\frac{y^2}{x^2}$$

2 The reqd expression

$$=\frac{x^8+x^4y^4+y^8}{x^2-xy+y^2}=\frac{(x^4-x^2y^2+y^4)(x^2+xy+y^2)(x^2-xy+y^2)}{x^2-xy+y^2}$$

$$=(x^4-x^2y^2+y^4)(x^2+xy+y^2)=x^6+x^5y-x^3y^3+xy^5+y^6$$

3 (i) $x^3+x^2-x-1 = x^2(x+1)-(x+1)$
$= (x+1)(x^2-1) = (x+1)(x+1)(x-1) = (x+1)^2(x-1)$

(ii) $a^2b^2-a^2-b^2+1 = a^2(b^2-1)-(b^2-1)$
$= (a^2-1)(b^2-1) = (a+1)(a-1)(b+1)(b-1)$

4 $(ax+by)^2+(bx-ay)^2$
$= a^2x^2+b^2y^2+2abxy+b^2x^2+a^2y^2-2abxy$
$= a^2(x^2+y^2)+b^2(x^2+y^2) = (x^2+y^2)(a^2+b^2)$

5 $8x^3+27=(2x)^3+(3)^3=(2x+3)(4x^2-6x+9)$
$16x^4+36x^2+81=(4x^2+9)^2-36x^2=(4x^2+9+6x)(4x^2+9-6x)$
and $6x^2-5x-6=6x^2-9x+4x-6$
$=3x(2x-3)+2(2x-3)=(2x-3)(3x+2)$
∴ the reqd L. C. M.
$=(2x+3)(4x^2-6x+9)(4x^2+6x+9)(2x-3)(3x+2)$
$=(4x^2-9)(16x^4+36x^2+81)(3x+2)$
$=\{(4x^2)^3-(9)^3\}(3x+2)=(64x^6-729)(3x+2)$

6 $\frac{3x-4}{x}+\frac{2}{4x+3}=3$ or, $3-\frac{4}{x}+\frac{2}{4x+3}=3$

or, $\frac{2}{4x+3}=\frac{4}{x}$ or, $2x=16x+12$

or, $-14x=12$ ∴ $x=-\frac{6}{7}$

7 We have $bx+ay=2a^2b^2$
or, $bx+ay-2a^2b^2=0$ (1)
also $by-ax=ab(b^2-a^2)$
or, $-ax+by-ab(b^2-a^2)=0$ (2)
From (1) and (2), by cross multiplication,

$$\frac{x}{-a^2b(b^2-a^2)+2a^2b^3}=\frac{y}{2a^3b^2+ab^2(b^2-a^2)}=\frac{1}{b^2+a^2}$$

$$x=\frac{-a^2b^3+a^4b+2a^2b^3}{b^2+a^2}=\frac{a^2b(a^2+b^2)}{b^2+a^2}=a^2b$$

and $y=\dfrac{2a^3b^2+ab^4-a^3b^2}{b^2+a^2}=\dfrac{ab^2(a^2+b^2)}{b^2+a^2}=ab^2$

8 $\frac{x}{a+b-c}=\frac{y}{a-b+c}=\frac{z}{b+c-a}=k$ (suppose)

$$x=k(a+b-c),\ y=k(a-b+c) \text{ and } z=k(b+c-a)$$
$$x+y+z=k(a+b+c)$$

Thus $\dfrac{x+y+z}{a+b+c}=k=$ each of the ratios

V

1 The given expression

$$=\frac{(x-5y)(x+5y)}{(x+5y)(x-2y)}\times\frac{(x-2y)(x+2y)}{(x+2y)(x-5y)}=1$$

2 Quotient $=a^2(b-c)+a(b^2-c^2)+bc\,(b-c)$

$$\begin{array}{l|l}
a+(b+c) & a^3(b-c)-a(b^3-c^3)+bc(b^2-c^2) \\
 & \underline{a^3(b-c)+a^2(b^2-c^2)} \\
 & -a^2(b^2-c^2)-a(b^3-c^3)+bc(b^2-c^2) \\
 & \underline{-a^2(b^2-c^2)-a(b^3+b^2c-bc^2-c^3)} \\
 & abc(b-c)+bc(b^2-c^2) \\
 & \underline{abc(b-c)+bc(b^2-c^2)}
\end{array}$$

The quotient is again

$$=(b-c)\{a^2-a(b+c)+bc\}$$
$$=(b-c)\{a(a-b)-c(a-b)\}$$
$$=(b-c)(a-c)(a-b)=-(b-c)(c-a)(a-b)$$

3 $$\frac{x^3-y^3}{x^3+y^3}=\frac{(x-y)(x^2+xy+y^2)}{(x+y)(x^2-xy+y^2)}=\frac{6\{(a+3)^2+(a-3)^2+(a^2-9)\}}{2a\{(a+3)^2+(a-3)^2-(a^2-9)\}}$$
$$=\frac{3\{2(a^2+9)+(a^2-9)\}}{a\{2(a^2+9)-(a^2-9)\}}=\frac{3(3a^2+9)}{a(a^2+27)}=\frac{9(a^2+3)}{a(a^2+27)}$$

4 $$\begin{array}{l|l}
 & \dfrac{x^2}{y}-\dfrac{8x}{\sqrt{y}}+24-\dfrac{32\sqrt{y}}{x}+\dfrac{16y}{x^2}\left(\dfrac{x}{\sqrt{y}}-4+\dfrac{4\sqrt{y}}{x}\right. \\
 & \underline{\dfrac{x^2}{y}} \\
\dfrac{2x}{\sqrt{y}}-4 & -\dfrac{8x}{\sqrt{y}}+24-\dfrac{32\sqrt{y}}{x}+\dfrac{16y}{x^2} \\
 & \underline{-\dfrac{8x}{\sqrt{y}}+16} \\
\dfrac{2x}{\sqrt{y}}-8+\dfrac{4\sqrt{y}}{x} & 8-\dfrac{32\sqrt{y}}{x}+\dfrac{16y}{x^2} \\
 & \underline{8-\dfrac{32\sqrt{y}}{x}+\dfrac{16y}{x^2}}
\end{array}$$

Thus the reqd root $=\dfrac{x}{\sqrt{y}}-4+\dfrac{4\sqrt{y}}{x}$

5 $(a^2+b^2+c^2)(x^2+y^2+z^2)-(ax+by+cz)^2$

$=a^2x^2+a^2(y^2+z^2)+b^2(x^2+z^2)+b^2y^2+c^2(x^2+y^2)$

$+c^2z^2-(a^2x^2+b^2y^2+c^2z^2)-2(abxy+acxz+bcyz)$

$=(a^2y^2-2abxy+b^2x^2)+(b^2z^2-2bcyz+c^2y^2)$

$+(c^2x^2-2acxz+a^2z^2)$

$=(ay-bx)^2+(bz-cy)^2+(cx-az)^2$

6 $\frac{7-2\sqrt{5}}{4-\sqrt{5}}=\frac{(7-2\sqrt{5})(4+\sqrt{5})}{16-5}=\frac{28-\sqrt{5}-10}{11}=\frac{18-\sqrt{5}}{11}$

and $\frac{15+6\sqrt{5}}{2+\sqrt{5}}=\frac{3\sqrt{5}(\sqrt{5}+2)}{2+\sqrt{5}}=3\sqrt{5}$

Hence, their difference

$=3\sqrt{5}-\frac{18-\sqrt{5}}{11}=\frac{33\sqrt{5}-18+\sqrt{5}}{11}=\frac{34\sqrt{5}-18}{11}$

7 $2^x\times 4^y=32$ (i), $3^x-9^y=3$ (ii)

From (i), we have $2^x\times 2^{2y}=2^5$, $x+2y=5$ (iii)

From (ii), we have $3^x\times 9^{-y}=3$ or, $3^x\times 3^{-2y}=3$

or, $3^{x-2y}=3$, $x-2y=1$ (iv)

From (iii) and (iv), we have

$2x=6$ or, $x=3$

and $4y=4$ or, $y=1$

8 $\frac{a}{b}=\frac{c}{d}=k$ (suppose),

$a=bk$ and $c=dk$,

$(a^2+c^2)(b^2+d^2)=(b^2k^2+d^2k^2)(b^2+d^2)=k^2(b^2+d^2)^2$

Also $(ab+cd)^2=(b^2k+d^2k)^2=k^2(b^2+d^2)^2$,

$(a^2+c^2)(b^2+d^2)=(ab+cd)^2$

VI

1 The given expression

$=\frac{2a(1-x)^2(1+x)^2}{yz}\times\frac{y^3}{(1+x)^2(1-x)}\times\frac{z}{2ay^2(1-x)}=1$

2 $\left(a+b+\frac{b^2}{a}+\frac{a^2}{b}\right)\left(a-b+\frac{b^2}{a}-\frac{a^2}{b}\right)$

$$=\left\{\left(a+\frac{b^2}{a}\right)+\left(b+\frac{a^2}{b}\right)\right\}\left\{\left(a+\frac{b^2}{a}\right)-\left(b+\frac{a^2}{b}\right)\right\}$$

$$=\left(a+\frac{b^2}{a}\right)^2-\left(b+\frac{a^2}{b}\right)^2$$

$$=a^2+2b^2+\frac{b^4}{a^2}-\left(b^2+2a^2+\frac{a^4}{b^2}\right)$$

$$=b^2-a^2+\frac{b^4}{a^2}-\frac{a^4}{b^2}$$

3 Quotient $=x^2+(a-b)x-ab$

$$x^2-(a+b)x+ab)\,x^4-2bx^3-(a^2-b^2)x^2+2a^2bx-a^2b^2$$
$$\underline{x^4-(a+b)x^3+abx^2}$$
$$(a-b)x^3-(a^2+ab-b^2)x^2+2a^2bx-a^2b^2$$
$$\underline{(a-b)x^3-(a^2-b^2)x^2+(a^2b-ab^2)x}$$
$$-abx^2+ab(a+b)x-a^2b^2$$
$$\underline{-abx^2+ab(a+b)x-a^2b^2}$$

4 $\quad a^2+b^2+c^2-bc-ca-ab$

$$=\tfrac{1}{2}(2a^2+2b^2+2c^2-2bc-2ac-2ab)$$
$$=\tfrac{1}{2}\{(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)\}$$
$$=\tfrac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}$$
$$=\tfrac{1}{2}\{(y+z-z-x)^2+(z+x-x-y)^2+(x+y-y-z)^2\}$$
$$=\tfrac{1}{2}\{(y-x)^2+(z-y)^2+(x-z)^2\}$$
$$=\tfrac{1}{2}(2x^2+2y^2+2z^2-2xy-2yz-2xz)$$
$$=x^2+y^2+z^2-xy-yz-xz$$

5 $\quad 5x^3-14x^2+16=5x^3-10x^2-4x^2+16=5x^2(x-2)-4(x^2-4)$

$$=(x-2)\{5x^2-4(x+2)\}=(x-2)(5x^2-4x-8)\,,$$

and $\quad 3x^3-2x^2+16x-48=3x^3-6x^2+4x^2-8x+24x-48$

$$=3x^2(x-2)+4x(x-2)+24(x-2)=(x-2)(3x^2+4x+24)\,,$$

$$\frac{5x^3-14x^2+16}{3x^3-2x^2+16x-48}=\frac{5x^2-4x-8}{3x^2+4x+24}$$

6 $\quad$ We have $\frac{2}{x}+\frac{7}{y}-29=0$ and $\frac{5}{x}-\frac{6}{y}-2=0$,

by cross multiplication

$$\frac{1}{x(-14-174)}=-\frac{1}{y(-145+4)}=\frac{1}{-12-35}$$

or $\quad \frac{1}{188x}=\frac{1}{141y}=\frac{1}{47}$, $\quad x=\frac{1}{4}$ and $y=\frac{1}{3}$

7 Multiplying the 2nd equation by 8, we have

$$56x-32y+8z-64=0$$

and $\quad 2x+3y-8z+35=0 \quad$ 1st equation

$$58x-29y-29=0$$

or, $\quad 2x-y-1=0 \quad (4)$

Again by multiplying the 2nd equation by 3, we have

$$21x-12y+3z-24=0$$

From (3) $\quad 12x-5y-3z+10=0$

$$33x-17y-14=0$$

From (4) $\quad 34x-17y-17=0$

$$x-3=0\,, \qquad x=3\,,$$

from (4), $\quad 6-y-1=0 \qquad y=5\,,$

and $\quad$ from (2) $21-20+z-80=0. \qquad z=7$

8 $\quad \dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ (suppose),

$$a=bk,\ c=dk \text{ and } e=fk\,, \qquad \frac{a}{b}=\frac{bk}{b}=k$$

Also, $\quad \dfrac{\sqrt{m^2a^2+n^2c^2-p^2e^2}}{\sqrt{m^2b^2+n^2d^2-p^2f^2)}}=\dfrac{\sqrt{m^2k^2b^2+n^2k^2d^2-p^2k^2f^2}}{\sqrt{m^2b^2+n^2d^2-p^2f^2}}=k,$

$$a \quad b \quad \sqrt{m^2a^2+n^2c^2-p^2e^2} \quad \sqrt{m^2b^2+n^2d^2-p^2f^2}$$

VII

1 $\quad$ Quotient $=2x^3y^{-3}-3x^4y$

$$\begin{array}{r|l} -x^2y^{-5}+7x^3y^{-1}+8x^4y^3 & -2x^5y^{-8}+17x^6y^{-4}-5x^7-24x^8y^4 \\ & \underline{-2x^5y^{-8}+14x^6y^{-4}+16x^7} \\ & \quad 3x^6y^{-4}-21x^7-24x^8y^4 \\ & \quad \underline{3x^6y^{-4}-21x^7-24x^8y^4} \end{array}$$

2 $\quad e^{2x}a^3+e^{2x}-a^3-1=e^{2x}(a^3+1)-(a^3+1)$

$$=(a^3+1)(e^{2x}-1)=(a+1)(a^2-a+1)(e^x-1)(e^x+1)\,,$$

and $\quad e^{2x}a^2+2e^xa^2-e^{2x}-2e^x+a^2-1$

$$=e^{2x}a^2+2e^xa^2+a^2-e^{2x}-2e^x-1$$

$$=a^2(e^{2x}+2e^x+1)-(e^{2x}+2e^x+1)$$

$$=(a^2-1)(e^{2x}+2e^x+1)=(a+1)(a-1)(e^x+1)^2\,,$$

the required H C F $=(a+1)(e^x+1)=ae^x+e^x+a+1$

3 The left-hand expression

$$=\frac{2ab+2cd-a^2-b^2+c^2+d^2}{2(ab+cd)}$$

$$=\frac{(c^2+d^2+2cd)-(a^2+b^2-2ab)}{2(ab+cd)}$$

$$=\frac{(c+d)^2-(a-b)^2}{2(ab+cd)}=\frac{(c+d+a-b)(c+d-a+b)}{2(ab+cd)}$$

$$=\frac{(a+c+d-b)(b+c+d-a)}{2(ab+cd)}$$

4 The numerator $=abx^2+xya^2+xyb^2+aby^2$

$$=ax(bx+ay)+by(bx+ay)=(bx+ay)(ax+by)$$

The denominator $=abx^2+xya^2-xyb^2-aby^2$

$$=ax(bx+ay)-by(bx+ay)=(ax-by)(bx+ay)$$

the required result $=\dfrac{ax+by}{ax-by}$

5 $$\frac{x-4}{x-5}-\frac{x-5}{x-6}=\frac{x-7}{x-8}-\frac{x-8}{x-9}$$

or, $$1+\frac{1}{x-5}-1-\frac{1}{x-6}=1+\frac{1}{x-8}-1-\frac{1}{x-9}$$

or, $$\frac{1}{x-5}-\frac{1}{x-6}=\frac{1}{x-8}-\frac{1}{x-9}$$

or, $$\frac{x-6-x+5}{(x-5)(x-6)}=\frac{x-9-x+8}{(x-8)(x-9)}$$

or, $(x-5)(x-6)=(x-8)(x-9)$

or, $x^2-11x+30=x^2-17x+72$

or, $6x=42$, $\quad x=7$

6 $$\begin{array}{r|l} x+a \;)\; x^2+px+q & (\,x+(p-a) \\ x^2+ax & \\ \hline x(p-a)+q & \\ x(p-a)+a(p-a) & \\ \hline a(a-p)+q & \end{array}$$

In order that x^2+px+q may be divisible by $x+a$

$$a(a-p)+q=0$$

Similarly $$a(a-p')+q'=0$$

$$a(p'-p)+q-q'=0 \qquad \text{or,} \quad a=\frac{q-q'}{p-p'}$$

7 Let x be the number of guineas, then the number of half crowns $= x+48$

$$\text{we have} \quad \frac{21x}{20}+\frac{(x+48)}{8}=100$$

or, $$42x+5x+240=4000$$

or, $$47x=3760, \qquad x=80$$

Thus, the number of guinea $=80$,

and the number of half crowns $=128$

8 $\frac{a}{b}=\frac{c}{d}=k$ (suppose), $\qquad a=bk$ and $c=dk$

$$\frac{4a^6+5b^6}{4c^6+5d^6}=\frac{4b^6k^6+5b^6}{4d^6k^6+5d^6}=\frac{b^6(4k^6+5)}{d^6(4k^6+5)}=\frac{b^6}{d^6}$$

Also, $$\frac{a^3b^3}{c^3d^3}=\frac{b^3k^3b^3}{d^3k^3d^3}=\frac{b^6}{d^6}$$

$$4a^6+5b^6 \quad 4c^6+5d^6 \qquad a^3b^3 \quad c^3d^3$$

VIII

1 We have

$$(ax+by+cz)+(cx-by+az)$$
$$=x(a+c)+z(a+c)$$
$$=(x+z)(a+c)$$

Putting m for $ax+by+cz$ and n for $cx-by+az$ we have the given expression

$=m^3+n^3$, of which $m+n$ is a factor

Thus the given expression is divisible by $m+n$ i e $(x+z)(a+c)$

2 (i) $(b+c)^2-6a(b+c)+5a^2=(b+c)^2-a(b+c)-5a(b+c)+5a^2$
$=(b+c)(b+c-a)-5a(b+c-a)=(b+c-a)(b+c-5a)$

(ii) $x^2+2xy-a^2-2ay=(x^2+2xy+y^2)-(y^2+a^2+2ay)$
$=(x+y)^2-(y+a)^2=(x+y+y+a)(x+y-y-a)$
$=(x+2y+a)(x-a)$

3 $$\frac{(a+b)\{(a+b)^2-c^2\}}{4b^2c^2-(a^2-b^2-c^2)^2}=\frac{(a+b)(a+b+c)(a+b-c)}{(2bc+a^2-b^2-c^2)(2bc-a^2+b^2+c^2)}$$
$$=\frac{(a+b)(a+b+c)(a+b-c)}{\{a^2-(b-c)^2\}\{(b+c)^2-a^2\}}$$

$$=\frac{(a+b)(a+b+c)(a+b-c)}{(a+b-c)(a-b+c)(b+c+a)(b+c-a)}=\frac{a+b}{(a-b+c)(b+c-a)}$$

4 $a+b+c=0$, $\therefore$ $a+b=-c$, $\therefore$ $(a+b)(a-b)=-c(a-b)$

$a^2-b^2=-ac+bc$, $\therefore$ $a^2-bc=b^2-ac$

Again $b+c=-a$, $\therefore$ $(b+c)(b-c)=-a(b-c)$,

$\therefore$ $b^2-c^2=-ab+ac$, $\therefore$ $b^2-ac=c^2-ab$.

$a^2-bc=b^2-ac=c^2-ab$

5 $$3(x+3)^2+5(x+5)^2=8(x+8)^2$$

or, $$3\{(x+3)^2-(x+8)^2\}=5\{(x+8)^2-(x+5)^2\}$$

or, $$3(x+3+x+8)(x+3-x-8)=5(x+8+x+5)(x+8-x-5)$$

or, $$3(2x+11)-5=5(2x+13)3$$

or, $$-2x-11=2x+13$$

or, $$4x=-24, \quad x=-6$$

6. $$\begin{array}{r|l}
 & 4x^4-12x+25x^{-2}-24x^{-5}+16x^{-8}\ \big(2x^2-3x^{-1}+4x^{-4} \\
 & 4x^4 \\
\hline
4x^2-3x^{-1} & -12x+25x^{-2}-24x^{-5}+16x^{-8} \\
 & -12x+9x^{-2} \\
\hline
4x^2-6x^{-1}+4x^{-4} & 16x^{-2}-24x^{-5}+16x^{-8} \\
 & 16x^{-2}-24x^{-5}+16x^{-8} \\
\hline
\end{array}$$

Thus the required root $=2x^2-3x^{-1}+4x^{-4}$

7 $$yz=4 \quad \cdot \quad (1)$$
$$zx=9 \quad (2)$$
$$xy=25 \quad (3)$$

Multiplying (1), (2) and (3) together, we have

$$x^2y^2z^2=36\times 25, \quad \therefore \quad xyz=6\times 5=30$$

Hence, from (1) $x=7\frac{1}{2}$, from (2) $y=3\frac{1}{3}$, and from (3) $z=1\frac{1}{5}$

8 $$\frac{a}{b}=\frac{c}{d}=k \text{ (suppose)},$$

$a=bk$ and $c=dk$,

$a(a+b+c+d)=bk(bk+b+dk+d)$

$=bk\{b(k+1)+d(k+1)\}=bk(k+1)(b+d)$.

Also, $(a+b)(a+c)=(bk+b)(bk+dk)=b(k+1)k(b+d)$

$=bk(k+1)(b+d)$

$a(a+b+c+d)=(a+b)(a+c)$

IX.

1 The given expression

$$=\{1-(2+3)^2\}-\{4-(-3-1)^2\}-\{9-(1-2)^2\}$$
$$=\{1-(5)^2\}-\{4-(-4)^2\}-\{9-(-1)^2\}$$
$$=(1-25)-(4-16)-(9-1)=-24+12-8=-20$$

2 The given expression

$$=\frac{x(x+1)^2-(x-1)^2-x(x^2+3)}{(x^2-1)^2}$$

$$=\frac{x(x^2+2x+1-x^2-3)-(x^2-2x+1)}{(x^2-1)^2}$$

$$=\frac{x(2x-2)-x^2+2x-1}{(x^2-1)^2}=\frac{2x^2-2x-x^2+2x-1}{(x^2-1)^2}$$

$$=\frac{x^2-1}{(x^2-1)^2}=\frac{1}{x^2-1}$$

3 $$a^3-b^3+3ab+1=(a^3+(-b)^3+(1)^3-3\,a(-b)\,1$$
$$=(a-b+1)(a^2+b^2+1+ab-a+b)$$

4 $$\frac{4x+3}{9}+\frac{7x-29}{5x-12}=\frac{8x+19}{18}$$

or, $$\frac{7x-29}{5x-12}=\frac{8x+19}{18}-\frac{4x+3}{9}=\frac{8x+19-8x-6}{18}=\frac{13}{18}$$

or, $$126x-522=65x-156$$

or, $$61x=366\,,\qquad x=6$$

5 Putting a for $y-z$, b for $z-x$ and c for $x-y$, the right hand side of the identity

$$=\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+2\left(\frac{1}{bc}+\frac{1}{ca}+\frac{1}{ab}\right)$$

$$=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{2(a+b+c)}{abc}$$

$$=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$ [Since $a+b+c=0$

$$=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}.$$

6 $$\frac{x+y}{x-y}=\frac{5}{3} \qquad (1)$$

and $$x+5y=36 \qquad (2)$$

From (1), by componendo, and dividendo, $\frac{x}{y}=\frac{8}{2}=4$,

$$\therefore x=4y$$

from (2), $$4y+5y=36$$

or, $$9y=36$$

$$\therefore y=4, \text{ and } x=16$$

7 At 8 o'clock the hands are 40 minute-divisions apart, therefore they will be at right angles when the minute hand will have travelled $(40-15)=25$ minute divisions more than the hour hand. If x minutes past 8 o'clock be the required time, we get

$$x=25+\frac{x}{12}$$

or, $$\frac{11x}{12}=25 \qquad \therefore x=\frac{300}{11}=27\frac{3}{11}$$

Thus the required time is $27\frac{3}{11}$ minutes past 8 o'clock

8 $$\frac{a}{b}=\frac{b}{c}=k \text{ (suppose)} \qquad \therefore a=bk \text{ and } b=ck,$$

$$\therefore a=ck^2,$$

$$(a+b+c)(a-b+c)=(ck^2+ck+c)(ck^2-ck+c)$$
$$=c^2(k^2+k+1)(k^2-k+1)=c^2(k^4+k^2+1)$$

Also, $$a^2+b^2+c^2=c^2k^4+c^2k^2+c^2=c^2(k^4+k^2+1)$$
$$\therefore (a+b+c)(a-b+c)=a^2+b^2+c^2$$

X.

1 $$27a^3-8b^3-27c^3-54abc$$
$$=(3a)^3+(-2b)^3+(-3c)^3-3\cdot 3a\cdot(-2b)\cdot(-3c)$$
$$=(3a-2b-3c)(9a^2+4b^2+9c^2+6ab+9ac-6bc),$$

$\therefore$ the quotient $=9a^2+4b^2+9c^2+6ab-6bc+9ac$

2 $$\begin{array}{r|l}
x^5+11x+12 & x^5+11x^3-54 \;(1 \\
 & \underline{x^5 \qquad\quad +11x+12} \\
 & 11\,\big|\,11x^3-11x-66 \\
 & \qquad x^3-x-6
\end{array}$$

$$
\begin{array}{r|l|l}
x^3-x-6 & x^6 \qquad\qquad + 11x + 12 & x^3+1 \\
 & x^6 - x^4 - 6x^3 & \\ \hline
 & x^4 + 6x^3 + 11x + 12 & \\
 & x^4 \qquad\quad - x - 6 & \\ \hline
 & 6 \,|\, 6x^3 + 12x + 18 & \\ \hline
 & x^3 + 2x + 3 &
\end{array}
$$

$$
\begin{array}{r|l|l}
x^2+2x+3 & x^3 \qquad - x - 6 & x-2 \\
 & x^3 + 2x^2 + 3x & \\ \hline
 & -2x^2 - 4x - 6 & \\
 & -2x^2 - 4x - 6 & \\ \hline
\end{array}
$$

Thus the reqd H C F $= x^2+2x+3$

3 $$(a^2-b^2)(x^2+y^2)+2(a^2+b^2)xy$$
$$= a^2(x^2+y^2+2xy) - b^2(x^2+y^2-2xy) = a^2(x+y)^2 - b^2(x-y)^2$$
$$= \{a(x+y)+b(x-y)\}\{a(x+y)-b(x-y)\}$$
$$= \{(a+b)x+(a-b)y\}\{(a-b)x+(a+b)y\}$$

4 The given expression

$$= \frac{\dfrac{a^4+b^4-2a^2b^2}{a^2b^2}}{\dfrac{a^4+b^4+2a^2b^2}{a^2b^2}} - \frac{\dfrac{a^2-b^2}{ab}}{\dfrac{(a+b)^2-2ab}{ab}}$$

$$= \frac{(a^2-b^2)^2}{(a^2+b^2)^2} - \frac{a^2-b^2}{a^2+b^2} = \frac{(a^2-b^2)^2}{(a^2+b^2)^2} \times \frac{a^2+b^2}{a^2-b^2} = \frac{a^2-b^2}{a^2+b^2}$$

5 $$a^2(b+c)+b^2(c+a)+c^2(a+b)+abc(a+b+c)$$
$$= a^2(b+c)+a(b^3+b^2c+bc^2+c^3)+a^2bc+b^3c+bc^3$$
$$= a^2(b+c)+a\{b^2(b+c)+c^2(b+c)\}+bc(a^2+b^2+c^2)$$
$$= a^2(b+c)+(b+c)(b^2+c^2)a+bc(a^2+b^2+c^2)$$
$$= a(b+c)(a^2+b^2+c^2)+bc(a^2+b^2+c^2)$$
$$= (a^2+b^2+c^2)(ab+ac+bc)$$

6 $$\sqrt{9+2x} - \sqrt{2x} = \frac{5}{\sqrt{9+2x}}$$

Multiplying both sides by $\sqrt{9+2x}$, we have

$$9+2x-\sqrt{18x+4x^2} = 5$$

or, $$4+2x = \sqrt{18x+4x^2}$$

or, $$(4+2x)^2 = 18x+4x^2$$

or, $$16+16x+4x^2 = 18x+4x^2$$

$$2x = 16, \qquad x = 8$$

7 Let one man do the work in x days and one boy in y days

$$\frac{1}{x}+\frac{2}{y}=\frac{1}{12} \qquad (1)$$

and $$\frac{3}{x}+\frac{1}{y}=\frac{1}{6} \qquad (2)$$

Multiplying (2) by 2, we have

$$\frac{6}{x}+\frac{2}{y}=\frac{1}{3} \quad \text{and} \quad \frac{1}{x}+\frac{2}{y}=\frac{1}{12} \qquad (1)$$

$$\frac{5}{x}=\frac{1}{4}, \qquad x=20$$

Thus one man can do the work in 20 days

8 $$\frac{a}{b}=\frac{b}{c}=k \text{ (suppose)},$$

$$a=bk \text{ and } b=ck, \qquad a=ck^2,$$

$$a^4+a^2c^2+c^4=c^4k^8+c^4k^4+c^4=c^4(k^8+k^4+1)$$

Also, $$b^2\left(\frac{b^2}{c^2}-1+\frac{b^2}{a^2}\right)\left(a^2+b^2+c^2\right)$$

$$=c^2k^2\left(\frac{c^2k^2}{c^2}-1+\frac{c^2k^2}{c^2k^4}\right)\left(c^2k^4+c^2k^2+c^2\right)$$

$$=(c^2k^4-c^2k^2+c^2)(c^2k^4+c^2k^2+c^2)$$

$$=c^4(k^4-k^2+1)(k^4+k^2+1)=c^4(k^8+k^4+1)$$

$$a^4+a^2c^2+c^4=b^2\left(\frac{b^2}{c^2}-1+\frac{b^2}{a^2}\right)\left(a^2+b^2+c^2\right)$$

XI

1 $$(x^2+xy+y^2)^2-4xy(x^2+y^2)=\{(x^2+y^2)+xy\}^2-4xy(x^2+y^2)$$

$$=(x^2+y^2)^2-2xy(x^2+y^2)+x^2y^2=\{(x^2+y^2)-xy\}^2$$

$$=(x^2-xy+y^2)^2$$

2. (i) $$a^2-b^2-c^2+d^2-2(ad-bc)=a^2-2ad+d^2-(b^2-2bc+c^2)$$

$$=(a-d)^2-(b-c)^2=\{(a-d)+(b-c)\}\{(a-d)-(b-c)\}$$

$$=(a+b-c-d)(a-b+c-d)$$

(ii) $$x^2-y^2-z^2+2yz+x+y-z=x^2-(y^2-2yz+z^2)+(x+y-z)$$

$$=x^2-(y-z)^2+(x+y-z)=(x+y-z)(x-y+z)+(x+y-z)$$

$$=(x+y-z)(x-y+z+1)$$

3

$$
\begin{array}{l|l}
 & \dfrac{9x^2}{a^2}-\dfrac{6x}{a}+3-\dfrac{2a}{3x}+\dfrac{a^2}{9x^2}\left(\dfrac{3x}{a}-1+\dfrac{a}{3x}\right. \\
 & \dfrac{9x^2}{a^2} \\
\hline
\dfrac{6x}{a}-1 & -\dfrac{6x}{a}+3-\dfrac{2a}{3x}+\dfrac{a^2}{9x^2} \\
 & -\dfrac{6x}{a}+1 \\
\hline
\dfrac{6x}{a}-2+\dfrac{a}{3x} & 2-\dfrac{2a}{3x}+\dfrac{a^2}{9x^2} \\
 & 2-\dfrac{2a}{3x}+\dfrac{a^2}{9x^2} \\
\hline
\end{array}
$$

Thus the reqd root $=\dfrac{3x}{a}-1+\dfrac{a}{3x}$

4 From (1), we have

$$2x+4y+6z=12$$

and $2x+4y+\ z=7$. (2)

$$5z=5\ ,\qquad z=1$$

From (1), $x+2y+3=6$

or, $x+2y=3$ (4)

From (3), $3x+2y+9=14$

or, $3x+2y=5$ (5)

From (4) and (5), $2x=2$, $x=1$

Hence also $z=1$

Thus $x=y=z=1$

5 $x^4y-x^3y^2-15x^2y^3+38xy^4-14y^5$

$=y(x^4-x^3y-15x^2y^2+38xy^3-14y^4)$

$x^5-7x^4y+21x^3y^2-34x^2y^3+28xy^4$

$=x(x^4-7x^3y+21x^2y^2-34xy^3+28y^4)$

$$
\begin{array}{r|l}
x^4-x^3y-15x^2y^2+38xy^3-14y^4 & x^4-7x^3y+21x^2y^2-34xy^3+28y^4\ (1 \\
 & x^4-x^3y-15x^2y^2+38xy^3-14y^4 \\
\hline
-6y & -6x^3y+36x^2y^2-72xy^3+42y^4 \\
\hline
 & x^3-6x^2y+12xy^2-7y^3
\end{array}
$$

$$
\begin{array}{r|l}
x^3-6x^2y+12xy^2-7y^3 & x^4-x^3y-15x^2y^2+38xy^3-14y^4 \;(x+5y \\
 & \underline{x^4-6x^3y+12x^2y^2-7xy^3} \\
 & 5x^3y-27x^2y^2+45xy^3-14y^4 \\
 & \underline{5x^3y-30x^2y^2+60xy^3-35y^4} \\
3y^2 & \underline{3x^2y^2-15xy^3+21y^4} \\
 & x^2-5xy+7y^2
\end{array}
$$

$$
\begin{array}{r|l}
x^2-5xy+7y^2 & x^3-6x^2y+12xy^2-7y^3 \;(x-y \\
 & \underline{x^3-5x^2y+7xy^2} \\
 & -x^2y+5xy^2-7y^3 \\
 & \underline{-x^2y+5xy^2-7y^3}
\end{array}
$$

Thus the reqd H C F $= x^2-5xy+7y^2$

6 Let him buy x oranges of the 1st sort, then he bought $(570-x)$ oranges of the other sort

We have $\dfrac{x}{16}+\dfrac{570-x}{18}+3=\dfrac{570}{15}$

or, $x\left(\dfrac{1}{16}-\dfrac{1}{18}\right)=570\left(\dfrac{1}{15}-\dfrac{1}{18}\right)-3$

or $x\left(\dfrac{9-8}{8\times 18}\right)=570\left(\dfrac{18-15}{15\times 18}\right)-3$

or, $\dfrac{x}{8\times 18}=\dfrac{19}{3}-3$

or $\dfrac{x}{8\times 18}=\dfrac{10}{3}$ $\qquad x=480$

Thus there were 480 oranges of the 1st sort and 90 oranges of the 2nd sort

7 The given expression

$$=\frac{a^2}{(a-c)(a-b)}+\frac{b^2}{(b-a)(b-c)}+\frac{c^2}{(c-b)(c-a)}$$

$$=\frac{-a^2(b-c)-b^2(c-a)-c^2(a-b)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}}{-\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}}=1$$

8 $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ (suppose)

$a=bk$, $c=dk$ and $e=fk$,

$\therefore\ (a^2+c^2+e^2)(b^2+d^2+f^2)$

$=(b^2k^2+d^2k^2+f^2k^2)(b^2+d^2+f^2)$

$=k^2(b^2+d^2+f^2)^2$

Also, $\quad (ab+cd+ef)^2$
$=(b^2k+d^2k+f^2k)^2$
$=k^2(b^2+d^2+f^2)^2$
$\therefore \quad (a^2+c^2+e^2)(b^2+d^2+f^2)=(ab+cd+ef)^2$

XII

1 $\quad x^2+y^2+z^2-yz-zx-xy$
$=\frac{1}{2}\{(x^2-2xy+y^2)+(y^2-2yz+z^2)+(z^2-2zx+x^2)\}$
$=\frac{1}{2}\{(x-y)^2+(y-z)^2+(z-x)^2\}$
$=\frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}$
$=\frac{1}{2}(2a^2+2b^2+2c^2-2ab-2bc-2ca)$
$=a^2+b^2+c^2-bc-ca-ab$

2 The given expression

$$=\frac{(y+z)(x^2-yz)+(z+x)(y^2-xz)+(x+y)(z^2-xy)}{(x^2-yz)(y^2-xz)(z^2-xy)}$$

$$=\frac{x^2(y+z-z-y)+y^2(z+x-z-x)+z^2(x+y-y-x)}{(x^2-yz)(y^2-xz)(z^2-xy)}=0$$

3 (i) $\quad x^2-2ax-b^2+2ab$
$=x^2-2ax+a^2-(a^2+b^2-2ab)$
$=(x-a)^2-(a-b)^2$
$=(x-a+a-b)(x-a-a+b)$
$=(x-b)(x-2a+b)$

(ii) $\quad x^2+(a+b+c)x+ab+ac$
$=x^2+ax+(b+c)x+a(b+c)$
$=x(x+a)+(b+c)(x+a)$
$=(x+a)(x+b+c)$

4 $\quad 9x^4+80x^2-9$
$=9x^4+81x^2-x^2-9$
$=9x^2(x^2+9)-(x^2+9)$
$=(9x^2-1)(x^2+9)$
$=(3x+1)(3x-1)(x^2+9)$

Also, $\quad 6x^4-2x^3+9x^2+9x-4$
$=6x^4-2x^3+9x^2-3x+12x-4$
$=2x^3(3x-1)+3x(3x-1)+4(3x-1)$
$=(2x^3+3x+4)(3x-1)$

Thus $\quad 3x-1 \quad$ is the reqd H C F

5 $$\frac{6x+13}{15}-\frac{3x+5}{5x-25}-\frac{2x}{5}=0$$

or, $$\frac{6x+13}{3}-\frac{3x+5}{x-5}-2x=0$$

or, $$\frac{6x+13-6x}{3}=\frac{3x+5}{x-5}$$

or, $9x+15=13x-65$

or, $4x=80$, $\qquad x=20$

6 Let A, B and C together do the work in x days

But A and B can do $\frac{1}{12}$ of the work in 1 day

A and C $\quad \frac{1}{15}$

B and C $\quad \frac{1}{20}$

A, B and C together can do

$\left(\frac{1}{12}+\frac{1}{15}+\frac{1}{20}\right)$ of the work in 2 days,

we have $\frac{2}{x}=\frac{1}{12}+\frac{1}{15}+\frac{1}{20}=\frac{5+4+3}{60}=\frac{12}{60}=\frac{1}{5}$,

$x=10$

Thus A, B and C can do the work in 10 days, all working together

7 $$4\sqrt{147}-3\sqrt{75}-6\sqrt{\tfrac{1}{3}}+18\sqrt{\tfrac{1}{27}}$$
$$=4\sqrt{49\cdot 3}-3\sqrt{25\cdot 3}-2\sqrt{3}+\frac{18}{\sqrt{9\cdot 3}}$$
$$=4\times 7\sqrt{3}-3\times 5\sqrt{3}-2\sqrt{3}+\frac{6\times 3}{3\sqrt{3}}$$
$$=28\sqrt{3}-15\sqrt{3}-2\sqrt{3}+2\sqrt{3}=13\sqrt{3}$$

8 $\frac{x}{y}=\frac{a}{b}=k$ (suppose), $\qquad x=ky$ and $a=kb$,

$$\frac{x^2+a^2}{x+a}+\frac{y^2+b^2}{y+b}=\frac{k^2y^2+k^2b^2}{ky+kb}+\frac{y^2+b^2}{y+b}$$
$$=\frac{k^2(y^2+b^2)}{k(y+b)}+\frac{y^2+b^2}{y+b}=k\,\frac{y^2+b^2}{y+b}+\frac{y^2+b^2}{y+b}$$
$$=(k+1)\left(\frac{y^2+b^2}{y+b}\right)$$

Also, $$\frac{(x+y)^2+(a+b)^2}{x+y+a+b}$$
$$=\frac{y^2(k+1)^2+b^2(k+1)^2}{y(k+1)+b(k+1)}=\frac{(y^2+b^2)(k+1)^2}{(k+1)(y+b)}$$
$$=(k+1)\left(\frac{y^2+b^2}{y+b}\right)$$
$$\frac{x^2+a^2}{x+a}+\frac{y^2+b^2}{y+b}=\frac{(x+y)^2+(a+b)^2}{x+y+a+b}$$

XIII

1 $$a(b-c)(s-a)^2+b(c-a)(s-b)^2+c(a-b)(s-c)^2$$
$$=a(b-c)(s^2-2as+a^2)+b(c-a)(s^2-2sb+b^2)+c(a-b)(s^2-2sc+c^2)$$
$$=s^2\{a(b-c)+b(c-a)+c(a-b)\}-2s\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}$$
$$+a^3(b-c)+b^3(c-a)+c^3(a-b)$$
$$=0-(a+b+c)\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}+a^3(b-c)+b^3(c-a)$$
$$+c^3(a-b)$$
$$=-\{a^3(b-c)+b^3(c-a)+c^3(a-b)\}$$
$$-\{a^2(b^2-c^2)+b^2(c^2-a^2)+c^2(a^2-b^2)\}$$
$$+a^3(b-c)+b^3(c-a)+c^3(a-b)=0$$

2 $$x^6+x^3a^3+a^6=\left(x^{\frac{3}{2}}\right)^4+\left(x^{\frac{3}{2}}a^{\frac{3}{2}}\right)^2+\left(a^{\frac{3}{2}}\right)^4$$
$$=\left\{\left(x^{\frac{3}{2}}\right)^2+x^{\frac{3}{2}}a^{\frac{3}{2}}+\left(a^{\frac{3}{2}}\right)^2\right\}\left\{\left(x^{\frac{3}{2}}\right)^2-x^{\frac{3}{2}}a^{\frac{3}{2}}+\left(a^{\frac{3}{2}}\right)^2\right\}$$
$$=\left(x^3+x^{\frac{3}{2}}a^{\frac{3}{2}}+a^3\right)\left(x^3-x^{\frac{3}{2}}a^{\frac{3}{2}}+a^3\right)$$

Again, $$x^3+x^{\frac{3}{2}}a^{\frac{3}{2}}+a^3$$
$$=\left(x^{\frac{3}{4}}\right)^4+\left(x^{\frac{3}{4}}a^{\frac{3}{4}}\right)^2+\left(a^{\frac{3}{4}}\right)^4$$
$$=\left\{\left(x^{\frac{3}{4}}\right)^2+x^{\frac{3}{4}}a^{\frac{3}{4}}+\left(a^{\frac{3}{4}}\right)^2\right\}\left\{\left(x^{\frac{3}{4}}\right)^2-x^{\frac{3}{4}}a^{\frac{3}{4}}+\left(a^{\frac{3}{4}}\right)^2\right\}$$
$$=\left(x^{\frac{3}{2}}+x^{\frac{3}{4}}a^{\frac{3}{4}}+a^{\frac{3}{2}}\right)\left(x^{\frac{3}{2}}-x^{\frac{3}{4}}a^{\frac{3}{4}}+a^{\frac{3}{2}}\right)$$

Lastly, $$x^{\frac{3}{2}}+x^{\frac{3}{4}}a^{\frac{3}{4}}+a^{\frac{3}{2}}$$
$$=\left(x^{\frac{3}{8}}\right)^4+\left(x^{\frac{3}{8}}\right)^2\left(a^{\frac{3}{8}}\right)^2+\left(a^{\frac{3}{8}}\right)^4$$
$$=\left\{\left(x^{\frac{3}{8}}\right)^2+x^{\frac{3}{8}}a^{\frac{3}{8}}+\left(a^{\frac{3}{8}}\right)^2\right\}\left\{\left(x^{\frac{3}{8}}\right)^2-x^{\frac{3}{8}}a^{\frac{3}{8}}+\left(a^{\frac{3}{8}}\right)^2\right\}$$
$$=\left(x^{\frac{3}{4}}+x^{\frac{3}{8}}a^{\frac{3}{8}}+a^{\frac{3}{4}}\right)\left(x^{\frac{3}{4}}-x^{\frac{3}{8}}a^{\frac{3}{8}}-a^{\frac{3}{4}}\right)$$

Therefore, $x^6+x^3a^3+a^6$ is divisible by $x^{\frac{3}{4}}-x^{\frac{3}{8}}a^{\frac{3}{8}}+a^{\frac{3}{4}}$

3 The given expression

$$=\frac{(x^2-yz)(y+z)+(y^2-zx)(z+x)+(z^2-xy)(x+y)}{(x+y)(y+z)(z+x)}$$

$$=\frac{x^2(y+z-z-y)+y^2(-z+x+z-x)+z^2(-y-x+x+y)}{(x+y)(y+z)(z+x)}=0$$

4 $$\frac{x^2-a^2}{x-a}+\frac{x^2-b^2}{x-b}+\frac{x^2-c^2}{x-c}=a+b+c-3x$$

or, $$x+a+x+b+x+c=a+b+c-3x$$

or, $$6x=0\,, \qquad x=0$$

5. Let x be the reqd number of gallons His total cost $=80\times15$ shillings, he must sell the mixture for $\frac{110}{100}\times80\times15=11\times8\times15$ shillings,

we have $12(80+x)=11\times8\times15$

or, $80+x=110\,, \qquad x=30$

Thus he must mix 30 gallons of water

6 $$\frac{x^{(a+b)2}x^{(b+c)2}x^{(c+a)2}}{(x^a x^b x^c)^4}$$

$$=\frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}}=\frac{x^{4a+4b+4c}}{x^{4a+4b+4c}}=1$$

7 $$3\sqrt[3]{128}-4\sqrt[3]{-686}+2\sqrt[3]{54}$$

$$=3\sqrt[3]{64\times2}-4\sqrt[3]{-343\times2}+2\sqrt[3]{27\times2}$$

$$=3\times4\sqrt[3]{2}-4\times-7\sqrt[3]{2}+2\times3\sqrt[3]{2}$$

$$=12\sqrt[3]{2}+28\sqrt[3]{2}+6\sqrt[3]{2}=46\sqrt[3]{2}$$

8 $\frac{a}{b}=\frac{b}{c}=k$ (suppose), $a=bk$ and $b=ck$, $\therefore$ $a=ck^2$,

$$\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{k^2b^2+kb\,kc+k^2c^2}{b^2+bc+c^2}$$

$$=\frac{k^2(b^2+bc+c^2)}{b^2+bc+c^2}=k^2$$

Also, $\frac{a}{c}=\frac{ck^2}{c}=k^2$ $\qquad a^2+ab+b^2 \quad b^2+bc+c^2 \quad a \quad c.$

XIV.

1 We have

$$a+b+c=2s \qquad c-s=s-(a+b)$$

Hence, $(a-s)^2+(b-s)^2+(c-s)^2$

$$=(a-s)^2+(b-s)^2+\{s-(a+b)\}^2$$

$$=a^2-2as+s^2+b^2-2bs+s^2+s^2+(a+b)^2-2s(a+b)$$

$$=2(a^2+b^2+ab+s^2)+s^2-4s(a+b)$$

$$=4s(a+b)+s^2-4s(a+b)=s^2$$

2

$$\begin{array}{l} x+a\,)\,x^2+px+q\,(\,x-(a-p) \\ \qquad\quad \underline{x^2+ax} \\ \qquad\quad -x(a-p)+q \\ \qquad\quad \underline{-x(a-p)-a(a-p)} \\ \qquad\qquad a(a-p)+q \end{array}$$

In order $x+a$ may be a factor of x^2+px+q, $a(a-p)+q=0$ (1)

Similarly $a(a-l)+m=0$ (2)

From (1) and (2), we have $a(l-p)-(m-q)=0$

$$a=\frac{m-q}{l-p}.$$

3 $$\frac{7+3\sqrt{5}}{7-3\sqrt{5}}+\frac{7-3\sqrt{5}}{7+3\sqrt{5}}=\frac{(7+3\sqrt{5})^2+(7-3\sqrt{5})^2}{49-45}$$

$$=\frac{2(49+45)}{4}=\frac{94}{2}=47$$

4 $$\frac{a-b}{x-a}+\frac{a-b}{x-b}=\frac{a}{x-a}-\frac{b}{x-b}$$

or, $$\frac{(a-b)(x-b)+(a-b)(x-a)}{(x-a)(x-b)}=\frac{a(x-b)-b(x-a)}{(x-a)(x-b)}$$

or, $$-b(x-b)+a(x-a)=0$$

or, $$(x-b)b=a(x-a)$$

or, $$xb-a^2=ax-a^2$$

or, $$x(b-a)=b^2-a^2$$

$$x=a+b$$

5 $$\frac{y+z-x}{b+c}=\frac{z+x-y}{c+a}=\frac{x+y-z}{a+b}=1$$

by addendo, we get

$$1=\frac{x+y+z}{2(a+b+c)}$$

$$x+y+z=2(a+b+c)$$

Also, $y+z-x=b+c$

$$2x=2a+b+c$$

$$x=\tfrac{1}{2}(2a+b+c)$$

Again we have $z+x-y=c+a$

and, $x+y+z=2(a+b+c)$

$$y=\tfrac{1}{2}(a+2b+c)$$

Similarly $z=\tfrac{1}{2}(a+b+2c)$

6 Suppose A worked for x days B worked for $(14-x)$ days, A finished $\frac{x}{20}$ of the work and B did the remaining portion which is equal to $\frac{14-x}{12}$

Therefore, we have $\frac{14-x}{12}=1-\frac{x}{20}$

or, $70-5x=60-3x$ or, $2x=10$

$$x=5$$

Thus A worked for 5 days

7 $$(x+a)(x+2a)(x+3a)(x+4a)$$
$$=\{(x+a)(x+4a)\}\{(x+2a)(x+3a)\}$$
$$=(x^2+5ax+4a^2)(x^2+5ax+6a^2)$$
$$=\{(x^2+5ax+5a^2)-a^2\}\{(x^2+5ax+5a^2)+a^2\}$$
$$=(x^2+5ax+5a^2)^2-a^4$$

8 $$a(y+z)=b(z+x)=c(x+y)$$
$$\frac{a(y+z)}{abc}=\frac{b(z+x)}{abc}=\frac{c(x+y)}{abc}$$

or, $\frac{y+z}{bc}=\frac{z+x}{ac}=\frac{x+y}{ab}=k$ (suppose),

$y+z=kbc$ (1)

$z+x=kac$ (2)

$x+y=kab$ (3)

From (1) and (2), $x-y=kc(a-b)$,

$$\frac{x-y}{c(a-b)}=k$$

Similarly from (1) and (3), $\dfrac{z-x}{b(c-a)}=\lambda$

and from (2) and (3) $\dfrac{y-z}{a(b-c)}=\lambda$

Hence, $\dfrac{y-z}{a(b-c)}=\dfrac{z-x}{b(c-a)}=\dfrac{x-y}{c(a-b)}$

XV

1

$$
\begin{array}{r|l}
 & x^4+2ax^3+(a^2+8)x^2+(4a+ab)x+4b\,\big(\,x^2+ax+4 \\
 & x^4 \\ \hline
2x^2+ax & 2ax^3+(a^2+8)x^2+(4a+ab)x+4b \\
 & 2ax^3+a^2x^2 \\ \hline
2x^2+2ax+4 & 8x^2+(4a+ab)x+4b \\
 & 8x^2+8ax+16 \\ \hline
 & -(4a-ab)x+4b-16
\end{array}
$$

In order the expression may be a perfect square, we must have

$$4-ab=0 \qquad (1)$$

also

$$4b-16=0 \qquad (2)$$

From both these conditions, we get $b=4$

2 The left-hand expression

$=(b-c)\{1+a(b+c)+a^2bc\}+(c-a)\{1+b(c+a)+b^2ca\}$
$\quad+(a-b)\{1+c(a+b)+c^2ab\}$

$=\{(b-c)+(c-a)+(a-b)\}+abc\{a(b-c)+b(c-a)+c(a-b)\}$
$\quad+\{a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)\}$

$=a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)$

$=-\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}$

$=-\{-(a-b)(b-c)(c-a)\}=(a-b)(b-c)(c-a)$

3 The given expression

$$=\frac{2x^2+2}{x^4+x^2+1}+\frac{x-\sqrt{x}+1+x+\sqrt{x}+1}{x^2+x+1}-\frac{1}{x^2-x+1}$$

$$=\frac{2x^2+2}{x^4+x^2+1}+\frac{2x+2}{x^2+x+1}-\frac{1}{x^2-x+1}$$

$$=\frac{2x^2+2+2x^3+2-x^2-x-1}{x^4+x^2+1}$$

$$=\frac{2x^3+x^2-x+3}{x^4+x^2+1}=\frac{2x^3+3x^2-2x^2-3x+2x+3}{x^4+x^2+1}$$

$$=\frac{x^2(2x+3)-x(2x-3)+(2x+3)}{x^4+x^2+1}$$

$$=\frac{(x^2-x+1)(2x+3)}{(x^2-x+1)(x^2+x+1)}=\frac{2x+3}{x^2+x+1}$$

4 $2x^3+(2a-3b)x^2-(2b+3ab)x+3b^2$
$=2x^3+2ax^2-2bx-3bx^2-3abx+3b^2$
$=2x(x^2+ax-b)-3b(x^2+ax-b)=(x^2+ax-b)(2x-3b)$
$2x^2-(3b-2c)x-3bc=2x^2+2cx-3bx-3bc$
$=2x(x+c)-3b(x+c)=(x+c)(2x-3b)$
the reqd H C F $=2x-3b$

5 We have $x=\frac{a^n+b^n}{2}$

$$2nx=na^n+nb^n$$

the given exp $=\frac{a^n}{2na^n-na^n-nb^n}+\frac{b^n}{2nb^n-na^n-nb^n}$

$$=\frac{a^n}{na^n-nb^n}+\frac{b^n}{nb^n-na^n}=\frac{a^n-b^n}{na^n-nb^n}=\frac{1}{n}$$

6 $$\frac{20x+36}{25}+\frac{5x+20}{9x-16}=\frac{4x}{5}+\frac{86}{25}$$

or, $$\frac{5x+20}{9x-16}=\frac{4x}{5}+\frac{86}{25}-\frac{20x+36}{25}$$

$$=\frac{20x+86-20x+36}{25}=\frac{50}{25}=2$$

or, $5x+20=18x-32$

or $13x=52$, $x=4$

7 Let x gallons be the contents of the vessel

there were $\frac{7x}{10}$ gallons of spirit

of 9 gallons of the mixture, there were $\frac{7\cdot 9}{10}$ or $\frac{63}{10}$ gallons of spirit

After 9 gallons have been taken there remains in the vessel $\left(\frac{7x}{10}-\frac{63}{10}\right)$ gallons of spirit

$$\text{we have,}\quad \frac{7x-63}{10} \quad x = 58\tfrac{1}{3} \quad 100$$

$$\text{or,}\quad \frac{7x-63}{10x}=\frac{175}{300}$$

$$\text{or,}\quad \frac{7x-63}{x}=\frac{35}{6}$$

$$\text{or,}\quad \frac{x-9}{x}=\frac{5}{6}$$

$$\text{or,}\quad 6x-54=5x\,, \qquad x=54$$

Thus the contents of the vessel $=54$ gallons

8 $$\frac{x-z}{y-z}=\frac{x^2}{y^2}\,,\qquad \frac{x-y}{y-z}=\frac{x^2-y^2}{y^2}$$

$$\text{or,}\quad \frac{1}{y-z}=\frac{x+y}{y^2}\,,\qquad \frac{y}{y^2-yz}=\frac{x+y}{y^2}=\frac{x}{yz}\ \left(\text{addendo}\right),$$

$$\frac{x+y}{y}=\frac{x}{z}\quad\text{or,}\quad \frac{x+2y}{y}=\frac{x+z}{z} \qquad (1)$$

$$\text{Again,}\quad \frac{x+y}{x}=\frac{y}{z}\,,\qquad \frac{2x+y}{x}=\frac{y+z}{z}\quad . \qquad (2)$$

Hence, from (1) and (2), we have

$$\frac{x+z}{y+z}=\frac{\frac{x+2y}{y}}{\frac{y+2x}{x}}=\frac{\frac{x}{y}+2}{\frac{y}{x}+2}$$

$$x+z \quad y+z=\frac{x}{y}+2 \quad \frac{y}{x}+2$$

XVI.

1 $$\begin{array}{l|l|l}
x^5+2x^4-5x^2-7x+3 & 3x^6-3x^4-18x^3+x^2+2x+3 & 1+3x \\
 & \underline{x^5+2x^4\qquad\quad -5x^2-7x+3} & \\
 & 3x^6-x^5-5x^4-18x^3+6x^2+9x & \\
 & \underline{3x^6+6x^5\qquad\quad -15x^3-21x^2+9x} & \\
 & -x^2\,|\,\underline{-7x^5-5x^4-3x^3+27x^2} & \\
 & \qquad 7x^3+5x^2+3x-27 &
\end{array}$$

$$
\begin{array}{r|l}
 & x^5+2x^4 \qquad -5x^2-7x+3 \\
 & \quad 7 \\
7x^3+5x^2+3x-27 & 7x^5+14x^4 \quad -35x^2-49x+21 \;(x^2 \\
 & 7x^5+5x^4+3x^3-27x^2 \\
 & \qquad 9x^4-3x^3-8x^2-49x+21 \\
 & \qquad 7 \\
 & \qquad 63x^4-21x^3-56x^2-343x+147 \;(9x \\
 & \qquad 63x^4+45x^3+27x^2-243x \\
 & \qquad\qquad -66x^3-83x^2-100x+147 \\
 & \qquad\qquad\quad 7 \\
 & \qquad -462x^3-581x^2-700x+1029 \;(-66 \\
 & \qquad -462x^3-330x^2-198x+1782 \\
 & \qquad -251 \mid -251x^2-502x-753 \\
 & \qquad\qquad\qquad x^2+2x-3
\end{array}
$$

$$
\begin{array}{r|l}
x^2+2x+3 & 7x^3+5x^2+3x-27 \;(7x-9 \\
 & 7x^3+14x^2+21x \\
 & \quad -9x^2-18x-27 \\
 & \quad -9x^2-18x-27
\end{array}
$$

Thus the reqd H C F $=x^2+2x+3$

2 $\quad \sqrt{2x-1}+\sqrt{3x-2}=\sqrt{4x-3}+\sqrt{5x-4}$

or, $\sqrt{2x-1}-\sqrt{5x-4}=\sqrt{4x-3}-\sqrt{3x-2}$

or, $2x-1+5x-4-2\sqrt{10x^2-13x+4}$

$\qquad =4x-3+3x-2-2\sqrt{12x^2-17x+6}$

or, $10x^2-13x+4=12x^2-17x+6$

or, $2x^2-4x+2=0$

or, $2(x-1)^2=0$, $\qquad x=1$

3 $\quad (a+b+c)x=(-a+b+c)y=(a-b+c)z=(a+b-c)w$

$$\frac{1}{y}=\frac{-a+b+c}{a+b+c}\ \frac{1}{x}$$

$$\frac{1}{z}=\frac{a-b+c}{a+b+c}\ \frac{1}{x}$$

$$\frac{1}{w}=\frac{a+b-c}{a+b+c}\ \frac{1}{x}$$

$$\frac{1}{y}+\frac{1}{z}+\frac{1}{w}=\frac{a+b+c}{a+b+c}\ \frac{1}{x}=\frac{1}{x}$$

4 $$\frac{5\sqrt{x+y}}{x} + \frac{5\sqrt{x+y}}{y} = 10\frac{2}{3} \qquad (1)$$

$$\frac{3\sqrt{x-y}}{y} - \frac{3\sqrt{x-y}}{x} = \frac{4}{5} \qquad (2)$$

From (1), we have $\dfrac{5\sqrt{x+y}(x+y)}{xy} = \dfrac{32}{3}$ (3)

From (2), we have $\dfrac{3\sqrt{x-y}(x-y)}{xy} = \dfrac{4}{5}$ (4)

From (3) and (4), we have $\dfrac{5(x+y)^{\frac{3}{2}}}{3(x-y)^{\frac{3}{2}}} = \dfrac{40}{3}$

or, $\left(\dfrac{x+y}{x-y}\right)^{\frac{3}{2}} = 8 = (4)^{\frac{3}{2}}$

or, $\dfrac{x+y}{x-y} = 4$

$\dfrac{x}{y} = \dfrac{5}{3}$ $\qquad x = \dfrac{5y}{3}$

from (4), we have $3\left(\dfrac{5}{3}y - y\right)^{\frac{3}{2}} = \dfrac{4}{5}\;\dfrac{5y^2}{3}$

or, $3\left(\dfrac{2y}{3}\right)^{\frac{3}{2}} = \dfrac{4y^2}{3}$ or, $\dfrac{9(2y)^3}{27} = \dfrac{16y^4}{9}$

$y = \frac{3}{2} = 1\frac{1}{2}$ $\qquad x = \frac{5}{3}\,\frac{3}{2} = \frac{5}{2} = 2\frac{1}{2}$

5 $ax(y^3+b^3)+by(bx^2+a^2y) = b^2x^2y + ab^3x + axy^3 + a^2by^2$

$= b^2x(xy+ab) + ay^2(xy+ab) = (xy+ab)(b^2x+ay^2)$

6 $(\sqrt{a}+\sqrt{b}+\sqrt{c})(\sqrt{a}+\sqrt{b}-\sqrt{c})$

$(\sqrt{a}-\sqrt{b}+\sqrt{c})(\sqrt{b}+\sqrt{c}-\sqrt{a})$

$= \{(\sqrt{a}+\sqrt{b})+\sqrt{c}\}\{(\sqrt{a}+\sqrt{b})-\sqrt{c}\}$

$\{\sqrt{c}+(\sqrt{a}-\sqrt{b})\}\{\sqrt{c}-(\sqrt{a}-\sqrt{b})\}$

$= (a+2\sqrt{ab}+b-c)(c-a+2\sqrt{ab}-b)$

$= \{2\sqrt{ab}+(a+b-c)\}\{2\sqrt{ab}-(a+b-c)\}$

$= 4ab-(a+b-c)^2$

$= 4ab-(a^2+b^2+c^2+2ab-2ac-2bc)$

$= -a^2-b^2-c^2+2ab+2ac+2bc$

7 $$\frac{a+b}{a-b}=\frac{c}{d} \qquad \frac{a}{b}=\frac{c+d}{c-d},$$

we have, $$\frac{a}{b}\cdot\frac{a+b}{a-b}=\frac{c+d}{c-d}\cdot\frac{c}{d}$$

Thus, $$\frac{a^2+ab}{ab-b^2}=\frac{c^2+cd}{cd-d^2}$$

8 Let there be x per cent wine in the 1st vessel and y per cent wine in the other vessel

from the 1st mixture, we get

$$x+y=(100-x)+(100-y)$$

or, $$2x+2y=200$$

or, $$x+y=100 \qquad (1)$$

From the second mixture, we have

$$\frac{4x+y}{4(100-x)+(100-y)}=\frac{2}{3}$$

or, $$\frac{4x+y}{500-(4x+y)}=\frac{2}{3}$$

or, $$3(4x+y)=1000-2(4x+y)$$

or, $$5(4x+y)=1000$$

or, $$4x+y=200 \qquad (2)$$

Also, $$x+y=100 \qquad (1)$$

$$3x=100, \qquad \frac{x}{100}=\frac{1}{3},$$

from (1), $$3y=300-100=200, \qquad \frac{y}{100}=\frac{2}{3},$$

in the 1st vessel the wine is $\frac{1}{3}$ of the whole, in the second $\frac{2}{3}$

XVII

1 $$x^5+x^2+2x+2=x^2(x^3+1)+2(x+1)$$
$$=(x+1)\{x^2(x^2-x+1)+2\}=(x+1)(x^4-x^3+x^2+2)$$
$$=(x+1)\{(x^4+x^2+1)-(x^3-1)\}$$
$$=(x+1)\{(x^2+x+1)(x^2-x+1)-(x-1)(x^2+x+1)\}$$
$$=(x+1)(x^2+x+1)(x^2-2x+2)$$
and $$x^4+x^2+1=(x^2+x+1)(x^2-x+1)$$

Thus the reqd H C F. $=x^2+x+1$

2
$$\sqrt{y}-\sqrt{y-x}=\sqrt{20-x} \qquad (1)$$

$$\sqrt{y-x}\cdot\sqrt{20-x}=3 \quad 2 \qquad (2)$$

From (2), $\dfrac{\sqrt{y-x}}{\sqrt{20-x}}=\dfrac{3}{2}$

or, $\dfrac{y-x}{20-x}=\dfrac{9}{4}$

or, $4y-4x=180-9x$

or, $5x+4y=180 \qquad (3)$

From (1), $\dfrac{\sqrt{y}}{\sqrt{20-x}}-\dfrac{\sqrt{y-x}}{\sqrt{20-x}}=1$

or, $\dfrac{\sqrt{y}}{\sqrt{20-x}}-\dfrac{3}{2}=1$

or, $\dfrac{\sqrt{y}}{\sqrt{20-x}}=\dfrac{5}{2}, \qquad \dfrac{y}{20-x}=\dfrac{25}{4},$

$$4y=500-25x$$

or, $25x+4y=500 \qquad (4)$

also, $5x+4y=180 \qquad (3)$

$$20x=320 \qquad x=16$$

from (4) $4y=500-400=100 \qquad y=25$

3
$$\left(\frac{x}{x-1}\right)^2+\left(\frac{x}{x+1}\right)^2=\frac{x^2\{(x+1)^2+(x-1)^2\}}{(x^2-1)^2}$$

$$=\frac{2x^2(x^2+1)}{(x^2-1)^2}$$

$$=\frac{\dfrac{2(n-1)}{n+1}\left(\dfrac{n-1}{n+1}+1\right)}{\left(\dfrac{n-1}{n+1}-1\right)^2}$$

$$=\frac{\dfrac{2(n-1)2n}{(n+1)^2}}{\left(\dfrac{-2}{n+1}\right)^2}=\frac{4n(n-1)}{4}=n(n-1)$$

4 The numerator

$$=a^3(b^2-c^2)+b^3(c^2-a^2)+c^3(a^2-b^2)$$
$$=a^3(b^2-c^2)+a^2bc(b-c)+b^3(c^2-a^2)+b^2ac(c-a)$$
$$+c^3(a^2-b^2)+c^2ab(a-b)$$
$$=a^2(b-c)(ab+ac-bc)+b^2(c-a)(bc-ab+ac)$$
$$+c^2(a-b)(ac+bc-ab)$$
$$=(ab+bc+ab)\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}$$

the given fraction $=ab+bc-ca$

5 $\quad a+b+c=0 \qquad a+b=-c \qquad (a+b)^2=c^2$

$$a^2+b^2=c^2-2ab \qquad a^2+b^2-c^2=2(c^2-ab)$$
$$(a^2+b^2+c^2)^2=4(c^2-ab)^2=4(c^2c^2+a^2b^2-2c^2ab)$$
$$=4\{c^2(a+b)^2-2c^2ab-a^2b^2\}$$
$$=4\{c^2(a^2+b^2+2ab)-2c^2ab-a^2b^2\}$$
$$=4(a^2b^2+a^2c^2+b^2c^2)$$

Putting a for $(y-z)$, b for $(z-x)$ and c for $(x-y)$,

We get $\quad a+b+c=0 \qquad 4(a^2b^2+b^2c^2+c^2a^2)=(a^2+b^2+c^2)^2$

$$4\{(y-z)^2(z-x)^2+(z-x)^2(x-y)^2+(x-y)^2(y-z)^2\}$$
$$=\{(y-z)^2+(z-x)^2+(x-y)^2\}^2$$
$$=(y^2+z^2-2yz+z^2+x^2-2xz+x^2+y^2-2xy)^2$$
$$=4(x^2+y^2+z^2-xy-yz-xz)^2$$
$$(y-z)^2(z-x)^2+(z-x)^2(x-y)^2+(x-y)^2(y-z)^2$$
$$=(x^2+y^2+z^2-xy-yz-xz)^2$$

6 Let x and $(x-5)$ be the digits of the required number

the number $=10x+(x-5)=11x-5$

we get $\quad 10(x-5)+x=\frac{3}{8}(11x-5)$

or, $\quad 88x-400=33x-15$

or, $\quad 55x=385 \qquad x=7$,

the required number is 72

7 The numerator

$$=3x^3-x^2-5x+21=3x^3+7x^2-6x^2-14x+9x+21$$
$$=x^2(3x+7)-2x(3x+7)+3(3x+7)=(3x+7)(x^2-2x+3)$$

The denominator

$$=6x^3+29x^2+26x-21=6x^3+14x^2+15x^2+35x-9x-21$$
$$=2x^2(3x+7)+5x(3x+7)-3(3x+7)$$
$$=(3x+7)(2x^2+5x-3)$$

Hence the expression $=\dfrac{x^2-2x+3}{2x^2+5x-3}$

8 $$3(a^2+b^2+c^2)=(a+b+c)^2$$
$$=a^2+b^2+c^2+2ab+2bc+2ca$$
$$2a^2+2b^2+2c^2-2ab-2bc-2ca=0$$
or, $$(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)=0$$
or, $$(a-b)^2+(b-c)^2+(c-a)^2=0$$

Each of the terms being a perfect square, none of the terms is negative, therefore their sum cannot be zero unless each of them be zero

We have $a-b=0$ or, $a=b$ and $b-c=0$ or, $b=c$

$$a=b=c$$

XVIII

1 Putting a for $(x-y)$, b for $(y-z)$ and c for $(z-x)$,

we get $a+b+c=x-y+y-z+z-x=0$

$$(a^2+b^2+c^2)^2=4(a^2b^2+b^2c^2+c^2a^2)$$

[See Ex 5 XVII Page 383]

$$a^4+b^4+c^4+2a^2b^2+2b^2c^2+2c^2a^2=4(a^2b^2+b^2c^2+c^2a^2)$$
$$a^4+b^4+c^4=2(a^2b^2+b^2c^2+c^2a^2)$$
$$2(a^4+b^4+c^4)=4(a^2b^2+b^2c^2+c^2a^2)=(a^2+b^2+c^2)^2$$
$$2\{(x-y)^4+(y-z)^4+(z-x)^4\}=\{(x-y)^2+(y-z)^2+(z-x)^2\}$$

2 $$\frac{x+\sqrt{x^2-a^2}}{x-\sqrt{x^2-a^2}}=4\sqrt{2}\left\{\frac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}\right\}^{\frac{1}{2}}$$

or, $$\frac{2x+2\sqrt{(x+a)(x-a)}}{2x-2\sqrt{(x+a)(x-a)}}=2^{\frac{5}{2}}\left\{\frac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}\right\}^{\frac{1}{2}}$$

or, $$\left\{\frac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}\right\}^2=2^{\frac{5}{2}}\left\{\frac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}\right\}^{\frac{1}{2}}$$

or, $$\left\{\frac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}\right\}^{\frac{5}{2}}=2^{\frac{5}{2}}$$

or, $$\frac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}=2$$

or, $$\frac{\sqrt{x+a}}{\sqrt{x-a}}=\frac{3}{1}$$

$$\frac{x+a}{x-a}=9 \quad \text{or,} \quad \frac{x}{a}=\frac{10}{8}=\frac{5}{4} \qquad x=\frac{5a}{4}$$

3 (i) $14x^2-37x+5=14x^2-2x-35x+5$

$=2x(7x-1)-5(7x-1)=(7x-1)(2x-5)$

(ii) $(1+a)^2(1+c^2)-(1+c)^2(1+a^2)$

$=(1+a^2+2a)(1+c^2)-(1+c^2+2c)(1+a^2)$

$=(1+a^2)(1+c^2)+2a(1+c^2)-(1+c^2)(1+a^2)-2c(1+a^2)$

$=2(a+ac^2-c-a^2c)=2\{(a-c)-ac(a-c)\}$

$=2(a-c)(1-ac)$

(iii) $m^4-n^4+2n(m^3+n^3)-(m+n)^2(m-n)^2$

$=m^4-n^4-(m^2-n^2)^2+2n(m^3+n^3)$

$=(m^2+n^2)(m^2+n^2-m^2+n^2)+2n(m^3+n^3)$

$=2n^2(m^2+n^2)+2n(m^3+n^3)$

$=2n(m+n)\{n(m-n)+m^2-mn+n^2\}$

$=2n(m+n)(nm-n^2+m^2-mn+n^2)$

$=2m^2n(m+n)$

4 Out of every 4 lbs sold he cleared $\frac{1}{4}$ lb of the whole quantity sold he cleared $\frac{1}{16}$ Now if x be the required number of loaves he cleared $\frac{19x}{2\times 12}+\frac{1}{16}$ shillings

we have $\frac{19x}{2\times 12\times 16}=100-5=95$

$x=5\times 2\times 12\times 16=1920$

5 $\frac{a^3}{(a-b)(a-c)}+\frac{b^3}{(b-c)(b-a)}+\frac{c^3}{(c-a)(c-b)}$

$=\frac{-\{a^3(b-c)+b^3(c-a)+c^3(a-b)\}}{(a-b)(b-c)(c-a)}$

$=\frac{-\{-(a-b)(b-c)(c-a)(a+b+c)\}}{(a-b)(b-c)(c-a)}$ [See page 125 Ex 9]

$=a+b+c$

6 $\frac{a-b}{ay+bx}=\frac{b-c}{bz+cx}=\frac{c-a}{cy+az}=\frac{a+b+c}{ax+by+cz}$

each of the ratios

$=\frac{(a-b)+(b-c)+(c-a)+(a+b+c)}{ay+bx+bz+cx+cy+az+ax+by+cz}$

$=\frac{a+b+c}{x(a+b+c)+y(a+b+c)+z(a+b+c)}=\frac{1}{x+y+z}$

7 $\quad x(x+y+z)=24 \qquad (1)$

$y(x+y+z)=48 \qquad (2)$

$z(x+y+z)=72 \qquad (3)$

By adding the equations, we have $(x+y+z)^2=144$

$x+y+z=12 \qquad (4)$

from (1) and (4), $x=2$,

from (2) and (4), $y=4$,

from (3) and (4), $z=6$

8 $\quad a+c=\dfrac{b}{x}-dx \qquad (1)$

$a-c=\dfrac{d}{x}-bx \qquad (2)$

From (1), $\quad dx^2+(a+c)x-b=0$

From (2), $\quad bx^2+(a-c)x-d=0$

we have

$$\frac{x^2}{-d(a+c)+b(a-c)}=\frac{x}{-b^2+d^2}=\frac{1}{d(a-c)-b(a+c)}$$

$$\frac{x^2}{\{b(a-c)-d(a+c)\}\{d(a-c)-b(a+c)\}}=\frac{x^2}{(b^2-d^2)^2}$$

$$db(a-c)^2-d^2(a^2-c^2)-b^2(a^2-c^2)+bd(a+c)^2=(b^2-d^2)^2$$

or, $\quad bd\{(a-c)^2+(a+c)^2-(a^2-c^2)(b^2+d^2)=(b^2-d^2)^2$

or, $\quad 2bd(a^2+c^2)-(a^2-c^2)(b^2+d^2)=(b^2-d^2)^2$

or, $\quad c^2(b^2+d^2+2bd)-a^2(b^2+d^2-2bd)=(b^2-d^2)^2$

or, $\quad c^2(b+d)^2-a^2(b-d)^2=(b^2-d^2)^2$

or, $\quad \dfrac{c^2}{(b-d)^2}-\dfrac{a^2}{(b+d)^2}=1$

XIX.

1 $\quad (x^2-2ax+3a^2)^{\frac{1}{2}}+(x^2-4ax+5a^2)^{\frac{1}{2}}$

$$=(x^2-5ax+7a^2)^{\frac{1}{2}}+(x^2-7ax+9a^2)^{\frac{1}{2}}$$

$$(x^2-2ax+3a^2)^{\frac{1}{2}}-(x^2-7ax+9a^2)^{\frac{1}{2}}$$

$$=(x^2-5ax+7a^2)^{\frac{1}{2}}-(x^2-4ax+5a^2)^{\frac{1}{2}}$$

or, $\quad x^2-2ax+3a^2+x^2-7ax+9a^2-2\{x^2-2ax+3a^2\}^{\frac{1}{2}}$

$$\{x^2-7ax+9a^2\}^{\frac{1}{2}}$$

5 $$x^3+y^3+z^3-3xyz$$
$$=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$=2(a+b+c)(x^2+y^2+z^2-xy-yz-zx)$$
$$=(a+b+c)\{(x^2+y^2-2xy)+(x^2+z^2-2zx)$$
$$+(y^2+z^2-2yz)\}$$
$$=(a+b+c)\{(x-y)^2+(z-x)^2+(y-z)^2\}$$
$$=(a+b+c)\{(b-a)^2+(a-c)^2+(c-b)^2\}$$
$$=2(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$
$$=2(a^3+b^3+c^3-3abc)$$

6 $$\frac{1}{b^3(a-c)}+\frac{1}{a^3(b-c)}=\frac{1}{ab(a-c)(b-c)}$$

or, $a^3(b-c)+b^3(a-c)=a^2b^2$

or, $a^3(b-c)-a^2bc-a^2b^2+a^2bc+ab^2c+ab^3-ab^2c-b^3c=0$

or, $a^2(ab-ac-bc)-ab(ab-ac-bc)+b^2(ab-ac-bc)=0$

or, $(ab-ac-bc)(a^2-ab+b^2)=0$

Hence either $\quad ab-ac-bc=0 \quad (1)$

or, $\quad a^2-ab+b^2=0 \quad (2)$

From (1), $$\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$$

and from (2) $$a^2+b^2=ab$$

7 The given expression

$$=\sqrt{\frac{(2\sqrt{3}-2\sqrt{2})(\sqrt{3}+\sqrt{2})}{5+2\sqrt{3}\sqrt{2}}}=\sqrt{\frac{2(3-2)}{(\sqrt{3}+\sqrt{2})^2}}$$

$$=\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{2}(\sqrt{3}-\sqrt{2})}{3-2}=\sqrt{6}-2$$

8 From the 1st and the 2nd equations, we have

$$\frac{x}{(c+a)(c+b)-(a+b)^2}=\frac{y}{(a+b)(a+c)-(b+c)^2}$$
$$=\frac{1}{(a+b)(b+c)-(c+a)^2}$$

$$x=\frac{(c+a)(c+b)-(a+b)^2}{(a+b)(b+c)-(c+a)^2}$$

and $$y=\frac{(a+b)(a+c)-(b+c)^2}{(a+b)(b+c)-(c+a)^2}$$

from the 3rd equation, we have

$$\frac{(a+b)(b+c)(c+a)-(a+b)^3}{(a+b)(b+c)-(c+a)^2}+\frac{(a+b)(b+c)(c+a)-(b+c)^3}{(a+b)(b+c)-(c+a)^2}$$
$$+(c+a)=0$$

we have $3(a+b)(b+c)(c+a)-(a+b)^3-(b+c)^3-(c+a)^3=0$

or, $\{(a+b)+(b+c)+(c+a)\}\{(a+b)^2+(b+c)^2+(c+a)^2$
$\quad -(a+b)(b+c)-(b+c)(c+a)-(c+a)(a+b)\}=0$

or, $2(a+b+c)\frac{1}{2}[\{(a+b)-(b+c)\}^2+\{(b+c)-(c+a)\}^2$
$\quad +\{(c+a)-(a+b)\}^2]=0$

or, $(a+b+c)\{(a-c)^2+(b-a)^2+(c-b)^2\}=0$

or, $(a+b+c)2(a^2+b^2+c^2-ab-bc-ca)=0$

Thus $a^3+b^3+c^3-3abc=0$ is the required Eliminant

XX

1 $\quad a(b+c)^2+b(c+a)^2+c(a+b)^2-4abc$

$=a(b+c)^2+b(c+a)^2-2abc+c(a+b)^2-2abc$

$=a(b+c)^2+b(c^2+a^2)+c(a^2+b^2)=a(b+c)^2+a^2(b+c)+bc(b+c)$

$=(b+c)\{a^2+a(b+c)+bc\}=(b+c)(a+b)(a+c)$

2
$$\begin{array}{l}
x+a\,)\,a^2x^3-b^3x^2+ac^3x+3a^3bc\,(\,a^2x^2-(a^3+b^3)x+a(a^3+b^3+c^3)\\
\qquad\;\; \underline{a^2x^3+a^3x^2}\\
\qquad\;\; -x^2(a^3+b^3)+ac^3x+3a^3bc\\
\qquad\;\; \underline{-x^2(a^3+b^3)-a(a^3+b^3)x}\\
\qquad\;\; xa(a^3+b^3+c^3)+3a^3bc\\
\qquad\;\; \underline{xa(a^3+b^3+c^3)+a^2(a^3+b^3+c^3)}\\
\qquad\;\; -a^2\{a^3+b^3+c^3-3abc\}
\end{array}$$

Hence, we have $\quad a^2(a^3+b^3+c^3-3abc)=0$

But since a is not zero, $\quad a^3+b^3+c^3-3abc=0$

or, $\quad a^3+b^3+c^3=3abc$

3 The given expression

$$=\frac{-\{b^2c^2(b^2-c^2)+a^2c^2(c^2-a^2)+a^2b^2(a^2-b^2)\}}{abc(a^2-b^2)(b^2-c^2)(c^2-a^2)}$$

$$=\frac{(a^2-b^2)(b^2-c^2)(c^2-a^2)}{abc(a^2-b^2)(b^2-c^2)(c^2-a^2)}=\frac{1}{abc}$$

4 $$a^4(b-c)+b^4(c-a)+c^4(a-b)$$
$$=a^4(b-c)-a(b^4-c^4)+bc(b^3-c^3)$$
$$=(b-c)\{a^4-a(b^3+b^2c+bc^2+c^3)+bc(b^2+bc+c^2)\}$$
$$=(b-c)\{b^3(c-a)+b^2c(c-a)+bc^2(c-a)-a(c^3-a^3)\}$$
$$=(b-c)(c-a)\{b^3+b^2c+bc^2-a(c^2+ac+a^2)\}$$
$$=(b-c)(c-a)\{-c^2(a-b)-c(a^2-b^2)-(a^3-b^3)\}$$
$$=(b-c)(c-a)(a-b)\{-c^2-c(a+b)-(a^2+ab+b^2)\}$$
$$=(a-b)(b-c)(c-a)(-a^2-b^2-c^2-ab-bc-ca)$$

the reqd quotient $=-(a^2+b^2+c^2+ab+bc+ca)$

5 $a+b+c=0$, $\quad a+b=-c$

$$(a+b)^5=(-c)^5=-c^5$$

or, $$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5=-c^5,$$
$$a^5+b^5+c^5=-5ab(a^3+2a^2b+2ab^2+b^3)$$
$$=-5ab\{(a^3+b^3)+2ab(a+b)\}$$
$$=-5ab(a+b)(a^2+b^2-ab+2ab)$$
$$=-5ab(a+b)\{(a+b)^2-ab\}$$
$$=5abc(c^2-ab)$$

6 $$\frac{a}{x+a-c}+\frac{b}{x+b-c}=2$$

or, $$1-\frac{a}{x+a-c}+1-\frac{b}{x+b-c}=0$$

or, $$\frac{x-c}{x+a-c}+\frac{x-c}{x+b-c}=0$$

or, $$(x-c)\left\{\frac{1}{x+a-c}+\frac{1}{x+b-c}\right\}=0$$

either $x-c=0$, whence $x=c$,

$$\frac{1}{x+a-c}+\frac{1}{x+b-c}=0$$
$$\frac{1}{x+a-c}=-\frac{1}{x+b-c}$$
$$-x-b+c=x+a-c$$

$2x=2c-b-a$, whence $x=\dfrac{2c-b-a}{2}$

$$=c-\frac{a+b}{2}$$

7 $$ax+by+cz=a+b+c \qquad (1)$$

$$\frac{ax}{b+c}+\frac{by}{a+c}=1. \qquad (2)$$

$$\frac{2x}{b+c}+\frac{2y}{a+c}=\frac{1}{a}+\frac{1}{b} \qquad (3)$$

From (2), $\frac{2ax}{b+c}+\frac{2by}{a+c}=2$

From (3), $\frac{2bx}{b+c}+\frac{2by}{a+c}=\frac{b}{a}+1$

$$\frac{2x(a-b)}{b+c}=1-\frac{b}{a}=\frac{a-b}{a}, \qquad x=\frac{b+c}{2a}$$

from (2) $\frac{1}{2}+\frac{by}{a+c}=1$

or, $$\frac{by}{a+c}=\frac{1}{2}$$

or, $$2by=a+c$$

$$y=\frac{a+c}{2b}$$

Lastly, from (1), $\frac{b+c}{2}+\frac{a+c}{2}+cz=a+b+c$

or, $cz=a+b+c-\frac{a+b}{2}-c$

$$=\frac{a+b}{2}, \qquad z=\frac{a+b}{2c}$$

8 Let $x+y$ miles per hour be their respective rates of walking, and z miles be the required distance

Then we have,

$$\frac{z-1\frac{1}{2}}{x}=\frac{1\frac{1}{2}}{y} \qquad (1)$$

and $$\frac{(z-1\frac{1}{2})+1}{y}=\frac{1\frac{1}{2}+(z-1)}{x}=1 \qquad (2)$$

From (2), we have

$$\frac{(z-1\frac{1}{2})+1+1\frac{1}{2}+(z-1)}{x+y}=1$$

or, $$\frac{2z}{x+y}=1$$

or, $$\frac{z}{x+y}=\frac{1}{2} \qquad (3)$$

Also from (1), $$\frac{z}{x+y}=\frac{3}{2y}$$

$$\therefore \frac{3}{2y}=\frac{1}{2}, \qquad y=3$$

$\therefore$ from (2) $$\frac{z-\frac{1}{2}}{3}=1$$

or, $$z=3\tfrac{1}{2},$$

and from (3), $$x=4$$

$\therefore$ the reqd rates are 4 and 3 miles per hour respectively, and the reqd distance $=3\frac{1}{2}$ miles

XXI

1 $$\{(b+c)^2+(c+a)^2+(a+b)^2\}\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}$$
$$=2(a^2+b^2+c^2+ab+bc+ca)\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}$$
$$=2\{a^2+a(b+c)+(b^2+bc+c^2)\}\{a^2(b-c)-a(b^2-c^2)+bc(b-c)\}$$
$$=2\{a^4(b-c)+a^3(b^2-c^2)+a^2(b^3-c^3)-a^3(b^2-c^2)-a^2(b^3+b^2c-bc^2-c^3)$$
$$-a(b^4-c^4)+a^2bc(b-c)+abc(b^2-c^2)+bc(b^3-c^3)\}$$
$$=2\{a^4(b-c)+b^4(c-a)+c^4(a-b)\}$$

2 The left-hand expression
$$=\frac{(b-c)^3+(c-a)^3+(a-b)^3}{(a-b)(b-c)(c-a)}$$
$$=\frac{b^3-c^3-3bc(b-c)+c^3-a^3-3ac(c-a)+a^3-b^3-3ab(a-b)}{(a-b)(b-c)(c-a)}$$
$$=\frac{-3\{bc(b-c)+ac(c-a)+ab(a-b)\}}{-\{bc(b-c)+ac(c-a)+ab(a-b)\}}=3$$

3 $$a+b+c=0$$
$$\therefore a+b=-c,\ b+c=-a \text{ and } c+a=-b$$
$$a^2+ab+b^2=b^2+a(a+b)$$
$$=b^2+(-b-c)(-c)=b^2+bc+c^2$$
$$=c^2+b(b+c)$$
$$=c^2-(c+a)(-a)$$
$$=c^2+ac+a^2,$$
$$\therefore a^2+ab+b^2=b^2+bc+c^2=c^2+ac+a^2$$

4. The left-hand expression

$$\begin{aligned}
&= s^2-6as+9a^2+s^2-6bs+9b^2+s^2-6cs+9c^2\\
&= 3\{s^2-2s(a+b+c)+3(a^2+b^2+c^2)\}\\
&= 3\{s^2-2s^2+3(a^2+b^2+c^2)\}\\
&= 3\{3(a^2+b^2+c^2)-(a+b+c)^2\}\\
&= 3(2a^2+2b^2+2c^2-2ab-2bc-2ca)\\
&= 3\{(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)\}\\
&= 3\{(a-b)^2+(b-c)^2+(c-a)^2\}
\end{aligned}$$

5. $a^2+2ab-2ac-3b^2+2bc$

$$\begin{aligned}
&=(a^2-b^2)+(2ab-2b^2)-(2ac-2bc)\\
&=(a+b)(a-b)+2b(a-b)-2c(a-b)\\
&=(a-b)(a+b+2b-2c)=(a-b)(a+3b-2c)
\end{aligned}$$

6.

$$\begin{array}{r|l|l}
x^4-2x^3+5x^2-4x+3 & 2x^4-x^3+6x^2+2x+3 & 1\\
 & x^4-2x^3+5x^2-4x+3 & \\ \hline
 & x \,|\, x^4+x^3+x^2+6x & \\
 & x^3+x^2+x+6 &
\end{array}$$

$$\begin{array}{r|l|l}
x^3+x^2+x+6 & x^4-2x^3+5x^2-4x+3 & x-3\\
 & x^4+x^3+x^2+6x & \\ \hline
 & -3x^3+4x^2-10x+3 & \\
 & -3x^3-3x^2-3x-18 & \\ \hline
 & 7 \,|\, 7x^2-7x+21 & \\
 & x^2-x+3 &
\end{array}$$

$$\begin{array}{r|l|l}
x^2-x+3 & x^3+x^2+x+6 & x+2\\
 & x^3-x^2+3x & \\ \hline
 & 2x^2-2x+6 & \\
 & 2x^2-2x+6 & \\ \hline
\end{array}$$

The required H. C. F. $=x^2-x+3$

7.

$$\begin{array}{r|l|l}
x+f & ax^3+bx+c & ax^2-afx+(af^2+b)\\
 & ax^3+afx^2 & \\ \hline
 & -afx^2+bx+c & \\
 & -afx^2-af^2x & \\ \hline
 & (af^2+b)x+c & \\
 & (af^2+b)x+f(af^2+b) & \\ \hline
 & -f(af^2+b)+c &
\end{array}$$

we must have $-f(af^2+b)+c=0$

or, $af^3+bf-c=0$

Similarly by dividing $a'x^3+b'x+c'$, we get $a'f^3+b'f-c'=0$

Hence, by cross multiplication, we have

$$\frac{f^2}{-bc'+cb'}=\frac{f}{-ca'+ac}=\frac{1}{ab'-ba'},$$

we get $\dfrac{f^2}{cb'-bc'}=\left(\dfrac{1}{ab'-ba'}\right)^2=\dfrac{f^2}{(ac'-ca')^2},$

$(ac'-ca')^2=(ab'-ba')^2(cb'-bc')$ is the required condition

8 $\dfrac{a}{b}=\dfrac{b}{c}=\dfrac{c}{d}=k$ (suppose) $\therefore$ $a=bk$, $b=ck$ and $c=dk$

$$a=ck^2=dk^3 \text{ and } b=dk^2$$

$$\frac{a}{d}=\frac{dk^3}{d}=k^3$$

Also, $\dfrac{\sqrt{a^5+b^2c^3+a^3c^2}}{\sqrt{b^4c+d^4+b^2cd^2}}=\sqrt{\dfrac{d^5k^{15}+d^2k^4\,d^3k^3+d^3k^9\,d^2k^2}{d^4k^8\,dk+d^4+d^2k^4\,dk\,d^2}}$

$$=\sqrt{\frac{d^5k^{15}+d^4k^6+d^5k^{11}}{d^5k^9+d^4+d^5k^5}}=\sqrt{\frac{k^6(d^5k^9+d^4+d^5k^5)}{d^5k^9+d^4+d^5k^5}}$$

$$=\sqrt{k^6}=k^3$$

Therefore $a:d=\sqrt{a^5+b^2c^3+a^3c^2}:\sqrt{b^4c+d^4+b^2cd^2}$

XXII

1 The left-hand expression

$$=a(b-c)\{1+a(b+c)+a^2bc\}+b(c-a)\{1+b(c+a)+b^2ca\}$$
$$+c(a-b)\{1+c(a+b)+c^2ab\}$$
$$=\{a(b-c)+b(c-a)+c(a-b)\}+\{a^2(b^2-c^2)+b^2(c^2-a^2)$$
$$+c^2(a^2-b^2)\}+abc\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}$$
$$=abc(a-b)(a-c)(b-c)$$

2 $a+b+c=0$ $\therefore$ $a+b=-c$ $\therefore$ $(a+b)^7=(-c)^7$

or, $a^7+7a^6b+21a^5b^2+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+b^7=-c^7$

or, $a^7+b^7+c^7=-7ab(a^5+3a^4b+5a^3b^2+5a^2b^3+3ab^4+b^5)$

$$=-7ab\{(a^5+b^5)+3ab(a^3+b^3)+5a^2b^2(a+b)\}$$
$$=-7ab(a+b)\{a^4-a^3b+a^2b^2-ab^3+b^4+3ab(a^2-ab+b^2)$$
$$+5a^2b^2\}$$
$$=7abc(a^4+2a^3b+3a^2b^2+2ab^3+b^4)$$
$$=7abc\{(a^4+2a^2b^2+b^4)+2ab(a^2+b^2)+a^2b^2\}$$

$$=7abc\{(a^2+b^2)^2+2ab(a^2+b^2)+a^2b^2\}$$
$$=7abc(a^2+b^2+ab)^2$$
$$=7abc\{(a^2+b^2+2ab)-ab\}^2=7abc\{(a+b)^2-ab\}$$
$$=7abc(c^2-b)^2$$

3
$$\begin{array}{r|l|l} x+f & ax^2+bx+c & ax-(af-b) \\ & \underline{ax^2+afx} & \\ & -x(af-b)+c & \\ & \underline{-x(af-b)-f(af-b)} & \\ & f(af-b)+c & \end{array}$$

we must have $\quad f(af-b)+c=0$

or, $\quad af^2-bf+c=0 \qquad (1)$

Similarly by dividing $a'x^2+b'x+c'$

we get $\quad a'f^2-b'f+c'=0 \qquad (2)$

From (1) and (2), by cross multiplication, we get

$$\frac{f^2}{-bc'+b'c}=\frac{f}{ca'-ac'}=\frac{1}{-ab'+ba'}$$

or, $$\frac{f^2}{b'c-bc'}\cdot\frac{1}{a'b-ab'}=\frac{f^2}{(ac'-ca')^2}$$

or, $$(ac'-ca')^2=(a'b-ab')(b'c-bc')$$

4 Let x yds be the length of the course Then A ran x yds while B ran $(x-1)-50$ yds

$$\frac{x}{20}=\frac{(x-1)-50}{19}$$

or, $$19x=20x-1020$$
$$x=1020$$

Thus the length of the course is 1020 yds

5
$$\frac{p+q+r}{p-q}-\frac{q(4p+3r)-r(p+r)}{p^2-q^2}$$
$$=\frac{(p+q)(p+q+r)-q(4p+3r)+r(p+r)}{p^2-q^2}$$
$$=\frac{(p+q)^2-4pq+r(p+q+p+r)-3qr}{p^2-q^2}$$
$$=\frac{(p+q)^2-4pq+2r(p-q)+r^2}{p^2-q^2}$$
$$=\frac{(p-q)^2+2r(p-q)+r^2}{p^2-q^2}=\frac{(p-q+r)^2}{p^2-q^2}$$

6 Putting x for a^2-b^2, y for b^2-c^2 and z for c^2-a^2, we have

the numerator $=x^3+y^3+z^3=3xyz$ $[\because x+y+z=0]$

$=3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

$=3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

Similarly putting x, y and z for $a-b$, $b-c$, and $c-a$ respectively we have

the denominator $=3(a-b)(b-c)(c-a)$

$\therefore$ the left-hand expression $=(a+b)(b+c)(c+a)$

7

$$x+y+z=2a+2b+2c \qquad (1)$$

$$ax+by+cz=2bc+2ca+2ab \qquad (2)$$

$$(b-c)x+(c-a)y+(a-b)z=0 \qquad (3)$$

From (1), $(x-2b)+(y-2c)+(z-2a)=0$

From (2), $a(x-2b)+b(y-2c)+c(z-2a)=0$

Hence, we have,

$$\frac{x-2b}{c-b}=\frac{y-2c}{a-c}=\frac{z-2a}{b-a}=\lambda \text{ (suppose)}$$

$\therefore$ $x=(c-b)\lambda+2b$, $y=\lambda(a-c)+2c$ and $z=\lambda(b-a)+2a$

$\therefore$ from (3), we get

$-\lambda(b-c)^2+2b(b-c)-\lambda(c-a)^2+2c(c-a)$

$-\lambda(a-b)^2+2a(a-b)=0$

or, $\lambda\{(a-b)^2+(b-c)^2+(c-a)^2\}=2(a^2+b^2+c^2-ab-cb-ca)$

$=2\{(a-b)^2+(b-c)^2+(c-a)^2\}$

$\therefore$ $\lambda=1$

$\therefore$ $x=c-b+2b=b+c$

$y=a-c+2c=a+c$

and $z=b-a+2a=a+b$

8

$$ax+cy+bz=0 \qquad (1)$$

$$cx+by+az=0 \qquad (2)$$

$$bx+ay+cz=0 \qquad (3)$$

From (1) and (2), we get

$$\frac{x}{ac-b^2}=\frac{y}{bc-a^2}=\frac{z}{ab-c^2}=\lambda \text{ (suppose)}$$

$\therefore$ $x=\lambda(ac-b^2)$, $y=\lambda(bc-a^2)$ and $z=\lambda(ab-c^2)$

$\therefore$ from (3), we get $\lambda\{b(ac-b^2)+a(bc-a^2)+c(ab-c^2)\}=0$

or, $a^3+b^3+c^3-3abc=0$

XXIII

1 The left-hand expression

$$=(b-c)\{1+a^3(b+c)+a^4bc\}+(c-a)\{1+b^3(c+a)+ab^4c\}$$
$$+(a-b)\{1+c^3(a+b)+abc^4\}$$
$$=\{(a-b)+(b-c)+(c-a)\}+\{a^3(b^2-c^2)+b^3(c^2-a^2)$$
$$+c^3(a^2-b^2)\}+abc\{a^3(b-c)+b^3(c-a)+c^3(a-b)\}$$
$$=0+0+abc\{a^3(b-c)+b^3(c-a)+c^3(a-b)\}$$
$$=abc(a+b+c)(a-b)(a-c)(b-c)$$

2 $21x^2-13x+2=21x^2-7x-6x+2=7x(3x-1)-2(3x-1)$
$$=(3x-1)(7x-2)$$

$$28x^2-15x+2=28x^2-8x-7x+2$$
$$=4x(7x-2)-(7x-2)=(7x-2)(4x-1)$$
$$12x^2-7x+1=12x^2-4x-3x+1$$
$$=4x(3x-1)-(3x-1)=(3x-1)(4x-1)$$

∴ the reqd. L. C. M. $=(3x-1)(7x-2)(4x-1)$

3 $(x+y)^7-x^7-y^7$

$$=7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6$$
$$=7xy(x^5+3x^4y+5x^3y^2+5x^2y^3+3xy^4+y^5)$$
$$=7xy\{(x^5+y^5)+3xy(x^3+y^3)+5x^2y^2(x+y)\}$$
$$=7xy(x+y)\{x^4-x^3y+x^2y^2-xy^3+y^4+3xy(x^2-xy+y^2)+5x^2y^2\}$$
$$=7xy(x+y)\{(x^4+2x^2y^2+y^4)+2xy(x^2+y^2)+x^2y^2\}$$
$$=7xy(x+y)\{(x^2+y^2)^2+2xy(x^2+y^2)+x^2y^2\}$$
$$=7xy(x+y)(x^2+y^2+xy)^2$$

∴ $(x+y)^7-x^7-y^7$ is divisible by $(x^2+xy+y^2)^2$

4 The left-hand expression

$$=3t^4-2t^2(a^2+b^2+c^2)+(a^2b^2+b^2c^2+c^2a^2)$$
$$=3t^4-4t^4+a^2b^2+b^2c^2+c^2a^2=(a^2b^2+b^2c^2+c^2a^2)-t^4$$
$$=(a^2b^2+b^2c^2+c^2a^2)-\tfrac{1}{4}(a^2+b^2+c^2)^2$$
$$=(a^2b^2+b^2c^2+c^2a^2)-\tfrac{1}{4}(a^4+b^4+c^4+2a^2b^2+2b^2c^2+2a^2c^2)$$
$$=\tfrac{1}{4}(2a^2b^2+2b^2c^2+2c^2a^2-a^4-b^4-c^4)$$
$$=\tfrac{1}{4}\{2b^2(a^2+c^2)-(a^2-c^2)^2-b^4\}$$
$$=\tfrac{1}{4}\{b^2\{(a+c)^2+(a-c)^2\}-(a^2-c^2)^2-b^4\}$$

$$=\tfrac{1}{4}\{b^2(a+c)^2-b^4-(a^2-c^2)^2+(a-c)^2b^2\}$$
$$=\tfrac{1}{4}[b^2\{(a+c)^2-b^2\}-(a-c)^2\{(a+c)^2-b^2\}]$$
$$=\tfrac{1}{4}[\{(a+c)^2-b^2\}\{b^2-(a-c)^2\}]$$
$$=\tfrac{1}{4}(a+c+b)(a+c-b)(b+a-c)(b-a+c)$$
$$=\tfrac{1}{4}(2s)(a+b+c-2b)(a+b+c-2c)(a+b+c-2a)$$
$$=\tfrac{1}{4}(2s)(2s-2b)(2s-2c)(2s-2a)$$
$$=4s(s-a)(s-b)(s-c)$$

5 We have

$$(1+xx'+yy')^2=(1+x^2+y^2)(1+x'^2+y'^2)$$

or, $1+x^2x'^2+y^2y'^2+2xx'+2yy'+2xx'yy'$

$$=1+x^2x'^2+y^2y'^2+x^2+y^2+x'^2+y'^2+x^2y'^2+x'^2y^2$$

or, $x^2-2xx'+x'^2+y^2-2yy'+y'^2+x^2y'^2+x'^2y^2-2xx'yy'=0$

or, $(x-x')^2+(y-y')^2+(xy'-x'y)^2=0$

In order that this condition be fulfilled it is necessary that each of these three terms be individually zero

$x-x'=0$ whence $x=x'$

$y-y'=0$ whence $y=y'$

6 The numerator

$$=a^4b-a^3b^2+a^2b^3-ab^4+b^4c-b^3c^2+b^2c^3-bc^4+c^4a-c^3a^2+c^2a^3-ca^4$$
$$=a^4(b-c)-a^3(b^2-c^2)+a^2(b^3-c^3)-a(b^4-c^4)+b^3c(b-c)+bc^3(b-c)$$
$$=(b-c)\{a^4-a^3(b+c)+a^2(b^2+bc+c^2)-a(b^3+b^2c+bc^2+c^3)+b^3c+bc^3\}$$
$$=(b-c)\{b^3(c-a)-b^2a(c-a)+b(c^3-a^3)-abc(c-a)+a^2c(c-a)-a(c^3-a^3)\}$$
$$=(b-c)(c-a)(b^3-b^2a+bc^2+abc+a^2b-abc+a^2c-ac^2-a^2c-a^3)$$
$$=(b-c)(c-a)\{c^2(b-a)+b^2(b-a)+a^2(b-a)\}$$
$$=(b-c)(c-a)(b-a)(c^2+b^2+a^2)$$

The denominator

$$=a^3b^2-a^2b^3+b^3c^2-b^2c^3+a^2c^3-a^3c^2$$
$$=a^3(b^2-c^2)-a^2(b^3-c^3)+b^2c^2(b-c)$$

$$=(b-c)\{a^3(b-c)-a^2(b^2+bc-c^2)+b^2c^2\}$$
$$=(b-c)\{b^2(c^2-a^2)-a^2b(c-a)-a^2c(c-a)\}$$
$$=(b-c)(c-a)(b^2c+b^2a-a^2b-a^2c)$$
$$=(b-c)(c-a)\{ab(b-a)+c(b^2-a^2)\}$$
$$=(b-c)(c-a)(b-a)(ab+bc+ca)$$

the required result

$$=\frac{a^2-b^2+c^2}{ab-bc+ca}$$

7 $a+b+c=0$, $\quad a+b=-c \quad (a+b)^5=-c^5$

or $a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5=-c^5$

or, $a^5+b^5+c^5=-5ab(a^3+2a^2b+2ab^2+b^3)$

$$\frac{a^5+b^5+c^5}{5}=-ab\{a^3+b^3+2ab(a+b)\}$$
$$=-ab(a+b)(a^2-ab+b^2+2ab)$$
$$=-ab(a+b)(a^2+ab+b^2) \qquad (A)$$

Again $a+b=-c \quad (a+b)^3=-c^3$

or $a^3+b^3+3ab(a+b)=-c^3$

or, $a^3+b^3+c^3=-3ab(a+b)$

$$\frac{a^3+b^3+c^3}{3}=-ab(a+b) \qquad (B)$$

Lastly $(a+b)^2=c^2$

or $a^2+2ab+b^2=c^2$

$$a^2+b^2+c^2=2c^2-2ab$$
$$\frac{a^2+b^2+c^2}{2}=c^2-ab$$
$$=(a+b)^2-ab$$
$$=a^2+ab+b^2 \qquad (C)$$

Thus from (A), (B) and (C), we have

$$\frac{a^5+b^5+c^5}{5}=\frac{a^3+b^3+c^3}{3}\cdot\frac{a^2+b^2+c^2}{2}$$

8 From the 2nd equation we get $q^2x^2+p^2y^2-\frac{p^2q^2}{a^2+b^2}=0$

From the 3rd equation we get $x^2+y^2-1=0$

by cross multiplication, we get

$$\frac{x^2}{-p^2+\frac{p^2q^2}{a^2+b^2}}=\frac{y^2}{-\frac{p^2q^2}{a^2+b^2}+q^2}=\frac{1}{q^2-p^2}$$

$$x^2 = \frac{p^2(q^2-a^2-b^2)}{(a^2+b^2)(q^2-p^2)} \qquad x = \frac{p\sqrt{q^2-(a^2+b^2)}}{\sqrt{(a^2+b^2)(q^2-p^2)}}$$

and $$y^2 = \frac{q^2(a^2+b^2-p^2)}{(a^2+b^2)(q^2-p^2)} \qquad y = \frac{q\sqrt{(a^2+b^2)-p^2}}{\sqrt{(a^2+b^2)(q^2-p^2)}}$$

from the 3rd equation, we get

$$ap\sqrt{q^2-(a^2+b^2)}+bq\sqrt{(a^2+b^2)-p^2}=(a^2+b^2)\sqrt{q^2-p^2}$$

or, $$a^2p^2q^2-a^2p^2(a^2+b^2)+b^2q^2(a^2+b^2)-b^2p^2q^2$$
$$+2apbq\sqrt{(a^2+b^2)(p^2+q^2)-p^2q^2-(a^2+b^2)^2}$$
$$=(a^2+b^2)^2(q^2-p^2)$$

or, $$p^2q^2(a^2-b^2)-(a^2+b^2)(a^2p^2-b^2q^2+a^2q^2-a^2p^2+b^2q^2-b^2p^2)$$
$$=-2apbq\sqrt{(a^2+b^2)(p^2+q^2)-p^2q^2-(a^2+b^2)^2}$$

or, $$p^4q^4(a^2-b^2)^2+(a^2+b^2)^2-(a^2q^2-b^2p^2)^2$$
$$-2p^2q^2(a^4-b^4)(a^2q^2-b^2p^2)$$
$$=4a^2b^2p^2q^2\{(a^2+b^2)(p^2+q^2)-p^2q^2-(a^2+b^2)^2\}$$

or, $$(a^2+b^2)^2\{p^4q^4+(a^2q^2+b^2p^2)^2\}$$
$$=2p^2q^2(a^4b^2p^2+2a^4b^2q^2+2a^2b^4p^2+2a^2b^4q^2$$
$$+a^6q^2-a^2b^2p^4-a^2b^4q^2+b^6p^2)$$
$$=2p^2q^2\{p^2b^2(a^4+2a^2b^2+b^4)+a^2q^2(a^4+2a^2b^2+b^4)\}$$
$$=2p^2q^2(a^2+b^2)^2(p^2b^2+a^2q^2)$$

or, $$(a^2q^2+b^2p^2)^2+p^4q^4-2p^2q^2(a^2q^2+b^2p^2)=0$$

or, $$(a^2q^2+b^2p^2-p^2q^2)^2=0$$

or, $$a^2q^2+b^2p^2=p^2q^2$$

$\dfrac{a^2}{p^2}+\dfrac{b^2}{q^2}=1$ is the reqd Eliminant

XXIV.

1 From the 1st equation, we get $(x-a)+(y-b)+(z-c)=0$

From the 2nd, we get $b(x-a)+c(y-b)+a(z-c)=0$

by cross multiplication, we get

$$\frac{x-a}{a-c}=\frac{y-b}{b-a}=\frac{z-c}{c-b}=k \text{ (suppose)}$$

From the 3rd equation, we get

$$c(x-a)+a(y-b)+b(z-c)=0$$
$$l\{c(a-c)+a(b-a)+b(z-c)\}=0$$

or $k=0$ (Supposing the other factor to be not zero)

we get $x-a=0$, or, $x=a$

$y-b=0$, or, $y=b$

$z-c=0$, or, $z=c$

2 Let x, y and z be the three parts

we get $x+y+z=243$

Also, $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}=k$ (suppose)

$x=2k$, $y=3k$ and $z=4k$

we get $k(2+3+4)=243$

or, $k=27$

$x=54$, $y=81$ and $z=108$

3 $4(a^2+b^2+c^2+d^2)=(a+b+c+d)^2$

$=a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$

or, $(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+d^2-2cd)+(d^2+a^2-2ad)+(a^2+c^2-2ac)+(d^2+b^2-2bd)=0$

or, $(a-b)^2+(b-c)^2+(c-d)^2+(d-a)^2+(a-c)^2+(d-b)^2=0$

In order that this expression may be equal to zero, each of these squares must be individually zero,

$a=b=c=d$

4 The given expression

$=s^2\{a(b-c)+b(c-a)+(a-b)\}-2s\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}+\{a^3(b-c)+b^3(c-a)+c^3(a-b)\}$

$=0-(a+b+c)(a-b)(a-c)(b-c)+(a+b+c)(a-b)(a-c)(b-c)$

$=0$

5 We have

$$a-bz-cy=0 \quad (1)$$

$$az-b+cx=0 \quad (2)$$

and $$ay+bx-c=0 \quad (3)$$

from (1) and (2), we get

$$\frac{a}{-xz-y}=\frac{b}{-yz-x}=\frac{c}{-1+z^2} \quad (4)$$

From (2) and (3), we get

$$\frac{a}{1-x^2}=\frac{b}{xy+z}=\frac{c}{zx+y} \qquad (5)$$

From (1) and (3), we get

$$\frac{a}{z+xy}=\frac{b}{-y^2+1}=\frac{c}{x+yz} \qquad (6)$$

from (4) and (5), we get by multiplication,

$$\frac{a^2}{(1-x^2)(xz+y)}=\frac{c^2}{(1-z^2)(zx+y)}$$

$$\frac{a^2}{1-x^2}=\frac{c^2}{1-z^2} \qquad (7)$$

Similarly from (5) and (6), we get

$$\frac{a^2}{1-x^2}=\frac{b^2}{1-y^2} \qquad (8)$$

from (7) and (8)

$$\frac{a^2}{1-x^2}=\frac{b^2}{1-y^2}=\frac{c^2}{1-z^2}$$

6 $$ax+by=0 \qquad (1)$$

$$x+y+xy=0 \qquad (2)$$

$$x^2+y^2-1=0 \qquad (3)$$

From (1) and (2), we get

$$\frac{x}{b}=\frac{y}{-a}=\frac{xy}{a-b}$$

we get $x=\frac{a-b}{-a}$ and $y=\frac{a-b}{b}$

from (3), we get

$$\frac{(a-b)^2}{a^2}+\frac{(a-b)^2}{b^2}-1=0$$

or, $b^2(a-b)^2+a^2(a-b)^2-a^2b^2=0$

or, $$(a-b)^2(a^2+b^2)=a^2b^2$$

or, $$\frac{a^2+b^2}{a^2b^2}=\frac{1}{(a-b)^2}$$

or, $$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{(a-b)^2}$$

Which is the reqd Eliminant

7 Let the common factor be $x-m$. Dividing the 1st expression by $x-m$, we get

$$
\begin{array}{r|l}
x-m & ax^2-bx+c \;\big(\, ax+(am-b) \\
 & \underline{ax^2-amx} \\
 & (am-b)x+c \\
 & \underline{(am-b)x-am^2+bm} \\
 & am^2-bm+c
\end{array}
$$

We must have $am^2-bm+c=0$ (1)

Again dividing the 2nd expression by $x-m$, we get

$$
\begin{array}{r|l}
x-m & dx^3-bx+c \;\big(\, dx^2+dmx+(dm^2-b) \\
 & \underline{dx^3-dmx^2} \\
 & dmx^2-bx+c \\
 & \underline{dmx^2-dm^2x} \\
 & (dm^2-b)x+c \\
 & \underline{(dm^2-b)x-dm^3+bm} \\
 & dm^3-bm+c
\end{array}
$$

We must have $dm^3-bm+c=0$ (2)

From (1) and (2), $am^2=dm^3$

or, $a=dm$

or, $m=\dfrac{a}{d}$

from (2), $d\left(\dfrac{a}{d}\right)^3-b\dfrac{a}{d}+c=0$

or, $\dfrac{a^3}{d^2}-\dfrac{ab}{d}+c=0$

or, $a^3-abd+cd^2=0$

8 $(a+b+c)^3=a^3+b^3+c^3+3(a+b)(b+c)(c+a)$ (Ex 6 Page 91)

$(a+b)(b+c)(c+a)=0$

Hence at least one of these factors must be zero

Suppose $(a+b)=0$, then $a+b+c=c$

Again $(a+b)$ is a factor of $a^{2n+1}+b^{2n+1}$

(Page 109—Cor 2 of Art 7)

$a^{2n+1}+b^{2n+1}=0$

$a^{2n+1}+b^{2n+1}+c^{2n+1}=c^{2n+1}$

$=(a+b+c)^{2n+1}$

XXV.

1 We get

$$\frac{x^2-yz}{x+y+z}=\frac{a^4+b^2c^2-2a^2bc-(b^2c^2-c^3a-ab^3+a^2bc)}{a^2+b^2+c^2-ab-bc-ca}$$

$$=\frac{a(a^3+b^3+c^3-3abc)}{a^2+b^2+c^2-ab-bc-ca}$$

$$=a(a+b+c)$$

$$\frac{x^2-yz}{a}=(a+b+c)(x+y+z)$$

Similarly it can be proved that $\dfrac{y^2-zx}{b}=(a+b+c)(x+y+z)$

and $\dfrac{z^2-xy}{c}=(a+b+c)(x+y+z)$

we get $\dfrac{x^2-yz}{a}=\dfrac{y^2-zx}{b}=\dfrac{z^2-xy}{c}=(a+b+c)(x+y+z)$

2

$$4(bc+ad)^2-(b^2+c^2-a^2-d^2)^2$$
$$=\{2(bc+ad)+(b^2+c^2-a^2-d^2)\}\{2(bc+ad)-(b^2+c^2-a^2-d^2)\}$$
$$=\{(b^2+c^2+2bc)-(a^2+d^2-2ad)\}\{(a^2+d^2+2ad)-(b^2+c^2-2bc)\}$$
$$=\{(b+c)^2-(a-d)^2\}\{(a+d)^2-(b-c)^2\}$$
$$=(b+c+a-d)(b+c-a+d)(a+d+b-c)(a+d-b+c)$$
$$=(2s-2d)(2s-2a)(2s-2b)(2s-2c)$$
$$=16(s-a)(s-b)(s-c)(s-d)$$

3 Putting x for $b+c-a$, y for $c+a-b$ and z for $a+b-c$, we get $x+y+z=a+b+c$ and $x-y=2(b-a)$, $y-z=2(c-b)$ and $z-x=2(a-c)$

$$(b+c-a)^3+(c+a-b)^3+(a+b-c)^3$$
$$-3(b+c-a)(c+a-b)(a+b-c)=x^3+y^3+z^3-3xyz$$
$$=\tfrac{1}{2}(x+y+z)\{(x-y)^2+(y-z)^2+(z-x)^2\}$$
$$=\tfrac{1}{2}(a+b+c)\{4(b-a)^2+4(c-b)^2+4(a-c)^2\}$$
$$=4[\tfrac{1}{2}(a+b+c)\{(b-a)^2+(c-b)^2+(a-c)^2\}]$$
$$=4(a^3+b^3+c^3-3abc)$$

4 $\left(\frac{b-c}{a}+\frac{c-a}{b}+\frac{a-b}{c}\right)\left(\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}\right)$

$$=\left\{\frac{bc(b-c)+ac(c-a)+ab(a-b)}{abc}\right\}$$

$$\left\{\frac{a(c-a)(a-b)+b(b-c)(a-b)+c(b-c)(c-a)}{(a-b)(b-c)(c-a)}\right\}$$

$$=\left\{\frac{-(a-b)(b-c)(c-a)}{abc}\right\}\left[\frac{a\{-a^2+a(b+c)-bc\}+b\{-b^2+b(a+c)-ac\}+c\{-c^2+c(a+b)-ab\}}{(a-b)(b-c)(c-a)}\right]$$

$$=\frac{-1}{abc}\left[-\{a^3+b^3+c^3-a^2(b+c)-b^2(c-a)-c^2(a+b)+3abc\}\right]$$

$$=\frac{(a^3+b^3+c^3-3abc)-\{a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc\}+9abc}{abc}$$

$$=\frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)-(a+b+c)(ab+bc+ca)+9abc}{abc}$$

$$=\frac{0-0+9abc}{abc}=9$$

5 $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$ (suppose)

$x=ak$, $y=bk$ and $z=ck$

$$\frac{x^2+a^2}{x+a}+\frac{y^2+b^2}{y+b}+\frac{z^2+c^2}{z+c}=\frac{a^2(k^2+1)}{a(k+1)}+\frac{b^2(k^2+1)}{b(k+1)}+\frac{c^2(k^2+1)}{c(k+1)}$$

$$=\frac{k^2+1}{k+1}(a+b+c)$$

Also $\frac{(x+y+z)^2+(a+b+c)^2}{x+y+z+a+b+c}$

$$=\frac{k^2(a+b+c)^2+(a+b+c)^2}{k(a+b+c)+(a+b+c)}=\frac{(k^2+1)(a+b+c)^2}{(k+1)(a+b+c)}$$

$$=\frac{k^2+1}{k+1}(a+b+c)$$

$$\frac{x^2+a^2}{x+a}+\frac{y^2+b^2}{y+b}+\frac{z^2+c^2}{z+c}=\frac{(x+y+z)^2+(a+b+c)^2}{x+y+z+a+b+c}$$

6 $ax+by+cz=0$ (1)

$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=0$ (2)

From (1), we get

$$(x^2+y^2+z^2+xy+yz+zx)(ax+by+cz)=0 \qquad (3)$$

From (2), we get

$$ayz+bxz+cxy=0$$

or, $$(x+y+z)(ayz+bxz+cxy)=0 \qquad (4)$$

From (3) and (4), we get

$$ax^3+by^3+cz^3+a(x^2y+x^2z+xy^2+xz^2+2xyz+y^2z+yz^2)$$
$$+b(x^2y+x^2z+xy^2+xz^2+2xyz+y^2z+yz^2)$$
$$+c(x^2y+x^2z+xy^2+xz^2+2xyz+y^2z+yz^2)=0$$

or, $ax^3+by^3+cz^3+(a+b+c)\{x^2(y+z)+x(y^2+z^2+2yz)$
$+yz(y+z)\}=0$

or, $ax^3+by^3+cz^3+(a+b+c)(y+z)\{x^2+x(y+z)+yz\}=0$

or, $ax^3+by^3+cz^3+(a+b+c)(y+z)(z+x)(x+y)=0$

7 (i) $ax+hy+gz=0$ (1)

$hx+by+fz=0$ (2)

$gx+fy+cz=0$ (3)

From (1) and (2), by cross multiplication,

we get $\dfrac{x}{hf-gb}=\dfrac{y}{gh-af}=\dfrac{z}{ab-h^2}=k$ (suppose)

$x=k(hf-gb)$, $y=k(gh-af)$ and $z=k(ab-h^2)$

from (3), we get

$k\{g(hf-gb)+f(gh-af)+c(ab-h^2)\}=0$

or, $fgh-bg^2+fgh-af^2+abc-ch^2=0$

or, $abc+2fgh-af^2-bg^2-ch^2=0$ is the reqd Eliminant

(ii) $a(y+z)=x$ (1)

$b(z+x)=y$ (2)

$c(x+y)=z$ (3)

From (1), we get $x-ay-az=0$ (4)

(2), $bx-y+bz=0$ (5)

(3), $cx+cy-z=0$ (6)

From (4), and (5), we get

$\dfrac{x}{-ab-a}=\dfrac{y}{-ab-b}=\dfrac{z}{-1+ab}=k$ (suppose)

$x=-k(ab+a)$, $y=-k(ab+b)$ and $z=k(ab-1)$

from (3), we get $k\{c(ab+a)+c(ab+b)+ab-1\}=0$

or, $ac+bc+ab+2abc=1$ is the reqd Eliminant

8 $al=bm=cn$ (1)

$l^2+m^2+n^2=1$ (2)

$a^2l^3+b^2m^3+c^2n^3=a'^2l+b'^2m+c'^2n$ (3)

From (1), we get $l^2=\dfrac{c^2n^2}{a^2}$ (4)

and $m^2=\dfrac{c^2n^2}{b^2}$ (5)

We get $l^2+m^2=c^2n^2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)=\dfrac{c^2n^2(a^2+b^2)}{a^2b^2}$

from (2), we get

$\dfrac{c^2n^2(a^2+b^2)}{a^2b^2}+n^2=1$ (6)

or, $n^2=\dfrac{a^2b^2}{a^2b^2+a^2c^2+b^2c^2}$ (7)

from (4), $l^2=\dfrac{c^2b^2}{a^2b^2+a^2c^2+b^2c^2}$ (8)

Again from (1) and (3), we get

$al^2+bm^2+cn^2=\dfrac{a'^2}{a}+\dfrac{b'^2}{b}+\dfrac{c'^2}{c}$ (9)

from (6), (7), (8), and (9), we get

$$\frac{ab^2c^2}{a^2b^2+a^2c^2+b^2c^2}+\frac{bc^2a^2}{a^2b^2+a^2c^2+b^2c^2}+\frac{ca^2b^2}{a^2b^2+a^2c^2+b^2c^2}$$

$$=\frac{a'^2}{a}+\frac{b'^2}{b}+\frac{c'^2}{c}$$

or, $abc(bc+ca+ab)=\left(\dfrac{a'^2}{a}+\dfrac{b'^2}{b}+\dfrac{c'^2}{c}\right)(a^2b^2+b^2c^2+c^2a^2)$

or, $\dfrac{abc(bc+ca+ab)}{a^2b^2c^2}=\left(\dfrac{a'^2}{a}+\dfrac{b'^2}{b}+\dfrac{c'^2}{c}\right)\left(\dfrac{a^2b^2+b^2c^2+c^2a^2}{a^2b^2c^2}\right)$

or, $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=\left(\dfrac{a'^2}{a}+\dfrac{b'^2}{b}+\dfrac{c'^2}{c}\right)\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}\right)$.

Exercise (99).

1 $8x+\dfrac{7}{x}=\dfrac{65}{7}x$ or, $56x^2+49=65x^2$

or, $9x^2=49$ or, $3x=\pm 7$ $x=\pm\frac{7}{3}=\pm 2\frac{1}{3}$

2 $$\frac{2x^2+10}{15}=7-\frac{50+x^2}{25} \quad \text{or,} \quad 10x^2+50=525-150-3x^2$$

or, $13x^2=325$ or, $x^2=25$ $x=\pm 5$

3 $$\frac{14x^2+16}{21}-\frac{2x^2+8}{8x^2-11}=\frac{2x^2}{3}$$

or, $112x^4-128x^2-154x^2-176-42x^2-168=102x^4-154x^2$

or, $128x^2-42x^2=176+168$ or, $86x^2=344$

or, $x^2=4$ $x=\pm 2$

4 $$\frac{x+7}{x(x-7)}-\frac{x-7}{x(x+7)}=\frac{7}{x^2-73}$$

or, $(x^2-73)\{(x+7)^2-(x-7)^2\}=7x(x^2-49)$

or, $(x^2-73)\,4\,7x=7x(x^2-49)$ or, $(x^2-73)\,4=x^2-49$

or, $4x^2-292=x^2-49$ or, $3x^2=292-49=243$

or, $x^2=81$ $x=\pm 9$

5 $$\frac{x^3-1}{(x-1)^2}-\frac{x^3+1}{(x+1)^2}=6$$

or, $$\frac{(x^2+1)+x}{x-1}-\frac{(x^2+1)-x}{x+1}=6$$

or, $(x^2+1)(x+1)+x(x+1)-(x^2+1)(x-1)+x(x-1)$
$=6(x^2-1)$

or, $2(x^2+1)+2x^2=6x^2-6$ or, $2x^2=8$

or, $x^2=4$ $x=\pm 2$

6 $$\frac{1}{\sqrt{1-x}+1}+\frac{1}{\sqrt{1+x}-1}=\frac{1}{x}$$

or, $$\frac{\sqrt{1-x}-1}{-x}+\frac{\sqrt{1+x}+1}{x}=\frac{1}{x}$$

or, $\sqrt{1+x}+1-\sqrt{1-x}+1=1$

or, $\sqrt{1-x}-\sqrt{1+x}=1$

or, $1-x+1+x-2\sqrt{1-x^2}=1$

or, $2\sqrt{1-x^2}=1$ or, $4(1-x^2)=1$

or, $4x^2=3$ or, $2x=\pm\sqrt{3}$ $x=\pm\sqrt{\frac{3}{4}}$

7

$$(1+x+x^2)^{\frac{1}{2}}=a-(1-x+x^2)^{\frac{1}{2}}$$

or, $$(1+x+x^2)^{\frac{1}{2}}+(1-x+x^2)^{\frac{1}{2}}=a$$

or, $$1+x+x^2+1-x+x^2+2(1+x^2+x^4)^{\frac{1}{2}}=a^2$$

or, $$2(1+x^2+x^4)^{\frac{1}{2}}=a^2-2(1+x^2)$$

or, $$4(1+x^2+x^4)=a^4+4(1+x^2)^2-4a^2(1+x^2)$$
$$=a^4+4+8x^2+4x^4-4a^2-4a^2x^2$$

or $$a^4+4x^2-4a^2-4a^2x^2=0$$

or, $$4x^2(a^2-1)=a^4-4a^2=a^2(a^2-4)$$

or, $$2x\sqrt{a^2-1}=\pm a\sqrt{(a^2-4)}$$

$$x=\pm\frac{a\sqrt{a^2-4}}{2\sqrt{a^2-1}}$$

8

$$\frac{(x-a)(x-b)}{(x-ma)(x-mb)}=\frac{(x+a)(x+b)}{(x+ma)(x+mb)}$$

or, $$\frac{x^2-x(a+b)+ab}{x^2-mx(a+b)+m^2ab}=\frac{x^2+x(a+b)+ab}{x^2+mx(a+b)+m^2ab}$$

or, $$x^4+x^3(a+b)(m-1)+x^2\{ab(m^2+1)-m(a+b)^2\}$$
$$+m^2a^2b^2-xmab(a+b)(m-1)$$
$$=x^4-x^3(a+b)(m-1)+x^2\{ab(m^2+1)-m(a+b)^2\}$$
$$+xmab(a+b)(m-1)$$

or, $$2x^3(a+b)(m+1)=2xmab(a+b)(m-1)$$

or, $$x^2=mab \qquad x=\pm\sqrt{mab}$$

9

$$\frac{ax+1+(a^2x^2-1)^{\frac{1}{2}}}{ax+1-(a^2x^2-1)^{\frac{1}{2}}}=\frac{b^2x}{2}$$

or, $$\frac{ax+1}{(a^2x^2-1)^{\frac{1}{2}}}=\frac{b^2x+2}{b^2x-2}$$

or, $$\frac{(ax+1)^2}{a^2x^2-1}=\frac{(b^2x+2)^2}{(b^2x-2)^2}$$

or, $$\frac{(ax+1)^2+(a^2x^2-1)}{(ax+1)^2-(a^2x^2-1)}=\frac{(b^2x+2)^2+(b^2x-2)^2}{(b^2x+2)^2-(b^2x-2)^2}$$

or, $$\frac{2ax(ax+1)}{2(ax+1)}=\frac{2(b^4x^2+4)}{8b^2x}$$

or, $ax = \frac{b^4x^2+4}{4b^2x}$ or, $4ab^2x^2 = b^4x^2+4$

or, $b^2x^2(4a-b^2)=4$ or, $bx\sqrt{4a-b^2} = \pm 2$

$$x = \frac{2}{\pm b\sqrt{4a-b^2}}$$

10 $(a+x)^{\frac{2}{3}} + (a-x)^{\frac{2}{3}} = 3(a^2-x^2)^{\frac{1}{3}}$

or, $(a+x)^2+(a-x)^2+9(a^2-x^2)^{\frac{1}{3}}(a^2-x^2)^{\frac{2}{3}} = 27(a^2-x^2)$

or, $2(a^2+x^2)+9(a^2-x^2)=27(a^2-x^2)$

or, $2(a^2+x^2)=18(a^2-x^2)$

or, $a^2+x^2=9(a^2-x^2)$

or, $10x^2=8a^2$

or, $x^2=\frac{4a^2}{5}$ $x=\pm\frac{2a}{\sqrt{5}}$

11 $\frac{5x^2+17}{x^2-11}+\frac{14x^2-117}{2x^2-9}=12$

or, $5+\frac{72}{x^2-11}+7-\frac{54}{2x^2-9}=12$

or, $\frac{72}{x^2-11}-\frac{54}{2x^2-9}=0$

or, $\frac{4}{x^2-11}-\frac{3}{2x^2-9}=0$

or, $8x^2-36=3x^2-33$

or, $5x^2=3$ $x=\pm\sqrt{\frac{3}{5}}$

12. $\frac{x^2-1}{x^2-4}-\frac{x^2-5}{x^2-8}=\frac{x^2-2}{x^2-5}-\frac{x^2-6}{x^2-9}$

or, $1+\frac{3}{x^2-4}-1-\frac{3}{x^2-8}=1+\frac{3}{x^2-5}-1-\frac{3}{x^2-9}$

or, $\frac{x^2-8-x^2+4}{(x^2-4)(x^2-8)}=\frac{x^2-9-x^2+5}{(x^2-5)(x^2-9)}$

or, $\frac{-4}{(x^2-4)(x^2-8)}=\frac{-4}{(x^2-5)(x^2-9)}$

or, $x^4-14x^2+45=x^4-12x^2+32$

or, $2x^2=13$ $x=\pm\sqrt{\frac{13}{2}}$

13 We get

$$\sqrt{2}(a+x)^{\frac{1}{2}} = n\left\{\frac{a+x}{a+(a^2-x^2)^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$$

or, $$\sqrt{2} = \frac{n}{\sqrt{a+(a^2-x^2)^{\frac{1}{2}}}}$$

or, $$2\{a+(a^2-x^2)^{\frac{1}{2}}\} = n^2$$

or, $$(a^2-x^2)^{\frac{1}{2}} = \frac{n^2}{2} - a$$

or, $$a^2 - x^2 = \frac{n^4}{4} - an^2 + a^2$$

or, $$x^2 = n^2\left(a - \frac{n^2}{4}\right)$$

$$x = \pm n\left(a - \frac{n^2}{4}\right)^{\frac{1}{2}}$$

14 $$\frac{(1+2x)^{\frac{1}{2}} - 1}{(1-2x^2)^{\frac{1}{2}} + 1} + \frac{(1-2x)^{\frac{1}{2}} + 1}{(1+2x)^{\frac{1}{2}} - 1} = 2\sqrt{2}$$

or $$\frac{(1-4x^2)^{\frac{1}{2}} - (1+2x)^{\frac{1}{2}} - (1-2x)^{\frac{1}{2}} - 1}{1-2x-1}$$

$$+\frac{(1-4x^2)^{\frac{1}{2}} + (1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} - 1}{1+2x-1} = 2\sqrt{2}$$

or, $$-(1-4x^2)^{\frac{1}{2}} + (1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} - 1 + (1-4x^2)^{\frac{1}{2}}$$

$$+(1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} + 1 = 4\sqrt{2}x$$

or, $$(1+2x)^{\frac{1}{2}} + (1-2x)^{\frac{1}{2}} = 2\sqrt{2}x$$

or, $$1+2x+1-2x+2(1-4x^2)^{\frac{1}{2}} = 8x^2$$

or, $$(1-4x^2)^{\frac{1}{2}} = 4x^2 - 1$$ or, $$1-4x^2 = 16x^4 - 8x^2 + 1$$

or, $$16x^4 = 4x^2$$ or, $$4x^2 = 1$$

or $$2x = \pm 1$$ $$x = \pm\frac{1}{2}$$

Exercise (100)

1 & 2—worked out in the book.

3 $$87-98x=30x-16x^2$$

or, $16x^2-128x=-87$

or, $x^2-8x=-\frac{87}{16}$

or, $x^2-8x+16=-\frac{87}{16}+16$

or, $(x-4)^2=(\frac{13}{4})^2$

$x-4=\pm\frac{13}{4}$ $\qquad x=4\pm\frac{13}{4}=7\frac{1}{4}$ or $\frac{3}{4}$

4 $$17x^2-85x+216=65x-8x^2$$

or, $25x^2-150x=-216$

or, $x^2-6x=-\frac{216}{25}$

or, $x^2-6x+9=-\frac{216}{25}+9$

or, $(x-3)^2=\frac{9}{25}$ $\qquad x-3=\pm\frac{3}{5}$

$x=3\pm\frac{3}{5}=3\frac{3}{5}$ or $2\frac{2}{5}$

5 $$\frac{x^2+8}{11}=5x-x^2-5$$

or, $x^2+8=55x-11x^2-55$

or, $12x^2-55x=-63$

or, $x^2-\frac{55}{12}x=-\frac{21}{4}$

or, $x^2-\frac{55}{12}x+(\frac{55}{24})^2=-\frac{21}{4}+(\frac{55}{24})^2$

or, $(x-\frac{55}{24})^2=\frac{1}{576}$

$x-\frac{55}{24}=\pm\frac{1}{24}$

$x=\frac{55}{24}\pm\frac{1}{24}=2\frac{1}{3}$ or $2\frac{1}{4}$

6 $$4\left(x^2-3\frac{3}{5}x\right)=10\left(x^2-4\frac{2}{5}x-6\right)+3\left(\frac{x}{5}-\frac{5}{3}\right)$$

or, $4x^2-14\frac{2}{5}x=10x^2-44x-60+\frac{3}{5}x-5$

or, $6x^2-29x=65$

or, $x^2-\frac{29}{6}x=\frac{65}{6}$

or, $x^2-\frac{29}{6}x+(\frac{29}{12})^2=\frac{65}{6}+(\frac{29}{12})^2$

or, $(x-\frac{29}{12})^2=\frac{2401}{144}$

$x-\frac{29}{12}=\pm\frac{49}{12}$ $\qquad x=\frac{29}{12}\pm\frac{49}{12}=6\frac{1}{2}$ or $-1\frac{2}{3}$

7 $$4(5x^2-3\tfrac{7}{9}x)=5(x^2-7x+12)+\frac{8(x-9)}{9}$$

or, $$20x^2-15\tfrac{1}{9}x=5x^2-35x+60+\tfrac{8}{9}x-8$$

or, $$15x^2+19x=52$$

or $$x^2+\frac{19x}{15}=\frac{52}{15}$$

or, $$x^2+\tfrac{19}{15}x+(\tfrac{19}{30})^2=\tfrac{52}{15}+(\tfrac{19}{30})^2$$

or, $$(x+\tfrac{19}{30})^2=\tfrac{3481}{900}$$

$$x+\tfrac{19}{30}=\pm\tfrac{59}{30}$$

$$x=-\tfrac{19}{30}\pm\tfrac{59}{30}=1\tfrac{1}{3}\text{ or }-2\tfrac{4}{5}.$$

8 $$2x+.02=2.45x-x^2$$

or, $$x^2-.45x=-.02$$

or, $$x^2-.45x+(.225)^2=-.02+(.225)^2$$

or, $$(x-.225)^2=.030625$$

$$x-.225=\pm .175$$

$$x=.225\pm .175=.4\text{ or }.05$$

9 $$4(x^2+23x-24)=29x^2-8x+1$$

or, $$4x^2+92x-96=29x^2-8x+1$$

or, $$25x^2-100x=-97$$

or, $$x^2-4x=-\tfrac{97}{25}$$

or, $$x^2-4x+4=-\tfrac{97}{25}+4$$

or, $$(x-2)^2=\tfrac{3}{25}$$

$$x-2=\pm\tfrac{1}{5}\sqrt{3}\qquad x=2\pm\tfrac{1}{5}\sqrt{3}.$$

10 Worked out in the text

11 $$(2x-5)(3x-7)-(x-1)(4x-5)=x^2-3(x+14)$$

or, $$6x^2-29x+35-4x^2+9x-5=x^2-3x-42$$

or, $$x^2-17x=-72$$

or, $$x^2-17+(\tfrac{17}{2})^2=-72+(\tfrac{17}{2})^2$$

or, $$(x-\tfrac{17}{2})^2=\tfrac{1}{4}$$

$$x-\tfrac{17}{2}=\pm\tfrac{1}{2}$$

$$x=\tfrac{17}{2}\pm\tfrac{1}{2}=9\text{ or }8$$

12 $$(3x-11)(x-2)+(2x-3)(x+4)+13x=10(2x-1)^2+12$$

or, $$3x^2-17x+22+2x^2+5x-12+13x=40x^2-40x+10+12$$

or, $$35x^2-41x=-12$$

or, $$x^2-\frac{41x}{35}=-\frac{12}{35}$$

or, $$x^2-\tfrac{41}{35}x+(\tfrac{41}{70})^2=-\tfrac{12}{35}+(\tfrac{41}{70})^2$$

or, $$(x-\tfrac{41}{70})^2=\tfrac{1}{4900}$$

$$x-\tfrac{41}{70}=\pm\tfrac{1}{70}$$

$$x=\tfrac{41}{70}\pm\tfrac{1}{70}=\tfrac{3}{5}\text{ or }\tfrac{4}{7}$$

13 $$(x-\tfrac{1}{2})(x-\tfrac{1}{3})+(x-\tfrac{1}{4})(x-\tfrac{1}{3})=(x-\tfrac{1}{4})(x-\tfrac{1}{5})$$

or, $$(x-\tfrac{1}{2})(x-\tfrac{1}{3})=(x-\tfrac{1}{4})\{(x-\tfrac{1}{5})-(x-\tfrac{1}{3})\}$$

or, $$(x-\tfrac{1}{2})(x-\tfrac{1}{3})=(x-\tfrac{1}{4})(\tfrac{1}{3}-\tfrac{1}{5})=\tfrac{2}{15}(x-\tfrac{1}{4})$$

or, $$x^2-\tfrac{5}{6}x+\tfrac{1}{6}=\tfrac{2}{15}x-\tfrac{1}{30}$$

or, $$x^2-\tfrac{29}{30}x=-\tfrac{6}{30}=-\tfrac{1}{5}$$

or, $$x^2-\tfrac{29}{30}x+(\tfrac{29}{60})^2=-\tfrac{1}{5}+(\tfrac{29}{60})^2$$

or, $$(x-\tfrac{29}{60})^2=\tfrac{121}{3600}$$

$$x-\tfrac{29}{60}=\pm\tfrac{11}{60}$$

$$x=\tfrac{29}{60}\pm\tfrac{11}{60}=\tfrac{2}{3}\text{ or }\tfrac{3}{10}$$

14 $$\frac{x}{15}+\frac{40}{3(10-x)}=\frac{3(10+x)}{95}$$

or, $$\frac{40}{3(10-x)}=\frac{3(10+x)}{95}-\frac{x}{15}=\frac{90+9x-19x}{15\times19}=\frac{90-10x}{3\times95}$$

or, $$\frac{4}{10-x}=\frac{9-x}{95}$$

or, $$380=90-19x+x^2$$

or, $$x^2-19x=290$$

or, $$x^2-19x+(\tfrac{19}{2})^2=290+(\tfrac{19}{2})^2$$

or, $$(x-\tfrac{19}{2})^2=\tfrac{1521}{4}$$

$$x-\tfrac{19}{2}=\pm\tfrac{39}{2}$$

$$x=\tfrac{19}{2}\pm\tfrac{39}{2}=29\text{ or }-10$$

15 $$\frac{2x}{15}+\frac{3x-50}{3(10+x)}=\frac{12x+70}{190}$$

or, $\dfrac{3x-50}{3(10+x)}=\dfrac{12x+70}{190}-\dfrac{2x}{15}=\dfrac{36x+210-76x}{190\times 3}=\dfrac{210-40x}{190\times 3}$

$$=\frac{21-4x}{19\times 3}$$

$$\frac{3x-50}{10+x}=\frac{21-4x}{19}$$

or, $57x-950=210+21x-40x-4x^2$

or, $4x^2+76x=1160$ or, $x^2+19x=290$

or, $x^2+19x+(\frac{19}{2})^2=290+(\frac{19}{2})^2$

or, $(x+\frac{19}{2})=\frac{1521}{4}$

$x+\frac{19}{2}=\pm\frac{39}{2}$ $x=-\frac{19}{2}\pm\frac{39}{2}=10$ or -29

16 Worked out in the book

17 $\dfrac{x}{x+1}+\dfrac{x+1}{x}=\dfrac{13}{6}$

or, $\dfrac{x}{x+1}-1+\dfrac{x+1}{x}-1=\dfrac{1}{6}$

or, $-\dfrac{1}{x+1}+\dfrac{1}{x}=\dfrac{1}{6}$

or, $\dfrac{-x+x+1}{x(x+1)}=\dfrac{1}{6}$

or, $x^2+x=6$

or, $x^2+x+(\frac{1}{2})^2=6+\frac{1}{4}$

or, $(x+\frac{1}{2})^2=\frac{25}{4}$ $x+\frac{1}{2}=\pm\frac{5}{2}$

$x=-\frac{1}{2}\pm\frac{5}{2}=2$ or -3

18 Worked out in the book

19 $\dfrac{x-2}{x+2}+\dfrac{x+2}{x-2}=\dfrac{2(x+3)}{x-3}$

or, $1-\dfrac{4}{x+2}+1+\dfrac{4}{x-2}=2+\dfrac{12}{x-3}$

or, $-\dfrac{1}{x+2}+\dfrac{1}{x-2}=\dfrac{3}{x-3}$

or, $\dfrac{-x+2+x+2}{x^2-4}=\dfrac{3}{x-3}$

or, $4x-12=3x^2-12$

$3x^2=4x$ $x=0$ or $\frac{4}{3}$

20 We have

$$\left(\frac{x+2}{x-2}-1\right)-\left(\frac{x-2}{x+2}-1\right)=\frac{5}{6}$$

or, $$\frac{4}{x-2}+\frac{4}{x+2}=\frac{5}{6}$$

or, $$\frac{4(x+2+x-2)}{x^2-4}=\frac{5}{6}$$

or, $$48x=5x^2-20$$

or, $$5x^2-48x=20$$

or, $$x^2-\tfrac{48}{5}x=4$$

or, $$x^2-\tfrac{48}{5}x+(\tfrac{24}{5})^2=4+(\tfrac{24}{5})^2$$

or, $$(x-\tfrac{24}{5})^2=\tfrac{676}{25}$$

$$x-\tfrac{24}{5}=\pm\tfrac{26}{5} \qquad x=\tfrac{24}{5}\pm\tfrac{26}{5}=10 \text{ or } -\tfrac{2}{5}$$

21 $$\frac{x-6}{x-12}-\frac{x-12}{x-6}=\frac{5}{6}$$

or, $$1+\frac{6}{x-12}-1+\frac{6}{x-6}=\frac{5}{6}$$

or, $$\frac{6(x-6+x-12)}{(x-12)(x-6)}=\frac{5}{6}$$

or, $$72x-648=5x^2-90x+360$$

or, $$5x^2-162x=-1008$$

or, $$x^2-\tfrac{162}{5}x=-\tfrac{1008}{5}$$

or, $$x^2-\tfrac{162}{5}x+(\tfrac{81}{5})^2=-\tfrac{1008}{5}+(\tfrac{81}{5})^2$$

or, $$(x-\tfrac{81}{5})^2=\tfrac{1521}{25} \qquad x-\tfrac{81}{5}=\pm\tfrac{39}{5}$$

$$x=\tfrac{81}{5}\pm\tfrac{39}{5}=24 \text{ or } 8\tfrac{2}{5}$$

22 $$\frac{2x-9}{2x-7}-\frac{2x-7}{2x-9}=\frac{7}{12}$$

or, $$1-\frac{2}{2x-7}-1-\frac{2}{2x-9}=\frac{7}{12}$$

or, $$-24(2x-9+2x-7)=7(2x-7)(2x-9)$$

or, $$-96x+384=28x^2-224x+441$$

or, $$28x^2-128x=-57$$

or, $$x^2-\tfrac{32}{7}x=-\tfrac{57}{28}$$

or, $$x^2-\tfrac{32}{7}x+(\tfrac{16}{7})^2=-\tfrac{57}{28}+(\tfrac{16}{7})^2$$

or, $$(x-\tfrac{16}{7})^2=\tfrac{625}{196}$$

$$x-\tfrac{16}{7}=\pm\tfrac{25}{14} \qquad x=\tfrac{16}{7}\pm\tfrac{25}{14}=4\tfrac{1}{14} \text{ or } \tfrac{1}{2}$$

23
$$\frac{x+6}{x+7}-\frac{x+1}{x+2}=\frac{1}{3x+1}$$
or, $$1-\frac{1}{x+7}-1+\frac{1}{x+2}=\frac{1}{3x+1}$$
or, $$\frac{-x-2+x+7}{(x+7)(x+2)}=\frac{1}{3x+1}$$
or, $$15x+5=x^2+9x+14$$
or, $$x^2-6x+9=0$$
$$(x-3)^2=0 \qquad x=3$$

24 We have
$$\left(\frac{2x}{x-4}-2\right)+\left(\frac{2x-5}{x-3}-2\right)=4\tfrac{1}{3}$$
or, $$\frac{8}{x-4}+\frac{1}{x-3}=\frac{13}{3}$$
or, $$24x-72+3x-12=13x^2-91x+156$$
or, $$13x^2-118x=-240$$
or, $$x^2-\tfrac{118}{13}x=-\tfrac{240}{13}$$
or, $$x^2-\tfrac{118}{13}x+\left(\tfrac{59}{13}\right)^2=-\tfrac{240}{13}+\left(\tfrac{59}{13}\right)^2$$
or, $$\left(x-\tfrac{59}{13}\right)^2=\tfrac{361}{169}$$
$$x-\tfrac{59}{13}=\pm\tfrac{19}{13} \qquad x=\tfrac{59}{13}\pm\tfrac{19}{13}=6 \text{ or } 3\tfrac{1}{13}.$$

25
$$\frac{x}{x+5}-\frac{11x}{11x-8}+\frac{7}{6-4x}=0$$
or, $$\frac{11x^2-8x-11x^2-55x}{(x+5)(11x-8)}+\frac{7}{6-4x}=0$$
or, $$\frac{-63x}{(x+5)(11x-8)}+\frac{7}{6-4x}=0$$
or, $$-9x(6-4x)+(x+5)(11x-8)=0$$
or, $$-54x+36x^2+11x^2-8x+55x-40=0$$
or, $$47x^2-7x=40$$
or, $$x^2-\tfrac{7}{47}x=\tfrac{40}{47}$$
or, $$x^2-\tfrac{7}{47}x+\left(\tfrac{7}{94}\right)^2=\tfrac{40}{47}+\left(\tfrac{7}{94}\right)^2$$
or, $$\left(x-\frac{7}{94}\right)^2=\frac{7569}{(94)^2}$$
$$x-\tfrac{7}{94}=\pm\tfrac{87}{94}$$
$$x=\tfrac{7}{94}\pm\tfrac{87}{94}=1 \text{ or } -\tfrac{40}{47}$$

26 We get

$$\frac{x+2a}{x+a}=-\frac{2x+5a}{x+2a}$$

or, $$x^2+4ax+4a^2=-(2x^2+7ax+5a^2)$$

or, $$3x^2+11ax=-9a^2$$

or, $$x^2+\frac{11ax}{3}=-3a^2$$

or $$x^2+\frac{11ax}{3}+\left(\frac{11a}{6}\right)^2=-3a^2+\left(\frac{11a}{6}\right)^2$$

or $$\left(x+\frac{11a}{6}\right)^2=\frac{13a^2}{36}$$

$$x+\frac{11a}{6}=\pm\frac{\sqrt{13}a}{6}$$

$$x=-\frac{11a}{6}\pm\frac{\sqrt{13}\,a}{6}$$

$$=\frac{-11\pm\sqrt{13}}{6}a$$

Exercise (101)

Note — The roots of an equation of the form $ax^2+bx+c=0$ are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, the following exercises are to be solved by applying this formula

1 $$3x^2-17x+24=0$$

$$x=\frac{17\pm\sqrt{289-288}}{6}=\frac{17\pm1}{6}$$

$$=3 \text{ or } 2\tfrac{2}{3}$$

2 $$x^2+9x+20=0$$

$$x=\frac{-9\pm\sqrt{81-80}}{2}=\frac{-9\pm1}{2}=-4 \text{ or } -5$$

3
$$6x^2=20-7x$$
$$6x^2+7x-20=0$$
$$x=\frac{-7\pm\sqrt{49+480}}{12}=\frac{-7\pm 23}{12}$$
$$=\tfrac{4}{3} \text{ or } -\tfrac{5}{2}$$

4
$$-9x^2+25=6x-10$$
or,
$$9x^2+6x-35=0$$
$$x=\frac{-6\pm\sqrt{36+1260}}{18}=\frac{-6\pm\sqrt{1296}}{18}$$
$$=\frac{-6\pm 36}{18}=\frac{5}{3} \text{ or } -\frac{7}{3}$$

5
$$8x^2=14x+15$$
or,
$$8x^2-14x-15=0$$
$$x=\frac{14\pm\sqrt{196+480}}{16}=14\pm\sqrt{676}$$
$$=\frac{14\pm 26}{16}=\frac{40}{19} \text{ or } -\frac{12}{16}=\frac{5}{2} \text{ or } -\frac{3}{4}$$

6
$$-3x^2+20x=25$$
or,
$$3x^2-20x+25=0$$
$$x=\frac{20\pm\sqrt{400-300}}{6}=\frac{20\pm 10}{6}=\frac{30}{6} \text{ or } \frac{10}{6}$$
$$=5 \text{ or } \tfrac{5}{3}$$

7
$$5+x-4x^2=0$$
or,
$$4x^2-x-5=0$$
$$x=\frac{1\pm\sqrt{1+80}}{8}=\frac{1\pm 9}{8}=\frac{10}{8} \text{ or } -\frac{8}{8}$$
$$=\tfrac{5}{4} \text{ or } -1$$

Exercise (102)

1
$$2x^2+9x=18$$
$$4(2x)^2+4\times(2x)\times 9+9^2=144+81=225$$
or,
$$(4x+9)^2=(15)^2$$
$$4x+9=\pm 15$$

$$4x = \pm 15 - 9 = 6 \text{ or } -24$$
$$x = \tfrac{3}{2} \text{ or } -6$$

2 $$15x^2 - 28 = x$$
or, $$15x^2 - x = 28$$
or, $$4(15x)^2 - 4 \times (15x) + 1 = 4 \times 15 \times 28 + 1 = 1681$$
$$(30x - 1)^2 = (41)^2$$
$$30x - 1 = \pm 41$$
$$30x = \pm 41 + 1 = 42 \text{ or } -40$$
$$x = \tfrac{7}{5} \text{ or } -\tfrac{4}{3}$$

3 $$16x^2 + 100x = 3x^2 + x + 40$$
or, $$13x^2 + 99x = 40$$
or, $$4(13x)^2 + 4 \times (13x) \times 99 + (99)^2$$
$$= 4 \times 13 \times 40 + 9801$$
$$= 2080 + 9801 = 11881$$
$$(26x + 99)^2 = (109)^2$$
$$26x + 99 = \pm 109$$
$$26x = -99 \pm 109 = -208 \text{ or } 10$$
$$x = -8 \text{ or } \tfrac{5}{13}$$

4 $$x^2 + 50x = 102 - 15x - x^2$$
or, $$2x^2 + 65x = 102$$
or, $$4(2x)^2 + 4 \times (2x) \times 65 + (65)^2 = 8 \times 102 + 4225$$
$$= 816 + 4225 = 5041$$
$$(4x + 65)^2 = (71)^2$$
$$4x + 65 = \pm 71$$
$$4x = -65 \pm 71 = 6 \text{ or } -136$$
$$x = \tfrac{3}{2} \text{ or } -34$$

5 $$17x^2 + 19x = 1848$$
$$4(17x)^2 + 4 \times (17x) \times 19 + (19)^2 = 4 \times 17(1848) + (19)^2$$
$$= 125664 + 361 = 126025$$
$$(34x + 19)^2 = (355)^2$$
$$34x + 19 = \pm 355$$
$$34x = \pm 355 - 19$$
$$= 336 \text{ or } -374$$
$$x = 9\tfrac{15}{17} \text{ or } -11$$

6
$$2cx^2 - acx = 3(2x - a)$$
or,
$$2cx^2 - x(ac+6) = -3a$$
$$4(2cx)^2 - 4(2cx)(ac+6) + (ac+6)^2$$
$$= (ac+6)^2 - 24ac$$
$$(4cx - ac - 6)^2 = (ac-6)^2$$
$$4cx - (ac+6) = \pm(ac-6)$$
$$4cx = 2ac \text{ or } 12$$
$$x = \frac{a}{2} \text{ or } \frac{3}{c}$$

7
$$x^2 + ax = ab(3x + a) - 2x^2$$
or,
$$3x^2 + x(a - 3ab) = a^2b$$
$$4(3x)^2 + 4(3x)(a - 3ab) + (a - 3ab)^2$$
$$= (a - 3ab)^2 + 4(3a^2b)$$
$$(6x + a - 3ab)^2 = (a + 3ab)^2$$
$$6x + a - 3ab = \pm(a + 3ab)$$
$$6x = 6ab \text{ or } -2a$$
$$x = ab \text{ or } -\frac{a}{3}$$

Exercise (103)

1
$$3x^2 - 12x + 1 = 6x - 23$$
or,
$$3x^2 - 18x + 24 = 0$$
or,
$$x^2 - 6x + 8 = 0$$
$$(x-2)(x-4) = 0$$
Hence either $x - 2 = 0$, $\quad x = 2$,
or, $x - 4 = 0$, $\quad x = 4$

2
$$4x^2 - 4x = 80$$
or,
$$x^2 - x - 20 = 0$$
$$(x+4)(x-5) = 0$$
Hence either $x + 4 = 0$ $\quad x = -4$,
or, $x - 5 = 0$ $\quad x = 5$

3 $$x+2-\frac{6}{x+2}=1$$

or, $(x+2)^2-(x+2)-6=0$

or, $\{(x+2)-3\}\{(x+2)+2\}=0$

$(x-1)(x+4)=0$

Hence either $x-1=0,$ $x=1,$

or, $x+4=0,$ $x=-4$

4 $x^2+9x-52=0$

$(x+13)(x-4)=0$

Hence either $x+13=0,$ $x=-13,$

or, $x-4=0,$ $x=4$

5 $x^2-\frac{5}{3}x-4=0$

or, $3x^2-5x-12=0$

$(x-3)(3x+4)=0$

Hence either $x-3=0,$ $x=3,$

or $3x+4=0,$ $x=-\frac{4}{3}$

6 $6x^2+5x-4=0$

$(2x-1)(3x+4)=0$

Hence either $2x-1=0,$ $x=\frac{1}{2},$

or, $3x+4=0,$ $x=-\frac{4}{3}$

7 $3(x-2)^2=18+(8x+1)$

or, $3x^2-12x+12=19+8x$

or, $3x^2-20x-7=0$

$(x-7)(3x+1)=0$

Hence either $x-7=0,$ $x=7,$

or, $3x+1=0,$ $x=-\frac{1}{3}$

8 $$x-\frac{x^3-8}{x^2+5}=2$$

or, $x^3+5x-x^3+8=2x^2+10$

or, $2x^2-5x+2=0$

$(x-2)(2x-1)=0$

Hence either $x-2=0,$ $x=2,$

or, $2x-1=0,$ $x=\frac{1}{2}$

9 $$\frac{21x^3-16}{3x^2-4}-7x=5$$

or, $$21x^3-16-21x^3+28x=15x^2-20$$

or, $$15x^2-28x-4=0$$

$$(x-2)(15x+2)=0$$

Hence either $x-2=0$, $x=2$,

or, $15x+2=0$, $x=-\frac{2}{15}$.

10 $$x^2-(a+b)x+ab=0$$

$$(x-a)(x-b)=0$$

Hence either $x-a=0$, $x=a$,

or, $x-b=0$, $x=b$

11 $$(a-b)x^2-(a+b)x+2b=0$$

or, $$(a-b)x^2-(a-b)x-2bx+2b=0$$

$$(x-1)\{(a-b)x-2b\}=0$$

Hence either $x-1=0$, $x=1$,

or, $(a-b)x-2b=0$, $x=\frac{2b}{a-b}$

12 $$\frac{x^2}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}-(a^{\frac{1}{2}}-b^{\frac{1}{2}})x=\frac{1}{(ab^2)^{-\frac{1}{2}}+(a^2b)^{-\frac{1}{2}}}$$

$$=\frac{1}{\frac{1}{a^{\frac{1}{2}}b}+\frac{1}{ab^{\frac{1}{2}}}}=\frac{ab}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}$$

or, $$x^2-(a-b)x-ab=0$$

$$(x-a)(x+b)=0$$

Hence either $x-a=0$, $x=a$,

or, $x+b=0$, $x=-b$

13 $$\frac{2x(a-x)}{3a-2x}=\frac{a}{4}$$

or, $$8ax-8x^2=3a^2-2ax$$

or, $$8x^2-10ax+3a^2=0$$

or, $$8x^2-6ax-4ax+3a^2=0$$

$$(4x-3a)(2x-a)=0$$

Hence either $\qquad 4x-3a=0 \qquad x=\frac{3a}{4},$

or, $\qquad 2x-a=0 \qquad x=\frac{a}{2}$

14 $$\frac{16}{x^{\frac12}}+\frac{x^{\frac32}}{2}=6x^{\frac12}$$

or $\qquad 32+x^2=12x$

or, $\qquad x^2-12x+32=0$

or, $\qquad x^2-8x-4x+32=0$

$$(x-8)(x-4)=0$$

Hence either $\qquad x-8=0 \qquad x=8$

or, $\qquad x-4=0 \qquad x=4$

15 $$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$$

or, $$\frac{a}{x-a}-\frac{c}{x-c}+\frac{b}{x-b}-\frac{c}{x-c}=0$$

or $$\frac{x(a-c)}{(x-a)(x-c)}+\frac{x(b-c)}{(x-b)(x-c)}=0$$

or, $$\frac{x}{x-c}\left\{\frac{a-c}{x-a}+\frac{b-c}{x-b}\right\}=0$$

Hence either $\qquad \frac{x}{x-c}=0, \qquad x=0,$

or, $$\frac{a-c}{x-a}+\frac{b-c}{x-b}=0$$

Whence $\qquad x(a-c+b-c)=b(a-c)+a(b-c)$

$$x=\frac{2ab-ac-bc}{a+b-2c}$$

16 $$\frac{a-\sqrt{2ax-x^2}}{a+\sqrt{2ax-x^2}}=\frac{x}{a-x}$$

$$-\frac{a}{-\sqrt{2ax-x^2}}=\frac{a}{2x-a}$$

or, $$\frac{1}{2ax-x^2}=\frac{1}{(2x-a)^2}$$

or, $$4x^2-4ax+a^2=2ax-x^2$$

or, $$5x^2-6ax+a^2=0$$

or, $$5x^2-5ax-ax+a^2=0$$

$$(x-a)(5x-a)=0$$

Hence either $x-a=0$ $\qquad x=a$,

or, $5x-a=0$ $\qquad x=\frac{a}{5}$